9-6 Analyzing Functions with Successive Differences

Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

45.

x	0	1	2	3	4
y	0	3	12	27	48

quadratic; $y = 3x^2$

46.

x	0	1	2	3	4
y	1	2	4	8	16

exponential; $y = 2^x$

47.

x	0	1	2	3	4
y	0	−1	−4	−9	−16

quadratic; $y = -x^2$

-1 -3 -5 -7

-2 -2 -2

x2!

0 | 3 | 12 | 27 | 48

3 9 15 21

6 6 6

2nd diff

Linear

$y = -3x$

-3, -6, -9

1st diff

-3 -3 -3

9-7 Special Functions

Graph each function. State the domain and range.

9-1 Graphing Quadratic Functions

Consider each equation.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

11. $y = x^2 - 4x + 4$ **11–14. See margin.**

12. $y = -x^2 + 3x$

13. $y = x^2 - 2x - 3$

14. $y = -x^2 + 2.$

15. **ROCKET** A toy rocket is launched with an upward velocity of 32 feet per second. The equation $h = -16t^2 + 32t$ gives the height of the ball t seconds after it is launched.

    a. Determine whether the function has a *maximum* or *minimum* value. **maximum**

    b. State the maximum or minimum value. **16**

    c. State a reasonable domain and range for this situation.
    $D = \{t \mid 0 \leq t \leq 2\}$; $R = \{h \mid 0 \leq h \leq 16\}$

Example 1

Consider $f(x) = x^2 + 6x + 5$.

a. Determine whether the function has a *maximum* or *minimum* value.

For $f(x) = x^2 + 6x + 5$, $a = 1$, $b = 6$, and $c = 5$.

Because a is positive, the graph opens up, so the function has a minimum value.

b. State the *maximum* or *minimum* value of the function.

The minimum value is the y-coordinate of the vertex.

The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{-6}{2(1)}$ or -3.

$f(x) = x^2 + 6x + 5$	Original function
$f(-3) = (-3)^2 + 6(-3) + 5$	$x = -3$
$f(-3) = -4$	Simplify.

The minimum value is -4.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or $\{y \mid y \geq -4\}$.

Additional Answers

11a. minimum

11b. 0

11c. D = {all real numbers}; $R = \{y \mid y \geq 0\}$

12a. maximum

12b. 2.25

12c. D = {all real numbers}; $R = \{y \mid y \leq 2.25\}$

13a. minimum

13b. -4

13c. D = {all real numbers}; $R = \{y \mid y \geq -4\}$

14a. maximum

14b. 2

14c. D = {all real numbers}; $R = \{y \mid y \leq 2\}$

9-2 Solving Quadratic Equations by Graphing

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

16. $x^2 - 3x - 4 = 0$ **−1, 4**

17. $-x^2 + 6x - 9 = 0$ **3**

18. $x^2 - x - 12 = 0$ **−3, 4**

19. $x^2 + 4x - 3 = 0$ **−4.6, 0.6**

20. $x^2 - 10x = -21$ **3, 7**

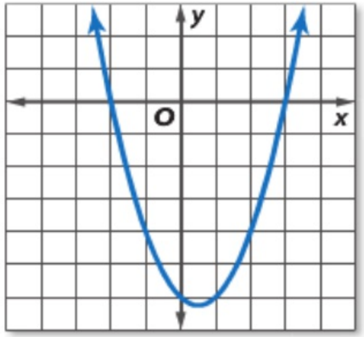
21. $6x^2 - 13x = 15$ **−0.8, 3**

22. **NUMBER THEORY** Find two numbers that have a sum of 2 and a product of −15. **−3 and 5**

Example 2

Solve $x^2 - x - 6 = 0$ by graphing.

Graph the related function $f(x) = x^2 - x - 6$.



The x-intercepts of the graph appear to be at −2 and 3, so the solutions are −2 and 3.

9-3 Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$. **23–28. See margin.**

23. $f(x) = x^2 + 8$

24. $f(x) = x^2 - 3$

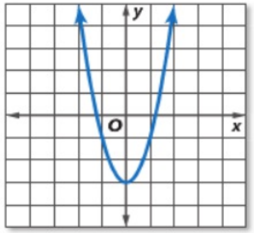
25. $f(x) = 2x^2$

26. $f(x) = 4x^2 - 18$

27. $f(x) = \frac{1}{3}x^2$

28. $f(x) = \frac{1}{4}x^2$

29. Write an equation for the function shown in the graph. $y = 2x^2 - 3$



30. **PHYSICS** A ball is dropped off a cliff that is 100 feet high. The function $h = -16t^2 + 100$ models the height h of the ball after t seconds. Compare the graph of this function to the graph of $h = t^2$. **See margin.**

Example 3

Describe how the graph of $f(x) = x^2 - 2$ is related to the graph of $f(x) = x^2$.

