

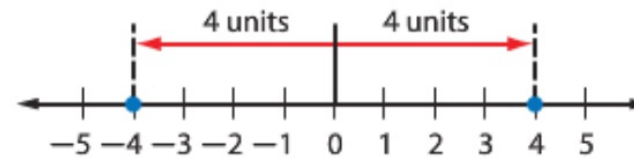
# 1-4 Solving Absolute Value Equations

## KeyConcept Absolute Value

**Words** For any real number  $a$ , if  $a$  is positive or zero, the absolute value of  $a$  is  $a$ .  
If  $a$  is negative, the absolute value of  $a$  is the opposite of  $a$ .

**Symbols** For any real number  $a$ ,  $|a| = a$  if  $a \geq 0$ , and  $|a| = -a$  if  $a < 0$ .

**Model**  $|-4| = 4$  and  $|4| = 4$



**central value**  $|x - c| = r$  **range**

**Example 1** Evaluate an Expression with Absolute Value

Evaluate  $8.4 - |2n + 5|$  if  $n = -7.5$ .

$$\begin{aligned}
 8.4 - |2n + 5| &= 8.4 - |2(-7.5) + 5| && \text{Replace } n \text{ with } -7.5. \\
 &= 8.4 - |-15 + 5| && \text{Multiply 2 and } -7.5. \\
 &= 8.4 - |-10| && \text{Add } -15 \text{ and } 5. \\
 &= 8.4 - 10 && |-10| = 10 \\
 &= -1.6 && \text{Subtract 10 from } 8.4.
 \end{aligned}$$

-2 · x

P E M D A S

④  $-2 | 3(-4) + 8 | - 4$

$-2 | (-12) + 8 | - 4$

$-2 | -4 | - 4$

$-2(4) - 4$

$-8 - 4$

$-12$

**Guided Practice**

- 1A. Evaluate  $|4x + 3| - 3\frac{1}{2}$  if  $x = -2$ . 1 $\frac{1}{2}$     1B. Evaluate  $1\frac{1}{3} - |2y + 1|$  if  $y = -\frac{2}{3}$ . 1

**Example 1** Evaluate each expression if  $x = -4$  and  $y = -9$ .

1.  $|x - 8|$  12    2.  $|7y|$  63    3.  $-3|xy|$  -108    4.  $-2|3x + 8| - 4$  -12

**Example 1** Evaluate each expression if  $a = -3$ ,  $b = -5$ , and  $c = 4.2$ .

14.  $|-3c|$            15.  $|5b|$  25    16.  $|a - b|$            17.  $|b - c|$  9.2
18.  $|3b - 4a|$            19.  $2|4a - 3c|$  49.2    20.  $-|3c - a|$            21.  $-|abc|$  -63

## Real-World Example 2 Solve an Absolute Value Equation

**TENNIS** A standard adult tennis racket has a 100-square-inch head, plus or minus 20 square inches. Write and solve an absolute value equation to determine the least and greatest possible sizes for the head of an adult tennis racket.

**Understand** We need to determine the greatest and least possible sizes for the head of a tennis racket given the middle size and the range in sizes.

**Plan** When writing an absolute value equation, the middle or *central* value is always placed inside the absolute value symbols. The *range* is always placed on the other side of the equality symbol.

$$|x - c| = r$$

Diagram illustrating the components of an absolute value equation:  $|x - c| = r$ . The expression  $c$  is labeled "central value" and the expression  $r$  is labeled "range".

**Solve**  $|x - c| = r$  Absolute value equation

$$|x - 100| = 20 \quad c = 100, \text{ and } r = 20$$

**Case 1**  $a = b$

$$x - 100 = 20$$

$$x - 100 + 100 = 20 + 100$$

$$x = 120$$

**Case 2**  $a = -b$

$$x - 100 = -20$$

$$x - 100 + 100 = -20 + 100$$

$$x = 80$$

**Check**  $|x - 100| = 20$

$$|120 - 100| \stackrel{?}{=} 20$$

$$|20| \stackrel{?}{=} 20$$

$$20 = 20 \quad \checkmark$$

$$|x - 100| = 20$$

$$|80 - 100| \stackrel{?}{=} 20$$

$$|-20| \stackrel{?}{=} 20$$

$$20 = 20 \quad \checkmark$$

22. **FOOD** To make cocoa powder, cocoa beans are roasted. The ideal temperature for roasting is  $300^{\circ}\text{F}$ , plus or minus  $25^{\circ}$ . Write and solve an equation describing the maximum and minimum roasting temperatures for cocoa beans.  $|x - 300| = 25$ ; maximum:  $325^{\circ}\text{F}$ ; minimum:  $275^{\circ}\text{F}$

$$|x - 78| = 2$$

5. **CCSS MODELING** Most freshwater tropical fish thrive if the water is within  $2^{\circ}\text{F}$  of  $78^{\circ}\text{F}$ .
- Write an equation to determine the least and greatest optimal temperatures.  $|x - 78| = 2$
  - Solve the equation you wrote in part a.
  - If your aquarium's thermometer is accurate to within plus or minus  $1^{\circ}\text{F}$ , what should the temperature of the water be to ensure that it reaches the minimum temperature? Explain.

