5-Minute Check

Over Lesson 7–4

5 Evaluate the quotient $\frac{6.33 \times 10^5}{2.11 \times 10^{-3}}$.

A. 3×10^6

B. 3×10^7

C. 3×10^8

D. 30×10^8

5-Minute Check

Over Lesson 7–4

5 Evaluate the quotient $\frac{6.33 \times 10^5}{2.11 \times 10^{-3}}$.

A. 3×10^6

B. 3×10^7

C. 3×10^8

D. 30×10^8

KeyConcept Graphs of Exponential Functions

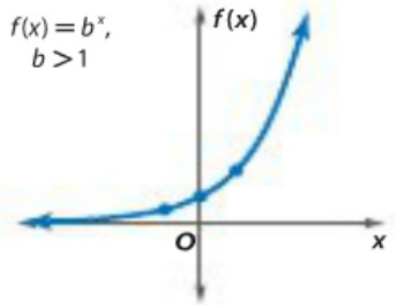
Exponential Growth Functions

Equation: $f(x) = ab^x$, $a > 0$, $b > 1$

Domain, Range: all reals; all positive reals

Intercepts: one y-intercept, no x-intercepts

End behavior: as x increases, $f(x)$ increases; as x decreases, $f(x)$ approaches 0



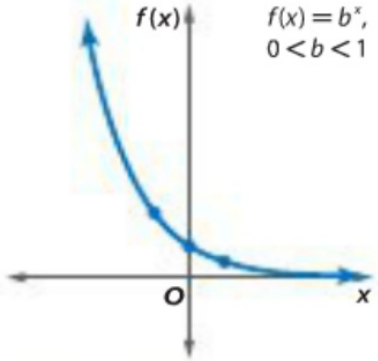
Exponential Decay Functions

Equation: $f(x) = ab^x$, $a > 0$, $0 < b < 1$

Domain, Range: all reals; all positive reals

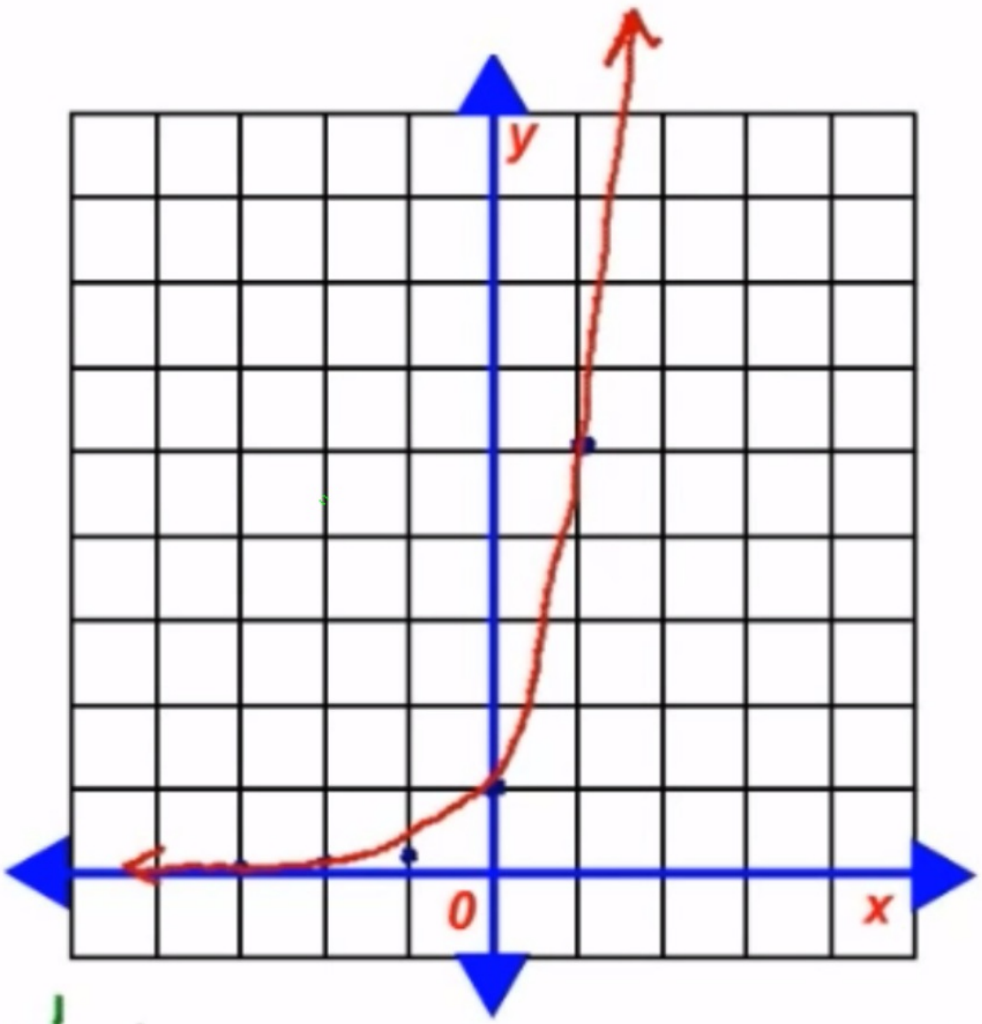
Intercepts: one y-intercept, no x-intercepts

End behavior: as x increases, $f(x)$ approaches 0; as x decreases, $f(x)$ increases



Graph $y = 5^x$. Find the y-intercept, and state the domain and range.

