

### What You'll Learn

Scan the lesson. List two headings you would use to make an outline of the lesson. **Sample answers:**

- **Direct Variation**
- **Compare Direct Variations**



### Real-World Link

**Charity** The amount of money David can raise for the Wish Upon A Rainbow Bike-a-thon is shown in the table.

Biking Time (h), $x$	Money Raised (\$), $y$
2	20
4	40
6	60

2 ↪

2 20

$\frac{20}{2} = 10$

"rate of change"  
"slope"  
y  
x

Recall that when the ratio of two variable quantities is constant, a proportional relationship exists. This relationship is called **direct variation**. The constant ratio is called the **constant of variation** or **constant of proportionality**.

Complete the steps below to derive the equation for a direct variation.

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

Slope formula

$$\frac{y - 0}{x - 0} = m$$

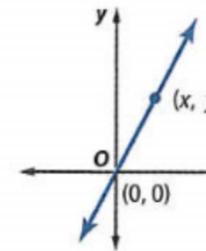
$(x_1, y_1) = (0, 0)$   
 $(x_2, y_2) = (x, y)$

$$\frac{y}{x} = m$$

Simplify.

$$y = m x$$

Multiplication Property of Equality



1. Use the table to find the rate of change. Then write an equation in  $y = mx$  form to represent the situation.

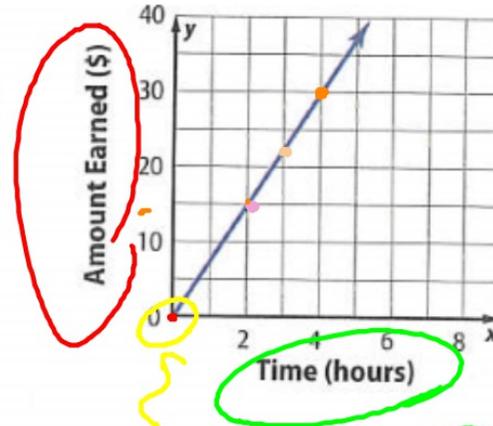
**\$10 per hour;  $y = 10x$**



## Example



1. The amount of money Robin earns while babysitting varies directly with the time as shown in the graph. Determine the amount that Robin earns per hour.



To determine the amount Robin earns per hour, or the unit rate, find the constant of variation.

Use the points (2, 15), (3, 22.5), and (4, 30).

$$\frac{\text{amount earned}}{\text{time}} \rightarrow \frac{15}{2} \text{ or } \frac{7.5}{1} \quad \frac{22.5}{3} \text{ or } \frac{7.5}{1} \quad \frac{30}{4} \text{ or } \frac{7.5}{1}$$

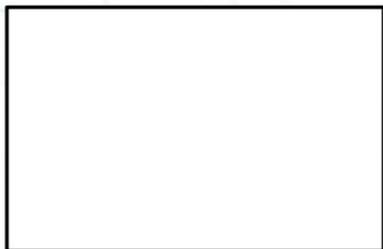
$$\frac{15}{2}$$

So, Robin earned \$7.50 for each hour she babysits.

**Got It?** Do this problem to find out.

- a. Two minutes after a skydiver opens his parachute, he has descended 1,900 feet. After 5 minutes, he descended 4,750 feet. If the distance varies directly with the time, at what rate is the skydiver descending?

Show  
your  
work.





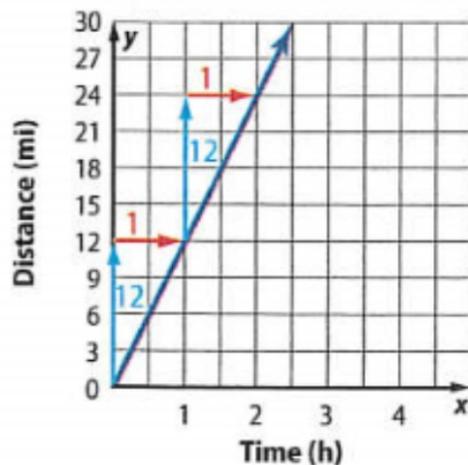
## Example



2. A cyclist can ride 3 miles in 0.25 hour. Assume that the distance biked in miles  $y$  varies directly with time in hours  $x$ . This situation can be represented by  $y = 12x$ . Graph the equation. How far can the cyclist ride per hour?

Make a table of values. Then graph the equation  $y = 12x$ . In a direct variation equation,  $m$  represents the slope. So, the slope of the line is  $\frac{12}{1}$ .

