

## 8-4 Graphing Rational Functions

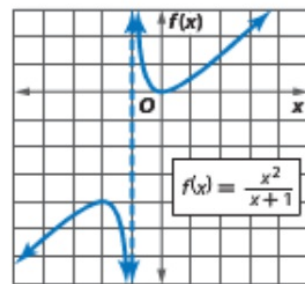
### Key Concept Vertical and Horizontal Asymptotes

**Words** If  $f(x) = \frac{a(x)}{b(x)}$ ,  $a(x)$  and  $b(x)$  are polynomial functions with no common factors other than 1, and  $b(x) \neq 0$ , then:

- $f(x)$  has a **vertical asymptote** whenever  $b(x) = 0$ .
- $f(x)$  has at most one **horizontal asymptote**.
  - If the degree of  $a(x)$  is greater than the degree of  $b(x)$ , there is no horizontal asymptote.
  - If the degree of  $a(x)$  is less than the degree of  $b(x)$ , the horizontal asymptote is the line  $y = 0$ .
  - If the degree of  $a(x)$  equals the degree of  $b(x)$ , the horizontal asymptote is the line  $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$ .

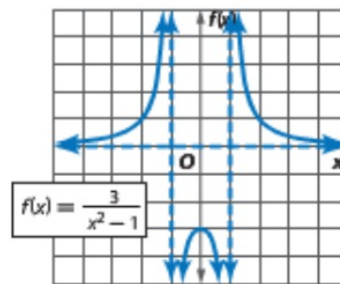
### Examples

No horizontal asymptote

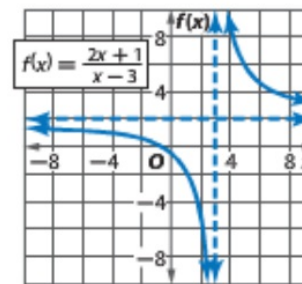


Vertical asymptote:  
 $x = -1$

One horizontal asymptote



Vertical asymptotes:  
 $x = -1, x = 1$   
Horizontal asymptote:  
 $f(x) = 0$



Vertical asymptote:  
 $x = 3$   
Horizontal asymptote:  
 $f(x) = 2$

in the Antelope data.

7.5, 7.6, 7.7, 8.9, 9.0, 9.5, 11.0

Q1

Med

Q3


## Guided Practice



- The average wind speeds for several cities in Pennsylvania are given in the table. (Examples 1 and 2)
  - Find the range of the data. 3.5
  - Find the median and the first and third quartiles.  
8.9; 7.6; 9.5      9.5-7.6
  - Find the interquartile range. 1.9
  - Identify any outliers in the data. none
- The heights of several types of palm trees, in feet, are 40, 25, 15, 22, 50, and 30. The heights of several types of pine trees, in feet, are 60, 75, 45, 80, 75, and 70. Compare and contrast

Wind Speed	
Pennsylvania City	Speed (mph)
Allentown	8.9
Erie	11.0
Harrisburg	7.5
Middletown	7.7
Philadelphia	9.5
Pittsburgh	9.0
Williamsport	7.6

2. The heights of several types of palm trees, in feet, are 40, 25, 15, 22, 50, and 30. The heights of several types of pine trees, in feet, are 60, 75, 45, 80, 75, and 70. Compare and contrast the measures of variation of both kinds of trees. (Example 3)
- ranges: 35 and 35; medians: 27.5 and 72.5;  $Q_3$ : 40 and 75;  $Q_1$ : 22 and 60; IQR: 18 and 15. Both sets of data have the same ranges of height and close interquartile ranges even though most of the palm trees are shorter than most of the pine trees.

3.  **Building on the Essential Question** Describe the difference between measure of center and measure of variation. Sample answer: A measure of center summarizes a set of data with a single number, but a measure of variation uses a single number to describe how the values vary.