x	5^x	y
-3	5^{-3}	$1/125$
-2	5^{-2}	$1/25$
-1	5^{-1}	$1/5$
0	5^0	1
1	5^1	5
2	5^2	25
3	5^3	125

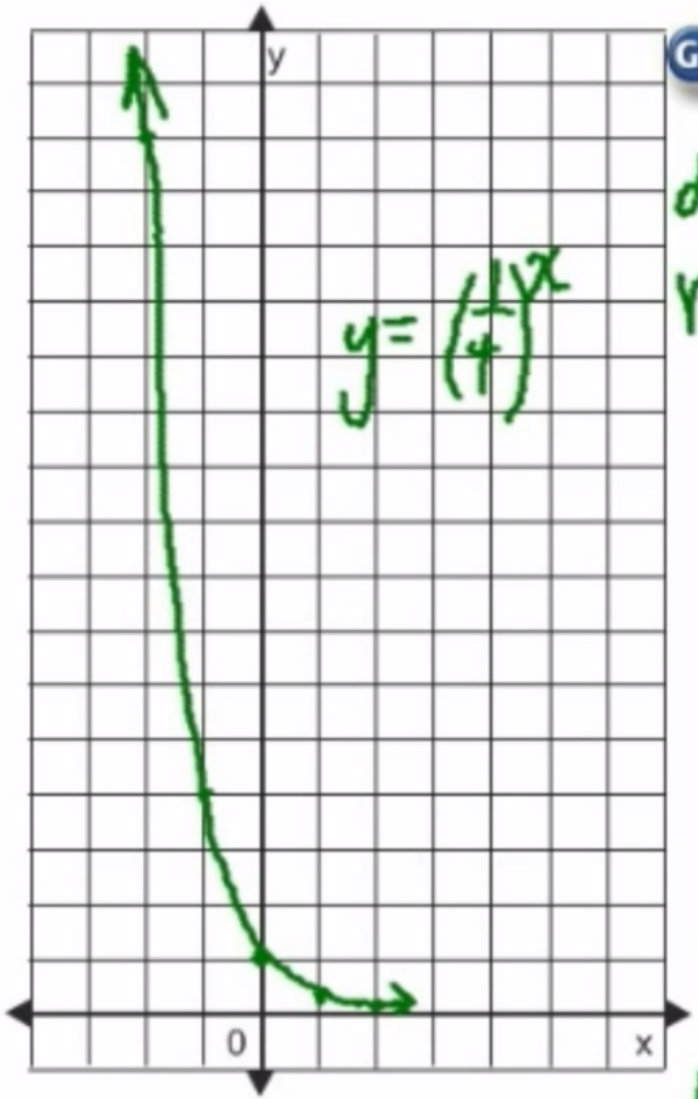


y-intercept: 1

Domain: real

Range: positive real

x	$\left(\frac{1}{4}\right)^x$	y
-2	$\left(\frac{1}{4}\right)^{-2}$	16
-1	$\left(\frac{1}{4}\right)^{-1}$	4
0	$\left(\frac{1}{4}\right)^{0}$	1
1	$\left(\frac{1}{4}\right)^{1}$	$\frac{1}{4}$
2	$\left(\frac{1}{4}\right)^{2}$	$\frac{1}{16}$



y-int = (0,1)

domain = reals

range = positive reals

$y = \left(\frac{1}{4}\right)^x$

$\left(\frac{1}{4}\right)^x \neq 0$

$y = \left(\frac{1}{4}\right)^1 = \frac{1}{4} = 0.25$

$y = \left(\frac{1}{4}\right)^{1.5} \approx 0.1$

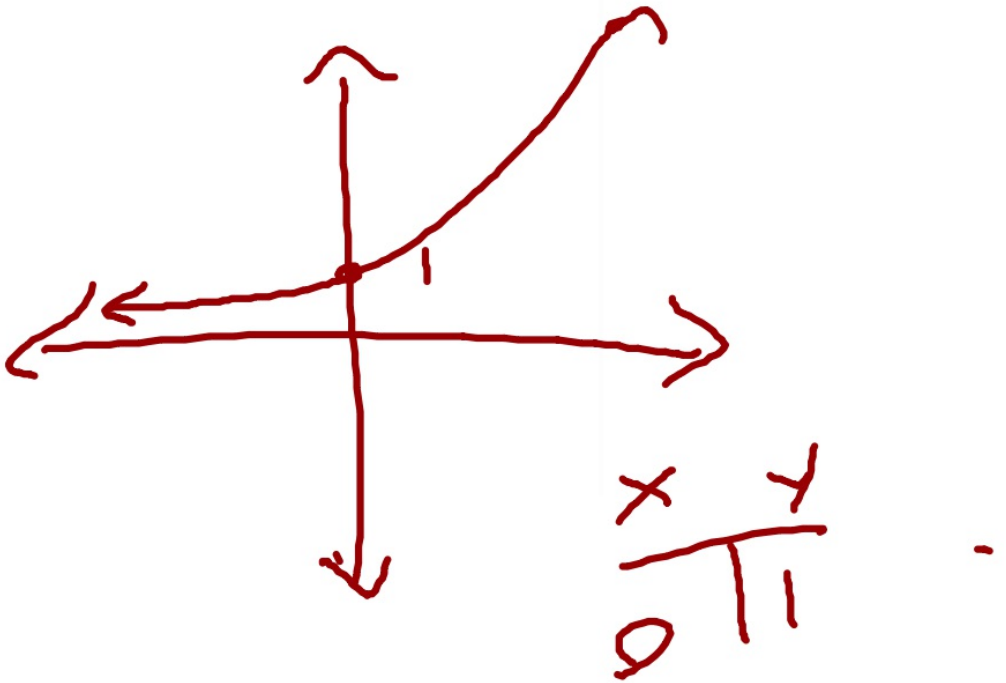
$y = \left(\frac{1}{4}\right)^{1.5} = 0.125$

Then use the graph to approximate the value of $\left(\frac{1}{4}\right)^{1.5}$.