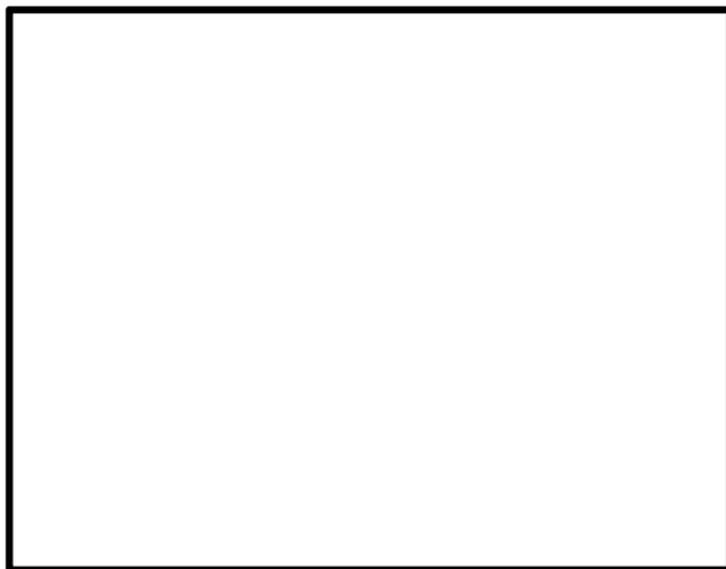
Hours, $x$	$y = 12x$	Miles, $y$
0	$y = 12(0)$	0
1	$y = 12(1)$	12
2	$y = 12(2)$	24



The unit rate is the slope of the line. So, the cyclist can ride 12 miles per hour.

**Got It? Do this problem to find out.**

- b. A grocery store sells 6 oranges for \$2. Assume that the cost of the oranges varies directly with the number of oranges. This situation can be represented by  $y = \frac{1}{3}x$ . Graph the equation. What is the cost per orange?



Number of oranges

Show your work.



## Compare Direct Variations

You can use tables, graphs, words, or equations to represent and compare proportional relationships.

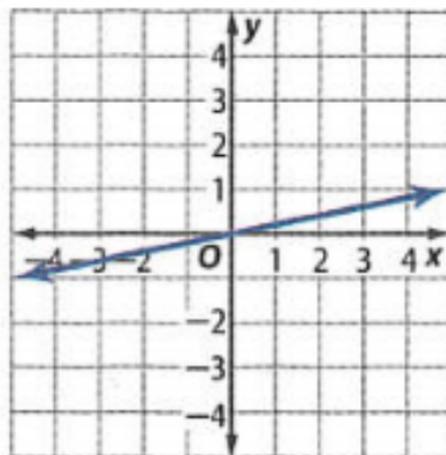
**Table**

$x$	15	20	25	30
$y$	3	4	5	6

**Words**  $y$  varies directly with  $x$

**Equation**  $y = \frac{1}{5}x$

**Graph**

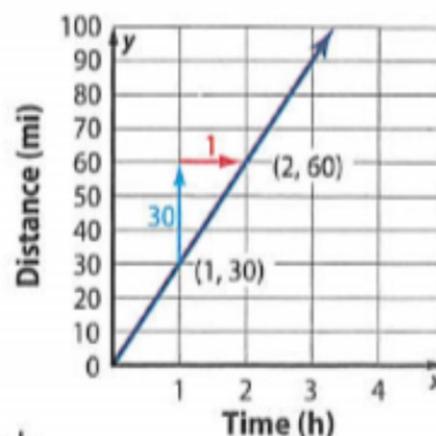




## Example



- 3.** The distance  $d$  in miles covered by a rabbit in  $t$  hours can be represented by the equation  $d = 35t$ . The distance covered by a grizzly bear is shown on the graph. Which animal is faster? Explain.



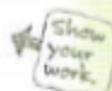
**Rabbit**  $d = 35t$

The slope or unit rate is 35 mph.

**Grizzly Bear** Find the slope of the graph.

$$\frac{\text{rise}}{\text{run}} = \frac{30}{1} \text{ or } 30$$

Since  $35 > 30$ , the rabbit is the faster animal.



**Got It?** Do this problem to find out.

- c. Financial Literacy** Damon's earnings for four weeks from a part time job are shown in the table. Assume that his earnings vary directly with the number of hours worked.

Time Worked (h)	15	12	22	9
Total Pay (\$)	112.50	90.00	165.00	67.50

He can take a job that will pay him \$7.35 per hour worked. Which job has the better pay? Explain.





## Example



4. A 3-year-old dog is often considered to be 21 in human years. Assume that the equivalent age in human years  $y$  varies directly with its age as a dog  $x$ . Write and solve a direct variation equation to find the human-year age of a dog that is 6 years old.

