


3-5 Arithmetic Sequences as Linear Functions

1 Recognize Arithmetic Sequences You can relate the pattern of team times to linear functions. A **sequence** is a set of numbers, called the **terms of the sequence**, in a specific order. Look for a pattern in the information given for the women's crew team. Make a table to analyze the data.

Distance (m)	400	800	1200	1600	2000
Time (min : sec)	1:32	3:04	4:36	6:08	7:40



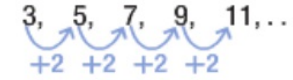

As the distance increases in regular intervals, the time increases by 1 minute 32 seconds. Since the difference between successive terms is constant, this is an **arithmetic sequence**. The difference between the terms is called the **common difference** d .

Key Concept Arithmetic Sequence

Words An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the *common difference*.

Examples

$3, 5, 7, 9, 11, \dots$ $33, 29, 25, 21, 17, \dots$

$d = 2$ $d = -4$

Example 1 Identify Arithmetic Sequences

Determine whether each sequence is an arithmetic sequence. Explain.

a. $-4, -2, 0, 2, \dots$

$$\begin{array}{cccc} -4 & -2 & 0 & 2 \\ \frown & \frown & \frown & \\ +2 & +2 & +2 & \end{array}$$

The difference between terms in the sequence is constant. Therefore, this sequence is arithmetic.

b. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}, \dots$

$$\begin{array}{cccc} \frac{1}{2} & \frac{5}{8} & \frac{3}{4} & \frac{13}{16} \\ \frown & \frown & \frown & \\ +\frac{1}{8} & +\frac{1}{8} & +\frac{1}{16} & \end{array}$$

This is not an arithmetic sequence. The difference between terms is not constant.

Example 2 Find the Next Term

Find the next three terms of the arithmetic sequence 15, 9, 3, -3,

Step 1 Find the common difference by subtracting successive terms.

$$\begin{array}{cccc} 15 & 9 & 3 & -3 \\ \curvearrowright & \curvearrowright & \curvearrowright & \\ -6 & -6 & -6 & \end{array}$$

The common difference is -6 .

Step 2 Add -6 to the last term of the sequence to get the next term.

$$\begin{array}{cccc} -3 & -9 & -15 & -21 \\ \curvearrowright & \curvearrowright & \curvearrowright & \\ -6 & -6 & -6 & \end{array}$$

The next three terms in the sequence are -9 , -15 , and -21 .

Each term in an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d .

Term	Symbol	In Terms of a_1 and d	Numbers
first term	a_1	a_1	8
second term	a_2	$a_1 + d$	8 + 1(3) = 11
third term	a_3	$a_1 + 2d$	8 + 2(3) = 14
fourth term	a_4	$a_1 + 3d$	8 + 3(3) = 17
\vdots	\vdots	\vdots	\vdots
n th term	a_n	$a_1 + (n - 1)d$	8 + $(n - 1)$ (3)

KeyConcept n th Term of an Arithmetic Sequence

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

n th term

a_n

$$a_1 + (n - 1)d$$

$$8 + (n - 1)(3)$$

Key Concept n th Term of an Arithmetic Sequence

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

the first five terms of the sequence. 5-6. See margin for graphs.

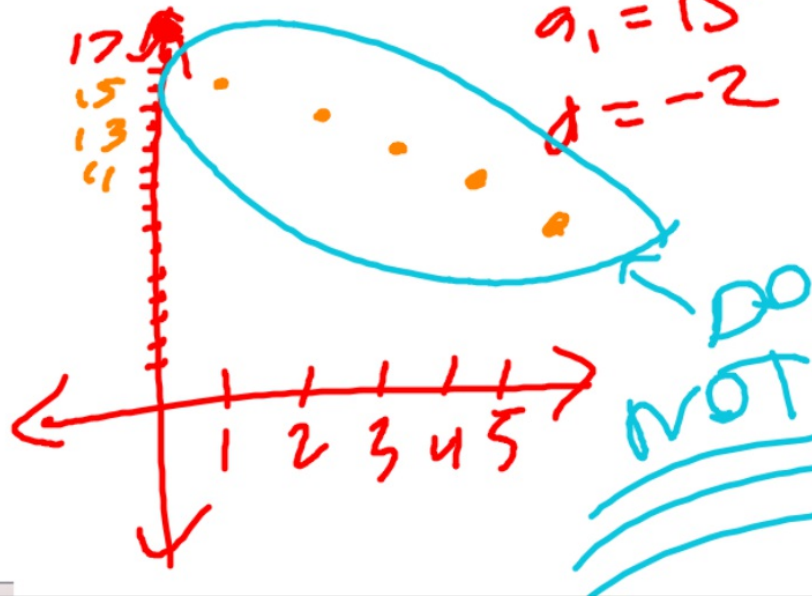
5. 15, 13, 11, 9, ... $a_n = 17 - 2n$

6. -1, -0.5, 0, 0.5, ... $a_n = 0.5n - 1.5$

Example 4

$$a_n = a_1 + (n - 1)d,$$

$a_1 = 15$
 $d = -2$



$$a_n = 15 + (n - 1)(-2)$$
$$= 15 + -2n + 2$$

$$a_n = 17 - 2n$$

connect $-2n + 17$ the dots!!



INVITATIONS Marisol is mailing invitations to her quinceañera. The arithmetic sequence \$0.42, \$0.84, \$1.26, \$1.68, ... represents the cost of postage.

a. Write a function to represent this sequence.