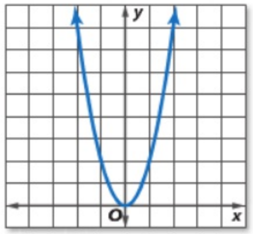
The graph of $f(x) = x^2 + c$ represents a translation up or down of the parent graph.

Since $c = -2$, the translation is down.

So, the graph is translated down 2 units from the parent function.

Example 4

Write an equation for the function shown in the graph.



Since the graph opens upward, the leading coefficient must be positive. The parabola has not been translated up or down, so $c = 0$. Since the graph is stretched vertically, it must be of the form of $f(x) = ax^2$ where $a > 1$. The equation for the function is $y = 2x^2$.

Additional Answers

23. shifted up 8 units

24. shifted down 3 units

25. vertical stretch

26. vertical stretch and shifted down 18 units

27. vertical compression

28. vertical compression

30. reflected across the x-axis, vertically stretched and shifted up 100 units

9-4 Solving Quadratic Equations by Completing the Square

Solve each equation by completing the square. Round to the nearest tenth if necessary.

31. $x^2 + 6x + 9 = 16$ **1, −7**

32. $-a^2 - 10a + 25 = 25$ **0, −10**

33. $y^2 - 8y + 16 = 36$ **10, −2**

34. $y^2 - 6y + 2 = 0$ **5.6, 0.4**

35. $n^2 - 7n = 5$ **−0.7, 7.7**

36. $-3x^2 + 4 = 0$ **−1.2, 1.2**

37. **NUMBER THEORY** Find two numbers that have a sum of −2 and a product of −48. **−8, 6**

Example 5

Solve $x^2 - 16x + 32 = 0$ by completing the square. Round to the nearest tenth if necessary.

Isolate the x^2- and x-terms. Then complete the square and solve.

$x^2 - 16x + 32 = 0$ Original equation

$x^2 - 16x = -32$ Isolate the x^2- and x-terms.

$x^2 - 16x + 64 = -32 + 64$ Complete the square.

$(x - 8)^2 = 32$ Factor.

$x - 8 = \pm\sqrt{32}$ Take the square root.

$x = 8 \pm \sqrt{32}$ Add 8 to each side.

$x = 8 \pm 4\sqrt{2}$ Simplify.

The solutions are about 2.3 and 13.7.

9-5 Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

38. $x^2 - 8x = 20$ **−2, 10**

39. $21x^2 + 5x - 7 = 0$ **−0.7, 0.5**

40. $d^2 - 5d + 6 = 0$ **2, 3**

41. $2f^2 + 7f - 15 = 0$ **−5, 1.5**

42. $2h^2 + 8h + 3 = 3$ **−4, 0**

43. $4x^2 + 4x = 15$ **−2.5, 1.5**

44. **GEOMETRY** The area of a square can be quadrupled by increasing the side length and width by 4 inches. What is the side length? **4 in.**

Example 6

Solve $x^2 + 10x + 9 = 0$ by using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-10 \pm \sqrt{10^2 - 4(1)(9)}}{2(1)}$ $a = 1, b = 10, c = 9$

$= \frac{-10 \pm \sqrt{64}}{2}$ Simplify.

$x = \frac{-10 + 8}{2}$ or $x = \frac{-10 - 8}{2}$ Separate the solutions.

$= -1$ $= -9$ Simplify.

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46. **exponential; $y = 2^x$**

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y	1	2	4	8	16

47. **quadratic; $y = -x^2$**

x	0	1	2	3	4
y	0	−1	−4	−9	−16

Example 7

Determine the model that best describes the data. Then write an equation for the function that models the data.

x	0	1	2	3	4
y	3	4	5	6	7

Step 1 First differences: 3 4 5 6 7 → 1 1 1 1

A linear function models the data.

Step 2 The slope is 1 and the y-intercept is 3, so the equation is $y = x + 3$.