Let  $x$  represent the dog's actual age and let  $y$  represent the human-equivalent age.

$$y = mx \quad \text{Direct variation}$$

$$21 = m(3) \quad y = 21, x = 3$$

$$7 = m \quad \text{Simplify.}$$

$$y = 7x \quad \text{Replace } m \text{ with } 7.$$

You want to know the human-year age or  $y$ -value when the dog is 6 years old.

$$y = 7x \quad \text{Write the equation.}$$

$$y = 7 \cdot 6 \quad x = 6$$

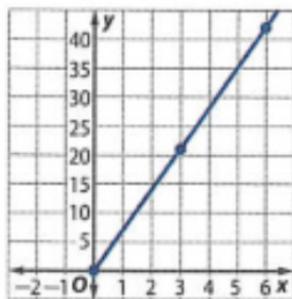
$$y = 42 \quad \text{Simplify.}$$

So, when a dog is 6 years old, the equivalent age in human years is 42.

### Check

Graph the equation  $y = 7x$ .

The  $y$ -value when  $x = 6$  is 42. ✓



**Got It?** Do these problems to find out.

- d. A charter bus travels 210 miles in  $3\frac{1}{2}$  hours. Assume the distance traveled is directly proportional to the time traveled. Write and solve a direct variation equation to find how far the bus will travel in 6 hours.
- e. A Monarch butterfly can fly 93 miles in 15 hours. Assume the distance traveled is directly proportional to the time traveled. Write and solve a direct variation equation to find how far the Monarch butterfly will travel in 24 hours.



# Guided Practice



1. A color printer can print 36 pages in 3 minutes and 108 pages in 9 minutes. If the number of pages varies directly with the time, at what rate is the color printer printing? (Example 1)

**12 pages per minute**

$(3, 36)$   $(9, 108)$

"y varies directly as x"

2. A new compact car can travel 288 miles on nine gallons of gas. The distance driven in miles y varies directly with the number of gallons of gas x. This situation can be represented by the equation  $y = 32x$ . (Examples 2 and 3)

- a. Graph the equation on the coordinate plane shown.  
b. How many miles per gallon does the car get?

**32 miles per gallon**

- c. The distance y traveled by a hybrid car using x gallons of gas can be represented by  $y = 42x$ . Which car gets better gas mileage? Explain.

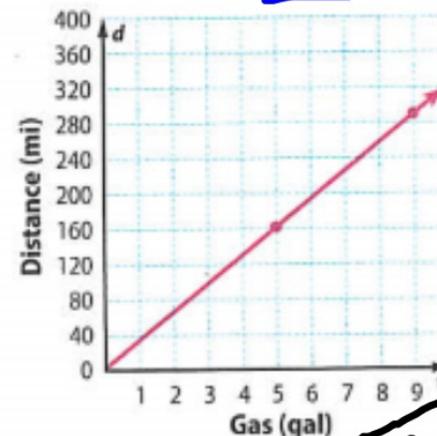
**hybrid; Sample answer: The unit rate for the hybrid is**

**42 mpg. The unit rate for the new car is 32 mpg.  $42 > 32$**

3. **Financial Literacy** Annie's current earnings are shown in the table. She was offered a new job that will pay \$7.25 per hour. Assume that her earnings vary directly with the number of hours worked.

Which job pays more an hour? (Example 3) **new job offer; Sample answer: The unit rate of her current job is \$6.50 per hour**

**which is lower than \$7.25 per hour.**



$\frac{y}{x}$   
**72**  
**6**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Hours, x	Money Earned (\$), y
2	13.00
3	19.50
4	26.00
5	32.50

4. The height of a wide-screen television screen varies directly with its width. A television screen that is 60 centimeters wide and 33.75 centimeters high. Write and solve a direct variation equation to find the height of a television screen that is 90 centimeters wide.

(Example 4)  $y = 0.5625x$ ; 50.625 cm

5.  **Building on the Essential Question** What is the relationship among the unit rate, slope, and constant rate of change of a proportional linear relationship?

**Sample answer: They all represent the same thing.**

### Rate Yourself!

How well do you understand direct variation? Circle the image that applies.



Clear



Somewhat  
Clear



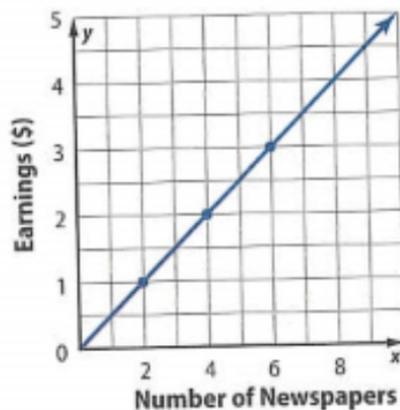
Not So  
Clear

For more help, go online to access a Personal Tutor.