EXAMPLE 1 Graph with $a > 0$ and $b > 1$

A. Graph $y = 4^x$. Find the y-intercept and state the domain and range.

x	4^x	y
-1	4^{-1}	$\frac{1}{4}$
0	4^0	1
1	4^1	4
2	4^2	16



This will not be given to you in the HW... you'll need to create it!

EXAMPLE 1 Graph with $a > 0$ and $b > 1$

A. Graph $y = 4^x$. Find the y-intercept and state the domain and range.

x	4^x	y
-1	4^{-1}	$\frac{1}{4}$
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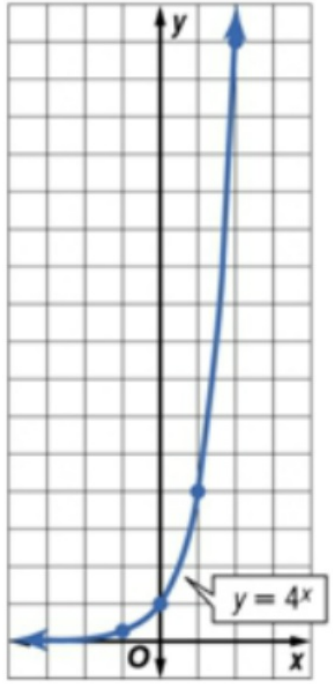


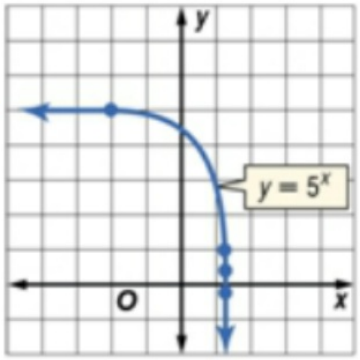
Graph the ordered pairs and connect the points with a smooth curve.

Answer: The graph crosses the y-axis at 1, so the y-intercept is 1. The domain is all real numbers and the range is all positive real numbers.

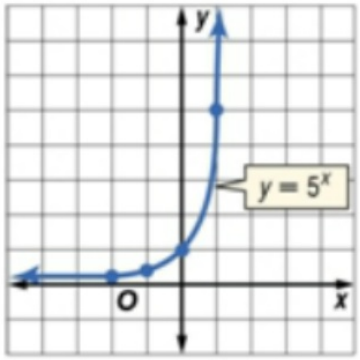
EXAMPLE 1 Check Your Progress

A. Graph $y = 5^x$.

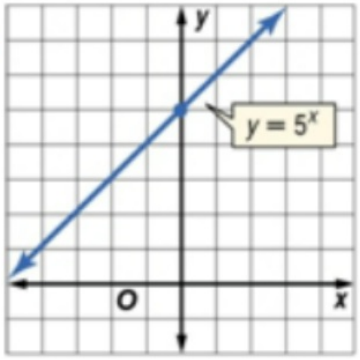
A.



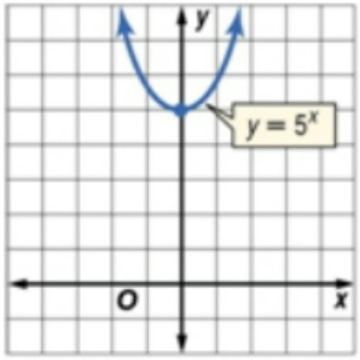
B.



C.



D.



EXAMPLE 1 Check Your Progress

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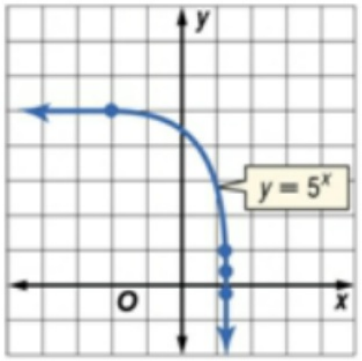
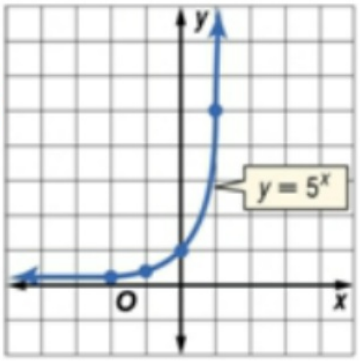
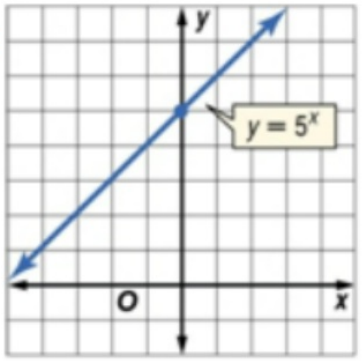


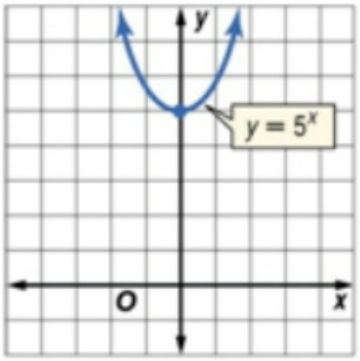
B.