5b.  
least:  $76^{\circ}\text{F}$ ,  
greatest:  $80^{\circ}\text{F}$



$77^{\circ}\text{F}$ ; This would ensure a minimum temperature of  $76^{\circ}\text{F}$ .



### Example 3 No Solution

Solve  $|3x - 2| + 8 = 1$ .

$$|3x - 2| + 8 = 1 \quad \text{Original equation}$$

$$|3x - 2| + 8 - 8 = 1 - 8 \quad \text{Subtract 8 from each side.}$$

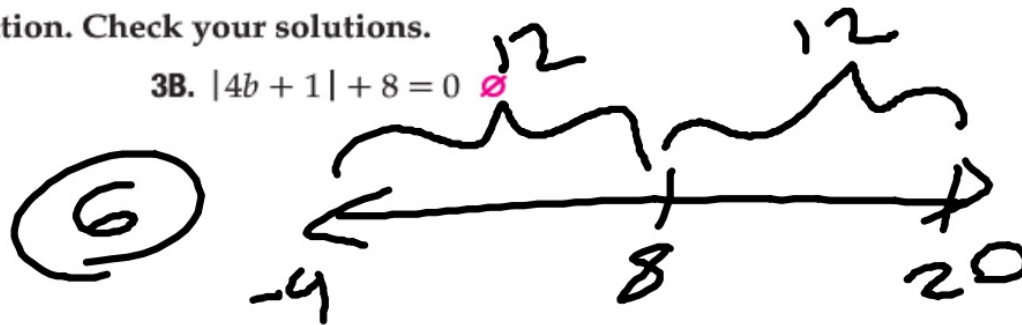
$$|3x - 2| = -7 \quad \text{Simplify.}$$

This sentence is *never* true. The solution set is  $\emptyset$ .

► **Guided Practice** Solve each equation. Check your solutions.

3A.  $-2|3a| = 6$   $\emptyset$

3B.  $|4b + 1| + 8 = 0$   $\emptyset$



**Examples 2-4** Solve each equation. Check your solutions.

6.  $|x + 8| = 12$   $\{4, -20\}$

8.  $|a - 5| + 4 = 9$   $\{10, 0\}$

10.  $3|2x - 3| - 5 = 4$   $\{3, 0\}$

12.  $|a - 4| = 3a - 6$

7.  $|y - 4| = 11$

9.  $|b - 3| + 8 = 3$

11.  $-2|5y - 1| = -10$

13.  $|b + 5| = 2b + 3$

**Examples 2-4** Solve each equation. Check your solutions.

6.  $|x + 8| = 12$  **{4, -20}**

8.  $|a - 5| + 4 = 9$  **{10, 0}**

10.  $3|2x - 3| - 5 = 4$  **{3, 0}**

12.  $|a - 4| = 3a - 6$  **{2.5}**

7.  $|y - 4| = 11$  **{15, -7}**

→ 9.  $|b - 3| + 8 = 3$  **∅**

11.  $-2|5y - 1| = -10$   **$\left\{\frac{6}{5}, -\frac{4}{5}\right\}$**

13.  $|b + 5| = 2b + 3$  **{2}**

8  $|a - 5| + 4 = 9$   
 $\quad\quad\quad -4 \quad -4$   


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 $|a - 5| = 5$   
 $a - 5 = 5$   
 $+5 \quad +5$   


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 $a = 10$

$a - 5 = -5$   
 $+5 \quad +5$   


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 $a = 0$

$a - 5 = 5$   
 $+5 \quad +5$   


---

 $a = 10$

$a - 5 = -5$   
 $+5 \quad +5$   


---

 $a = 0$

9  $|b - 3| + 8 = 3$   
 $\quad\quad\quad -8 \quad -8$   


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 $|b - 3| = -5$   
 no solution!