The first term, a_1 , is 0.42. Find the common difference.

$$\begin{array}{cccc}
 0.42 & 0.84 & 1.26 & 1.68 \\
 \curvearrowright & \curvearrowright & \curvearrowright & \\
 +0.42 & +0.42 & +0.42 &
 \end{array}$$

The common difference is 0.42.

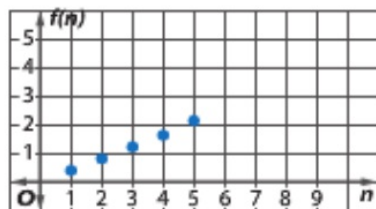
$$\begin{aligned}
 a_n &= a_1 + (n - 1)d && \text{Formula for the } n\text{th term} \\
 &= 0.42 + (n - 1)0.42 && a_1 = 0.42 \text{ and } d = 0.42 \\
 &= 0.42 + 0.42n - 0.42 && \text{Distributive Property} \\
 &= 0.42n && \text{Simplify.}
 \end{aligned}$$

The function is $f(n) = 0.42n$.

b. Graph the function and determine the domain.

The rate of change of the function is 0.42. Make a table and plot points.

n	$f(n)$
1	0.42
2	0.84
3	1.26
4	1.68
5	2.10



The domain of a function is the number of invitations Marisol mails. So, the domain is $\{1, 2, 3, 4, \dots\}$.

Check Your Understanding

Step-by-Step Solutions begin on page R13.



Example 1 Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain.

1. 18, 16, 15, 13, ... **No; there is no common difference.** 2. 4, 9, 14, 19, ... **Yes; the common difference is 5.**

Example 2 Find the next three terms of each arithmetic sequence.

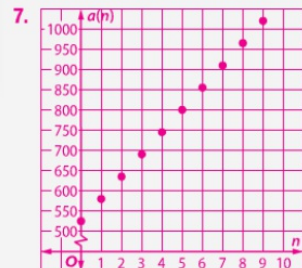
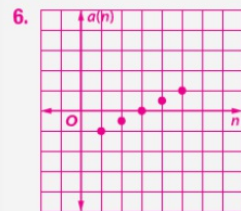
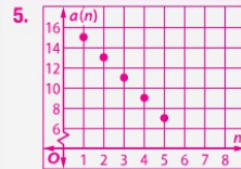
3. 12, 9, 6, 3, ... **0, -3, -6** 4. -2, 2, 6, 10, ... **14, 18, 22**

Example 3 Write an equation for the n th term of each arithmetic sequence. Then graph the first five terms of the sequence. **5-6. See margin for graphs.**

5. 15, 13, 11, 9, ... **$a_n = 17 - 2n$** 6. -1, -0.5, 0, 0.5, ... **$a_n = 0.5n - 1.5$**

Example 4 7. **SAVINGS** Kaia has \$525 in a savings account. After one month she has \$580 in the account. The next month the balance is \$635. The balance after the third month is \$690. Write a function to represent the arithmetic sequence. Then graph the function. **$a(n) = 55n + 525$; See margin for graph.**

Additional Answers



Example 1 Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain.

8. $-3, 1, 5, 9, \dots$ **Yes; the common difference is 4.**
9. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \dots$ **No; there is no common difference.**
10. $-10, -7, -4, 1, \dots$ **No; there is no common difference.**
11. $-12.3, -9.7, -7.1, -4.5, \dots$ **Yes; the common difference is 2.6.**

Example 2 Find the next three terms of each arithmetic sequence.

12. $0.02, 1.08, 2.14, 3.2, \dots$ **4.26, 5.32, 6.38**
13. $6, 12, 18, 24, \dots$ **30, 36, 42**
14. $21, 19, 17, 15, \dots$ **13, 11, 9**
15. $-\frac{1}{2}, 0, \frac{1}{2}, 1, \dots$ **$1\frac{1}{2}, 2, 2\frac{1}{2}$**
16. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \dots$ **$3\frac{2}{3}, 4, 4\frac{1}{3}$**
17. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, \dots$ **$3\frac{7}{12}, 4\frac{1}{3}, 5\frac{1}{12}$**

Example 3 Write an equation for the n th term of the arithmetic sequence. Then graph the first five terms in the sequence. **See Ch. 3 Answer Appendix for graphs.**

18. $-3, -8, -13, -18, \dots$ **$a_n = -5n + 2$**
19. $-2, 3, 8, 13, \dots$ **$a_n = 5n - 7$**
20. $-11, -15, -19, -23, \dots$ **$a_n = -4n - 7$**
21. $-0.75, -0.5, -0.25, 0, \dots$ **$a_n = 0.25n - 1$**

Example 4 22. **AMUSEMENT PARKS** Shiloh and her friends spent the day at an amusement park. In the first hour, they rode two rides. After 2 hours, they had ridden 4 rides. They had ridden 6 rides after 3 hours.

- a. Write a function to represent the arithmetic sequence. **$f(n) = 2n$**
- b. Graph the function and determine the domain. **See Ch. 3 Answer Appendix.**

23. **CCSS MODELING** The table shows how Ryan is paid at his lumber yard job.

Number of 10-ft 2×4 Planks Cut	1	2	3	4	5	6	7
Amount Paid in Commission (\$)	8	16	24	32	40	48	56

- a. Write a function to represent Ryan's commission. **$f(n) = 8n$**
- b. Graph the function and determine the domain. **See Ch. 3 Answer Appendix.**

Lesson 3-5