C.



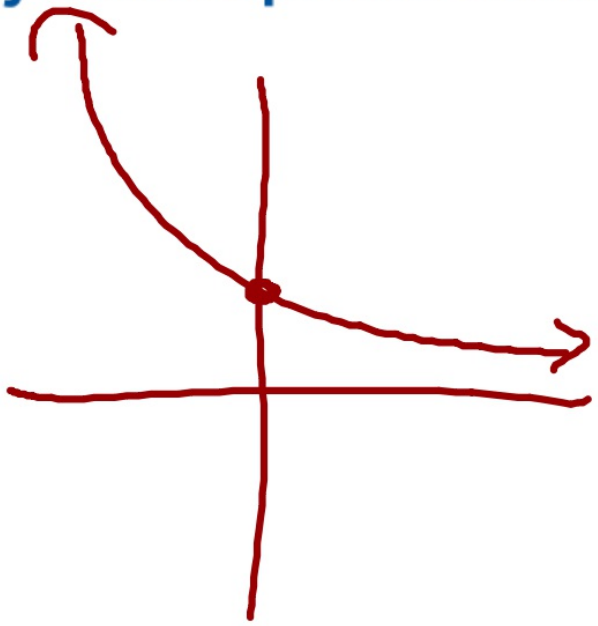
D.



EXAMPLE 2 Graph with $a > 0$ and $0 < b < 1$

A. Graph $y = \left(\frac{1}{4}\right)^x$. Find the *y*-intercept and state the domain and range.

x	$\left(\frac{1}{4}\right)^x$	y
-1	$\left(\frac{1}{4}\right)^{-1}$	4
0	$\left(\frac{1}{4}\right)^{0}$	1
1	$\left(\frac{1}{4}\right)^{1}$	$\frac{1}{4}$



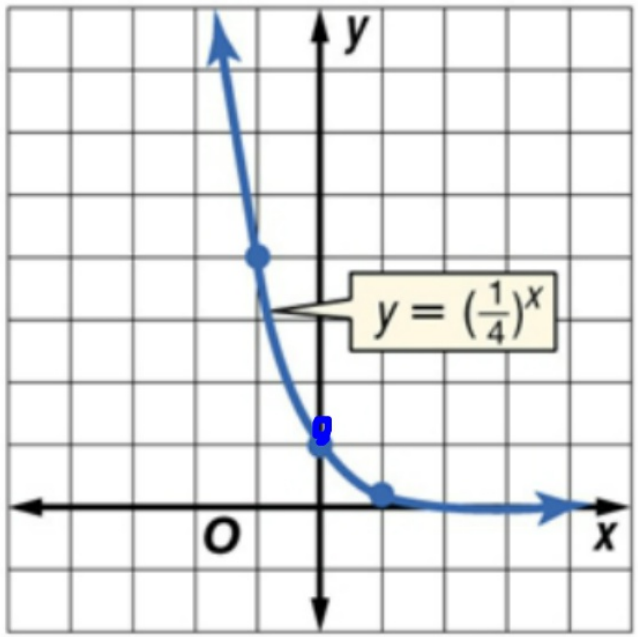
again, you'll have to create your own table.

EXAMPLE 2 Graph with $a > 0$ and $0 < b < 1$

A. Graph $y = \left(\frac{1}{4}\right)^x$. Find the *y*-intercept and state the domain and range.

x	$\left(\frac{1}{4}\right)^x$	y
-1	$\left(\frac{1}{4}\right)^{-1}$	4
0	$\left(\frac{1}{4}\right)^{0}$	1
1	$\left(\frac{1}{4}\right)^{1}$	$\frac{1}{4}$

Graph the ordered pairs and connect the points with a smooth curve.

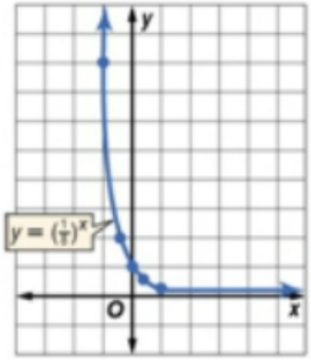


Answer: The *y*-intercept is 1. The domain is all real numbers and the range is all positive real numbers.

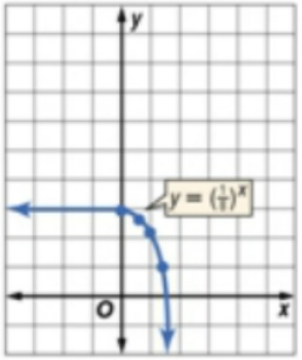
EXAMPLE 2 Check Your Progress

A. Graph $y = \left(\frac{1}{8}\right)^x$.

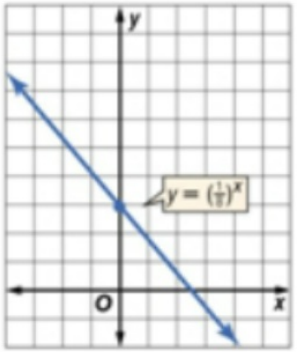
A.