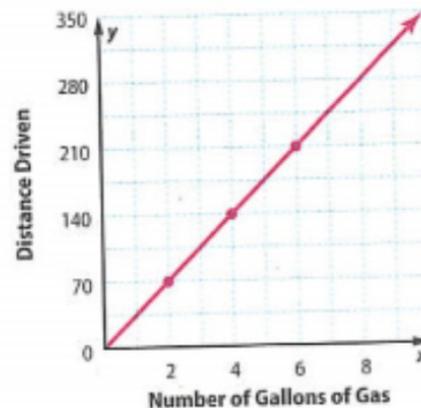
1. Dusty's earnings vary directly with the number of papers he delivers. The relationship is shown in the graph below. Determine the amount that Dusty earns for each paper he delivers. (Example 1)

**\$0.50 per paper**



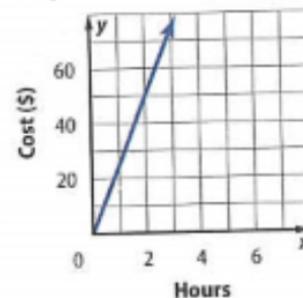
2. The Thompson family is buying a car that can travel 70 miles on two gallons of gas. Assume that the distance traveled in miles  $y$  varies directly with the amount of gas used  $x$ . This can be represented by  $y = 35x$ . Graph the equation on the coordinate plane. How many miles does the car get per gallon of gas? (Example 2)

**35 miles per gallon**



- 3 Tom was comparing computer repair companies. The cost  $y$  for Computer Access for  $x$  hours is shown in the graph. The cost for Computers R Us can be represented by the equation  $y = 23.5x$ . Which company's repair price is lower? Explain. (Example 3)

**Computers R Us; Sample answer: The unit cost for Computer Access is \$25 per hour. The unit cost for Computers R Us is \$23.50.  $23.5 < 25$**



4. The weight of an object on Mars varies directly with its weight on Earth. An object that weighs 50 pounds on Mars weighs 150 pounds on Earth. If an object weighs 120 pounds on Earth, write and solve a direct variation equation to find how much an object would weigh on Mars. (Sample 4)

$$y = \frac{1}{3}x; 40 \text{ lb}$$

Determine whether each linear function is a direct variation. If so, state the constant of variation. If not, explain why not.

5.

Pictures, $x$	5	6	7	8
Profit, $y$	20	24	28	32

yes; 4

6.

Age, $x$	10	11	12	13
Grade, $y$	5	6	7	8

no; Sample answer: The ratio of age to grade is not constant.

7 The number of centimeters varies directly with the number of inches. Find the measure of an object in centimeters if it is 50 inches long. 127 cm

Inches, $x$	6	9	12	15
Centimeters, $y$	15.24	22.86	30.48	38.10

$$y = \frac{-4}{3}x$$

$$y = \frac{-4}{3}(-4)$$

$$y = \frac{16}{3} = 5\frac{1}{3}$$

$$= 5\frac{1}{3}$$

CCSS **Persevere with Problems** If  $y$  varies directly with  $x$ , write an equation for the direct variation. Then find each value.

8. If  $y = -12$  when  $x = 9$ , find  $y$  when  $x = -4$ .  $y = -\frac{4}{3}x; 5\frac{1}{3}$

$$y = mx$$

$$-12 = m(9)$$

$$\frac{9m}{9} = \frac{-12}{9}$$

$$m = \frac{-12}{9} = -\frac{4}{3}$$

9. Find  $y$  when  $x = 10$  if  $y = 8$  when  $x = 20$ .  $y = \frac{2}{5}x; 4$

10. If  $y = -6$  when  $x = -14$ , find  $x$  when  $y = -4$ .  $y = \frac{3}{7}x; -9\frac{1}{3}$



## H.O.T. Problems Higher Order Thinking

11. **CCSS Model with Mathematics** Write three ordered pairs for a direct variation relationship where  $y = 12$  when  $x = 16$ . **Sample answer:**  
 $(4, 3), (8, 6), (0, 0)$
- 
12. **CCSS Persevere with Problems** The amount of stain needed to cover a wood surface is directly proportional to the area of the surface. If 3 pints are required to cover a square deck with a side of 7 feet, how many pints of stain are needed to paint a square deck with a side of 10 feet 6 inches?  
 $6\frac{3}{4}$  pt
- 
13. **CCSS Reason Inductively** Describe two real-world quantities that have a proportional linear relationship. Explain how you could change the situation to make the relationship nonproportional.  
**Sample answer: The total cost  $y$  of buying  $x$  boxes of popcorn is a proportional linear relationship. If you buy  $x$  boxes of popcorn and a drink for \$1, the relationship between the total cost and the boxes of popcorn becomes nonproportional.**