10.  $3|2x - 3| - 5 = 4$  **{3, 0}**

$$\begin{array}{r} +5 +5 \\ \hline 3|2x-3| = 9 \\ \hline 3 \end{array}$$

$$|2x - 3| = 3$$

$$\begin{array}{r} 2x - 3 = 3 \\ +3 +3 \\ \hline 2x = 6 \\ x = 3 \end{array}$$

$$\begin{array}{r} 2x - 3 = -3 \\ +3 +3 \\ \hline 2x = 0 \\ x = 0 \end{array}$$

13.  $|b + 5| = 2b + 3$  {2}

$$|b + 5| = 2b + 3$$

$$\begin{array}{r} b + 5 = 2b + 3 \\ -b - 3 \quad -b - 3 \\ \hline 2 = b \end{array}$$

$$\begin{array}{l} b + 5 = -(2b + 3) \\ b + 5 = -2b - 3 \\ -5 \quad \quad -5 \\ \hline b = -2b - 8 \\ +2b \quad +2b \end{array}$$

Check:

$$\left| \frac{-8}{3} + 5 \right|$$

$$\left| \frac{-8}{3} + \frac{15}{3} \right|$$

$$\left| \frac{7}{3} \right|$$

$$= 2\left(\frac{-8}{3}\right) + 3$$

$$= \frac{-16}{3} + \frac{9}{3}$$

$$\frac{-7}{3} \neq \frac{13}{3}$$

$$\frac{3b}{3} = \frac{-8}{3}$$

$$b = \frac{-8}{3}$$



7.  $|y - 4| = 11$  {15, -7}

check.

$$|-7 - 4| = 11$$

$$|-11| = 11$$

$$\textcircled{6} \quad |x+8| = 12$$

$$\begin{array}{r} x+8 = 12 \\ -8 \quad -8 \\ \hline x = 4 \end{array}$$

$$\left. \begin{array}{r} x+8 = -12 \\ -8 \quad -8 \\ \hline x = -20 \end{array} \right\}$$

### Example 4 One Solution

Solve  $|x + 10| = 4x - 8$ . Check your solutions.

**Case 1**

$$a = b$$

$$x + 10 = 4x - 8$$

$$10 = 3x - 8$$

$$18 = 3x$$

$$6 = x$$

**Case 2**

$$a = -b$$

$$x + 10 = -(4x - 8)$$

$$x + 10 = -4x + 8$$

$$5x + 10 = 8$$

$$5x = -2$$

$$x = -\frac{2}{5}$$

There appear to be two solutions, 6 and  $-\frac{2}{5}$ .



**CHECK** Substitute each value in the original equation.

$$|x + 10| = 4x - 8$$

$$|6 + 10| \stackrel{?}{=} 4(6) - 8$$

$$|16| \stackrel{?}{=} 24 - 8$$

$$16 = 16 \quad \checkmark$$

$$|x + 10| = 4x - 8$$

$$\left| -\frac{2}{5} + 10 \right| \stackrel{?}{=} 4\left(-\frac{2}{5}\right) - 8$$

$$\left| 9\frac{3}{5} \right| \stackrel{?}{=} -1\frac{3}{5} - 8$$

$$9\frac{3}{5} \neq -9\frac{3}{5} \quad \times$$

12.  $|a - 4| = 3a - 6$  {2.5}

13.  $|b + 5| = 2b + 3$  {2}

**Examples 2–4** Solve each equation. Check your solutions.

23.  $|z - 13| = 21$  **{34, -8}**

25.  $9 = |d + 5|$  **{4, -14}**

27.  $5|q + 6| = 20$  **{-2, -10}**

29.  $3|2a - 4| = 0$  **{2}**

31.  $2|3x - 4| + 8 = 6$  **∅**

33.  $-3|3t - 2| - 12 = -6$  **∅**

24.  $|w + 9| = 17$  **{8, -26}**

26.  $35 = |x - 6|$  **{-29, 41}**

28.  $-3|r + 4| = -21$  **{3, -11}**

30.  $8|5w - 1| = 0$  **{ $\frac{1}{5}$ }**

32.  $4|7y + 2| - 8 = -7$  **{ $-\frac{1}{4}, -\frac{9}{28}$ }**

34.  $-5|3z + 8| - 5 = -20$  **{ $-\frac{5}{3}, -\frac{11}{3}$ }**

- B** 35. **MONEY** The U.S. Mint produces quarters that weigh about 5.67 grams each. After the quarters are produced, a machine weighs them. If the quarter weighs 0.02 gram more or less than the desired weight, the quarter is rejected. Write and solve an equation to find the heaviest and lightest quarters the machine will approve.

**$|x - 5.67| = 0.02$ ;  
heaviest: 5.69 g;  
lightest: 5.65 g**

Evaluate each expression if  $q = -8$ ,  $r = -6$ , and  $t = 3$ .

Examples 2-4 Solve each equation. Check your solutions.

23.  $|z - 13| = 21$  {34, -8}

24.  $|w + 9| = 17$  {8, -26}

23  $|z - 13| = 21$

$$\begin{array}{r} z - 13 = 21 \\ + 13 \quad + 13 \\ \hline z = 34 \end{array}$$

$$\left\{ \begin{array}{r} z - 13 = -21 \\ + 13 \quad + 13 \\ \hline z = -8 \end{array} \right.$$