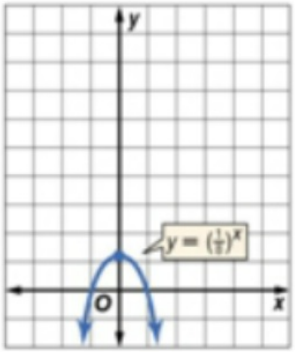
B.



C.



D.



EXAMPLE 2 Check Your Progress

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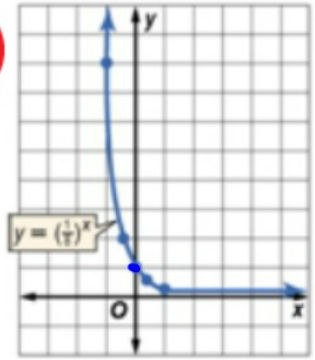
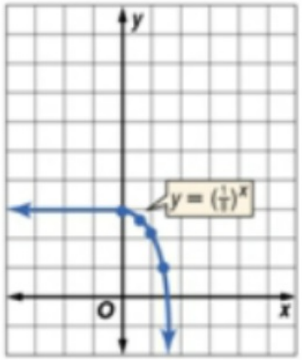
A.



B.



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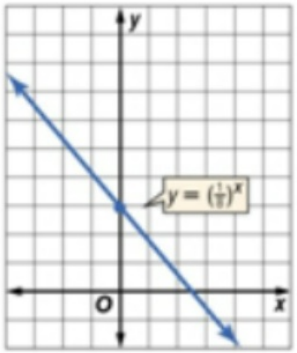
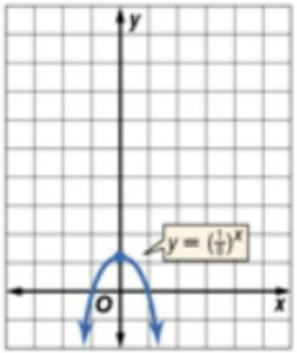


D.



LESSON 7–5 Exponential Functions

KeyConcept Exponential Function

Words An exponential function is a function that can be described by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

Examples $y = 2(3)^x$ $y = 4^x$ $y = \left(\frac{1}{2}\right)^x$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

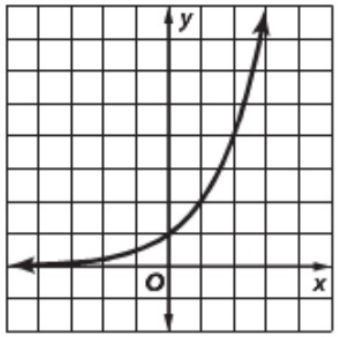
Examples 1–2 Graph each function. Find the y-intercept and state the domain and range.

1. $y = 2^x$
2. $y = -5^x$
3. $y = -\left(\frac{1}{5}\right)^x$
4. $y = 3\left(\frac{1}{4}\right)^x$
5. $f(x) = 6^x + 3$
6. $f(x) = 2 - 2^x$

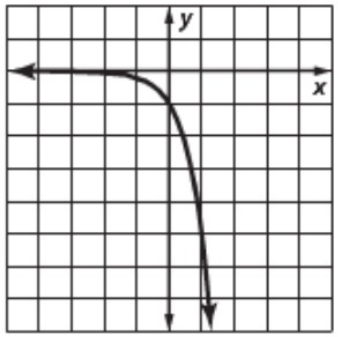
1–6. See Ch. 7 Answer Appendix.

Lesson 7-5

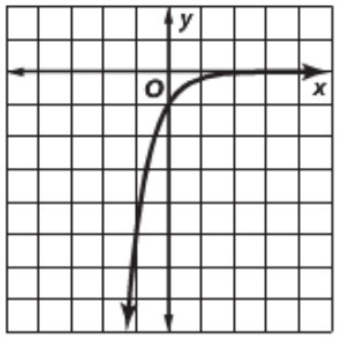
1. 1; D = {all real numbers}; R = $\{y \mid y > 0\}$



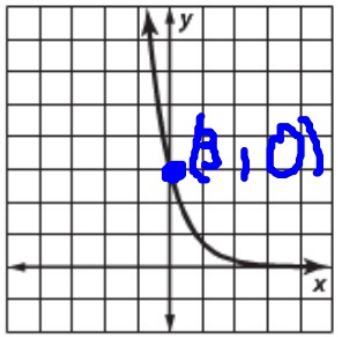
2. −1; D = {all real numbers}; R = $\{y \mid y < 0\}$



3. −1; D = {all real numbers}; R = $\{y \mid y < 0\}$



4. 3; D = {all real numbers}; R = $\{y \mid y > 0\}$



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5. $f(x) = 6^x + 3$
6. $f(x) = 2 - 2^x$

1–6. See Ch. 7 Answer Appendix.