# Independent Practice

Go online for Step-by-Step Solutions



**1** The table shows the number of golf courses in various states. (Examples 1 and 2)

- a. Find the range of the data. 1,028
- b. Find the median and the first and third quartiles.  
923.5; 513; 1,038
- c. Find the interquartile range. 525
- d. Name any outliers in the data. none

Number of Golf Courses			
California	1,117	New York	954
Florida	1,465	North Carolina	650
Georgia	513	Ohio	893
Iowa	437	South Carolina	456
Michigan	1,038	Texas	1,018

**For each data set, find the median, the first and third quartiles, and the interquartile range.** (Example 1)

2. texts per day: 24, 53, 38, 12, 31, 19, 26  
median: 26; Q<sub>1</sub>: 19; Q<sub>3</sub>: 38; IQR: 19

- 3** daily attendance at the water park: 346, 250, 433, 369, 422, 298  
median: 357.5; Q<sub>1</sub>: 298; Q<sub>3</sub>: 422; IQR: 124

4. The table shows the number of minutes of exercise for each person. Compare and contrast the measures of variation for both weeks. (Example 3) **ranges: 50 and 45; medians: both 50;  $Q_1$ : 45 and 35;  $Q_3$ : both 60; IQR: 15 and 25; Sample answer: Both sets of data have the same median number of minutes while the middle data in the first set are grouped more closely than the middle data in the second.**

Minutes of Exercise		
	Week 1	Week 2
Tanika	45	30
Tasha	40	55
Tyrone	45	35
Uniqua	55	60
Videl	60	45
Wesley	90	75

5. **STEM** The table shows the number of known moons for each planet in our solar system. Use the measures of variation to describe the data. **range: 63; median: 7.5;  $Q_3$ : 30.5;  $Q_1$ : 0.5; IQR: 30; Sample answer: The number of moons for each planet varies greatly. The IQR and range are both large.**

Known Moons of Planets			
Mercury	0	Jupiter	63
Venus	0	Saturn	34
Earth	1	Uranus	27
Mars	2	Neptune	13

6. **CCSS Use Math Tools** The *double stem-and-leaf plot*, where the stem is in the middle and the leaves are on either side, shows the high temperatures for two cities in the same week. Use the measures of variation to describe the data in the stem-and-leaf plot.

Minneapolis—range: 23 degrees;  $Q_3$ : 36 degrees; median: 25;

$Q_1$ : 21 degrees; IQR: 15 degrees; Columbus—range: 37 degrees;

$Q_3$ : 48 degrees; median: 29;  $Q_1$ : 27 degrees; IQR: 21 degrees;

Sample answer: The Minneapolis temperatures are closer

together than the Columbus temperatures.

<u>Minneapolis</u>		<u>Columbus</u>
5 3 1 0	2	5 7 9 9
6 4	3	7
3	4	8
	5	
	6	2
613 = 36°		215 = 25°

29, 21, 23, 25, 34, 36, 43  
 25, 27, 29, 29, 37, 48, 62



## H.O.T. Problems Higher Order Thinking

7. **CCSS Find the Error** Hiroshi was finding the measures of variation of the following set of data: 89, 93, 99, 110, 128, 135, 144, 152, and 159. Find his mistake and correct it.

Sample answer: The median is correct, but Hiroshi included it when finding the third and first quartiles. The first quartile is 96, the third quartile is 148, and the interquartile range is 52.

median = 128  
first quartile = 99  
third quartile = 144  
interquartile range = 45  
range = 70



8. **CCSS Reason Abstractly** Create a list of data with at least six numbers that has an interquartile range of 15 and two outliers.

Sample answer: 6, 30, 33, 41, 45, 71

9. **CCSS Persevere with Problems** How is finding the first and third quartiles

similar to finding the median? Sample answer: The third quartile is the median of the upper half of the data and the first quartile is the median of the lower half of the data.

10. **CCSS Reason Inductively** Explain why the median is not affected by very high or very low values in the data. Sample answer: The median is the middle number of a data set. Very high values and very low values typically fall at the beginning or end of a data set. Therefore, the median is not significantly affected by these values.

11. **CCSS Reason Inductively** Determine the range and IQR of each data set. Which measure of variation tells you more about the distribution of the data values? Explain.

Data Set A	Data Set B
1, 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 17, 19, 21	1, 2, 9, 17, 17, 17, 17, 17, 17, 18, 18, 18, 19, 20, 21

Set A—range: 20; IQR: 4; Set B—range: 20; IQR: 1; Sample

answer: The IQR tells more information, specifically that the middle half of the data in Set B are closer together than the middle half of the data in Set A.