5

x	y
-2	$3\frac{1}{36}$
-1	$3\frac{1}{6}$
0	4
1	9
2	39

6^1+3

6^0+3

$1+3$

$6^{-1}+3$

$\frac{1}{6}+3$

$6^{-2}+3$

$\frac{1}{6^2}+3 = \frac{1}{36}+3$

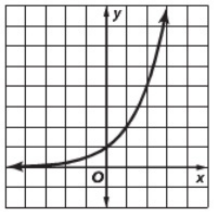
6^2+3

$36+3$

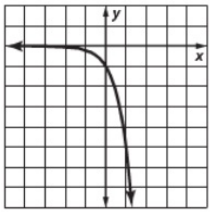
39

Lesson 7-5

1. 1; D = {all real numbers}; R = $\{y \mid y > 0\}$



2. −1; D = {all real numbers}; R = $\{y \mid y < 0\}$



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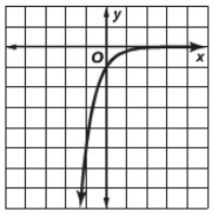

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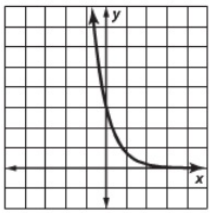

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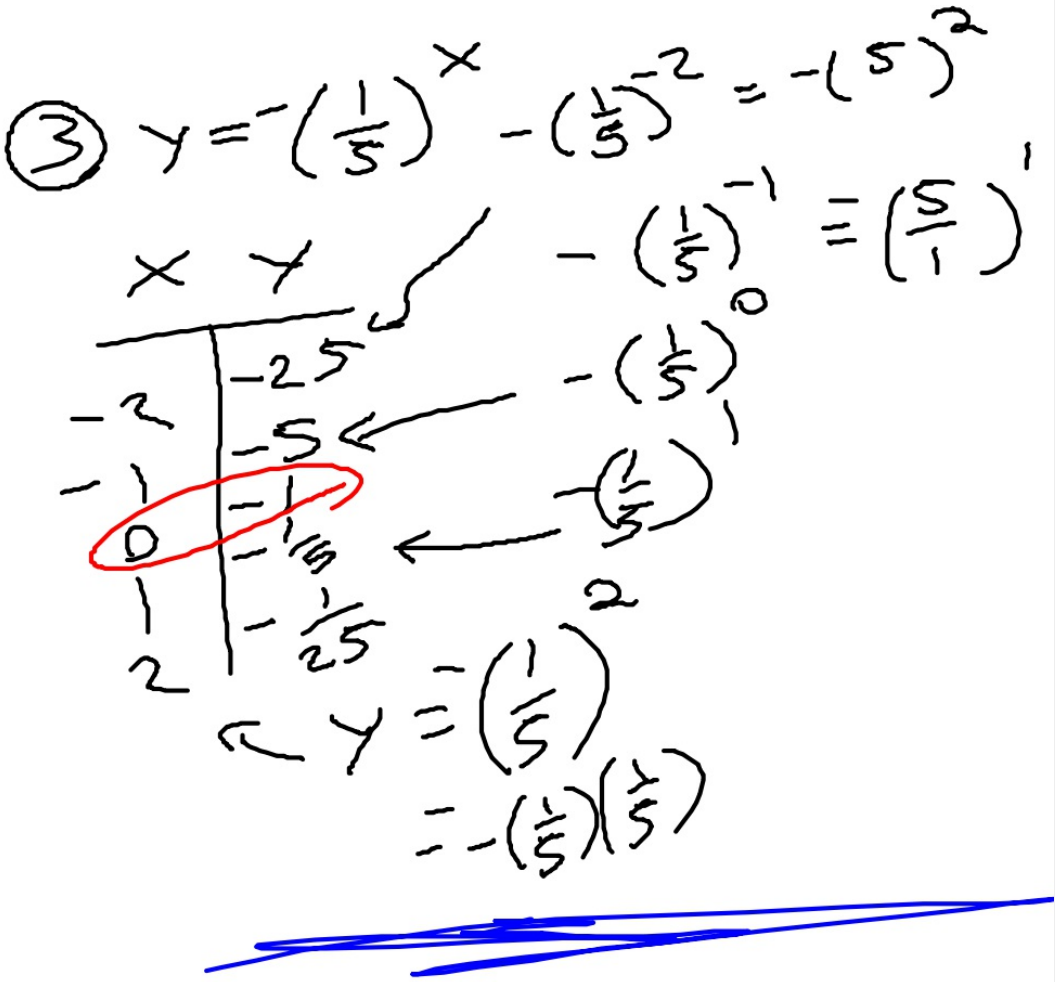

EXAMPLE 4 Identify Exponential Behavior

Determine whether the set of data displays exponential behavior. Explain why or why not.

x	0	10	20	30
y	10	25	62.5	156.25

EXAMPLE 4 Identify Exponential Behavior

Determine whether the set of data displays exponential behavior. Explain why or why not.

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For any exponential behavior, $y = ab^x$, you are either multiplying or dividing by a common factor (or ratio).

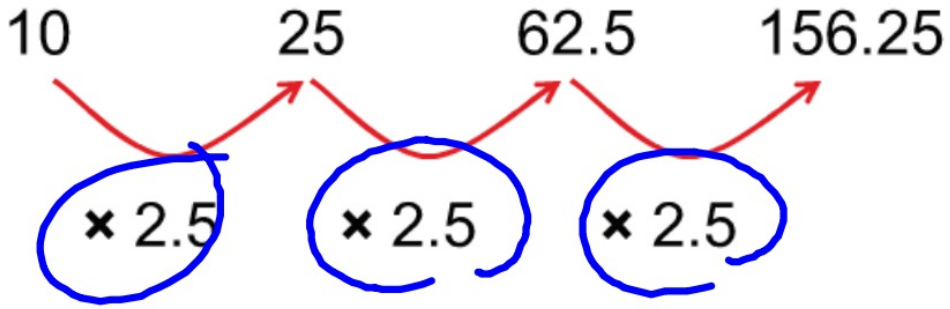
EXAMPLE 4 Identify Exponential Behavior

Determine whether the set of data displays exponential behavior. Explain why or why not.

x	0	10	20	30
y	10	25	62.5	156.25

Method 1 Look for a pattern.

The domain values are at regular intervals of 10. Look for a common factor among the range values.



EXAMPLE 4 Identify Exponential Behavior

Answer: Since the domain values are at regular intervals and the range values differ by a positive common factor, the data are probably exponential. The equation for the data may involve $(2.5)^x$.

EXAMPLE 4 Check Your Progress

Determine whether the set of data displays exponential behavior.

x	0	10	20	30
y	100	50	25	12.5

A. no

B. yes

C. cannot be determined

EXAMPLE 4 Check Your Progress

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x	0	10	20	30
y	100	50	25	12.5

A. no

B. yes

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5. $f(x) = 6^x + 3$
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1–6. See Ch. 7 Answer Appendix.

Example 3

7. **BIOLOGY** The function $f(t) = 100(1.05)^t$ models the growth of a fruit fly population, where $f(t)$ is the number of flies and t is time in days.

   a. What values for the domain and range are reasonable in the context of this situation? Explain. D = {d | d ≥ 0}, the number of days is greater than or equal to 0; R = {y | y ≥ 100}, the number of fruit flies is greater than or equal to 100.

   b. After two weeks, approximately how many flies are in this population? about 198 fruit flies

Example 4 Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not. 8–9. See margin.

8.

x	1	2	3	4	5	6
y	−4	−2	0	2	4	6

+2 +2 +2

No, not Exponential

9.

x	2	4	6	8	10	12
y	1	4	16	64	256	1024

×4 ×4 ×4

Yes!

10. $y = 2 \cdot 8^x$

11. $y = 2 \cdot \left(\frac{1}{6}\right)^x$

12. $y = \left(\frac{1}{12}\right)^x$

13. $y = -3 \cdot 9^x$

14. $y = -4 \cdot 10^x$

15. $y = 3 \cdot 11^x$

16. $y = 4^x + 3$

17. $y = \frac{1}{2}(2^x - 8)$

18. $y = 5(3^x) + 1$

19. $y = -2(3^x) + 5$

Example 3

20. CCSS **MODELING** A population of bacteria in a culture increases according to the model $p = 300(2.7)^{0.02t}$, where t is the number of hours and $t = 0$ corresponds to 9:00 A.M.

a. Use this model to estimate the number of bacteria at 11 A.M. **about 312**

b. Graph the function and name the p-intercept. Describe what the p-intercept represents, and describe a reasonable domain and range for this situation.
See Ch. 7 Answer Appendix.

Example 4

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not. **21–24. See margin.**

21.

x	−4	0	4	8	12
y	2	−4	8	−16	32

×−2 ×−2 ×−2 ×−2

22.

x	−6	−3	0	3
y	5	10	15	20

NO

23.

x	−8	−6	−4	−2
y	0.25	0.5	1	2

×2 ×2 ×2
yes.

24.

x	20	30	40	50	60
y	1	0.4	0.16	0.064	0.0256

×.4 ×.4 ×.4 ...
yes.

11. $y = 2 \cdot \left(\frac{1}{6}\right)^x$

$y = 2\left(\frac{1}{6}\right)^0$

$y = 2(1)$

x	y
−2	18
−1	3
0	2
1	$\frac{1}{3}$
2	$\frac{1}{18}$

$y = 2\left(\frac{1}{6}\right)^2$

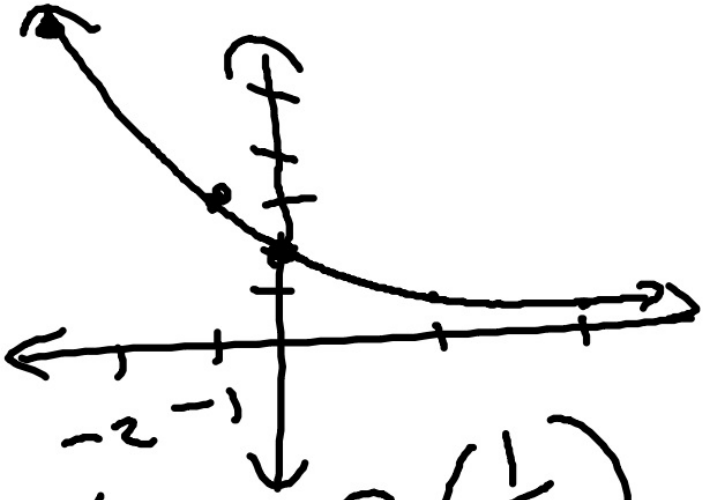
$= 2\left(\frac{1}{36}\right)$

$= \frac{2}{36} = \frac{1}{18}$

$y = 2\left(\frac{1}{6}\right)$

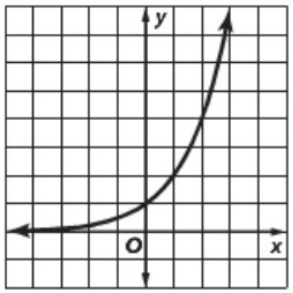
$= 2\left(\frac{1}{6}\right)$

$= \frac{2}{6} = \frac{1}{3}$

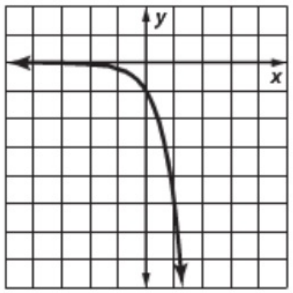


Lesson 7-5

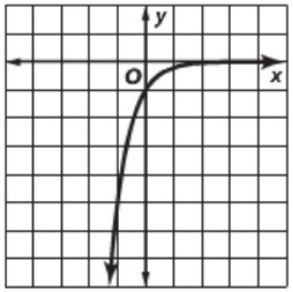
1. 1; D = {all real numbers}; R = $\{y \mid y > 0\}$



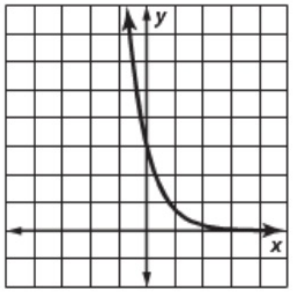
2. −1; D = {all real numbers}; R = $\{y \mid y < 0\}$



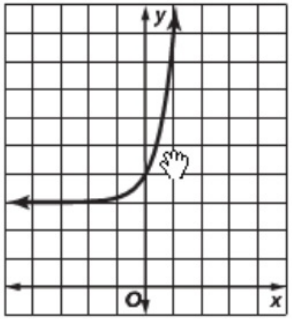
3. −1; D = {all real numbers}; R = $\{y \mid y < 0\}$



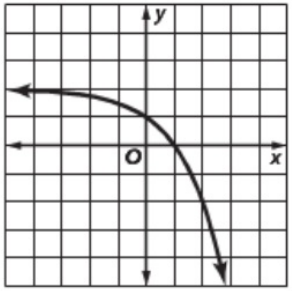
4. 3; D = {all real numbers}; R = $\{y \mid y > 0\}$



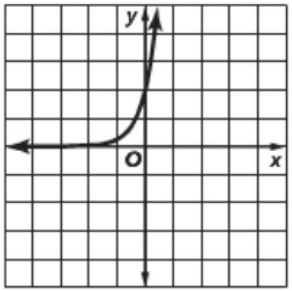
5. 4; D = {all real numbers}; R = $\{y \mid y > 3\}$



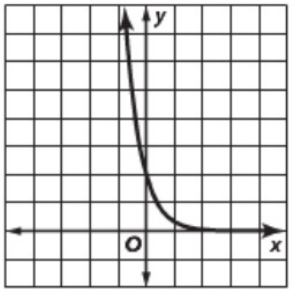
6. 1; D = {all real numbers}; R = $\{y \mid y < 2\}$



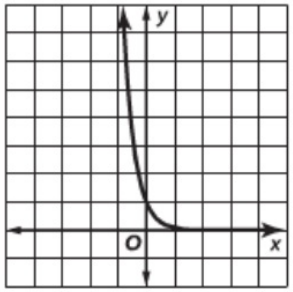
10. 2; D = {all real numbers}; R = $\{y \mid y > 0\}$



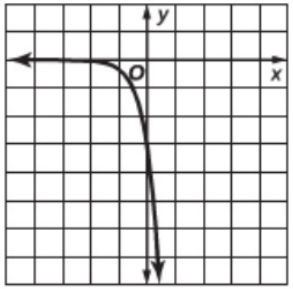
11. 2; D = {all real numbers}; R = $\{y \mid y > 0\}$



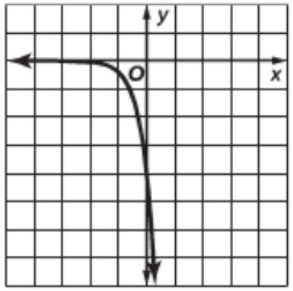
12. 1; D = {all real numbers};
R = $\{y \mid y > 0\}$



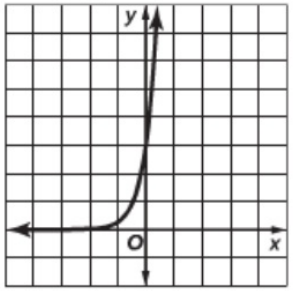
13. −3; D = {all real numbers};
R = $\{y \mid y < 0\}$



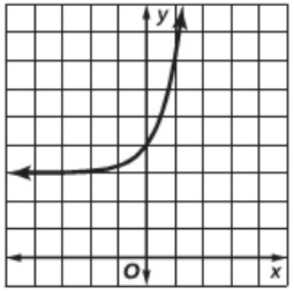
14. −4; D = {all real numbers};
R = $\{y \mid y < 0\}$



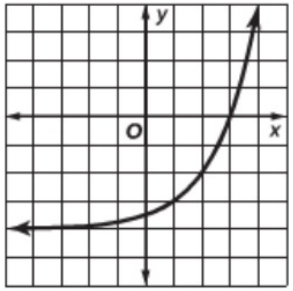
15. 3; D = {all real numbers};
R = $\{y \mid y > 0\}$



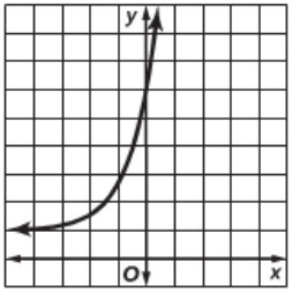
16. 4; D = {all real numbers}
R = $\{y \mid y > 3\}$



17. −3.5; D = {all real numbers}
R = $\{y \mid y > -4\}$



18. 6; D = {all real numbers}
R = $\{y \mid y > 1\}$



19. 3; D = {all real numbers};
R = $\{y \mid y < 5\}$