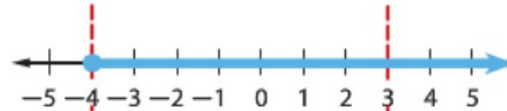# 1-6 Solving Compound and Absolute Value Inequalities

KeyConcept "And" Compound Inequalities

Words   A compound inequality containing the word *and* is true if and only if *both* inequalities are true.

Example $x \geq -4$
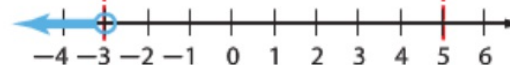

$x < 3$

$x \geq -4$ and $x < 3$

KeyConcept "Or" Compound Inequalities

Words   A compound inequality containing the word *or* is true if one or more of the inequalities is true.
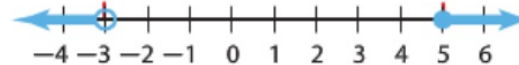
Example $x \geq 5$

$x < -3$

$x \geq 5$ or $x < -3$

Example 1 Solve an "And" Compound Inequality

Solve $8 < 3y - 7 \leq 23$. Graph the solution set on a number line.

Method 1 Solve separately.

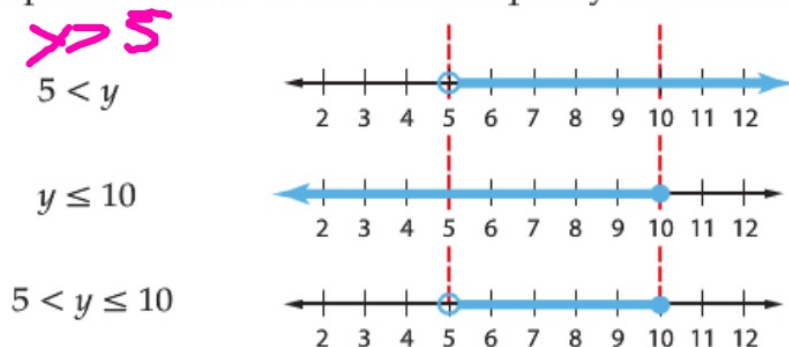Write the compound inequality using the word *and*. Then solve each inequality.

$$8 < 3y - 7 \quad \text{and} \quad 3y - 7 \leq 23$$
$$15 < 3y \qquad\qquad 3y \leq 30$$
$$5 < y \qquad\qquad y \leq 10$$
$$5 < y \leq 10$$

Method 2 Solve both together.

Solve both parts at the same time by adding 7 to each part. Then divide each part by 3.

$$8 < 3y - 7 \leq 23$$
$$15 < \quad 3y \quad \leq 30$$
$$5 < \quad y \quad \leq 10$$
$$5 < y \leq 10$$

Graph the solution set for each inequality and find their intersection.

y > 5

$5 < y$

$y \leq 10$

$5 < y \leq 10$

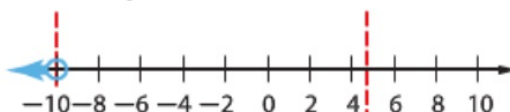The solution set is $\{y \mid 5 < y \leq 10\}$ or $(5, 10]$.

Example 2 Solve an "Or" Compound Inequality

Solve $k + 6 < -4$ or $3k \geq 14$. Graph the solution set.
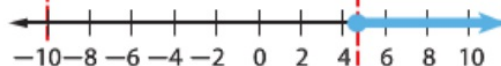
Solve each inequality separately.

$k + 6 < -4$ or $3k \geq 14$

$\quad k < -10$ $k \geq \dfrac{14}{3}$

$k < -10$

$k \geq \dfrac{14}{3}$

$k < -10$ or $k \geq \dfrac{14}{3}$
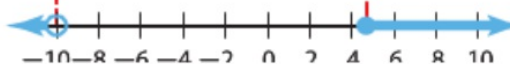
Examples 1–4 Solve each inequality. Graph the solution set on a number line. **1–10. See margin.**

1. $-4 < g + 8 < 6$

2. $-9 \leq 4y - 3 \leq 13$

3. $z + 6 > 3$ or $2z < -12$

4. $m - 7 \geq -3$ or $-2m + 1 \geq 11$

$6 < k < 10$

Example 3 Solve Absolute Value Inequalities

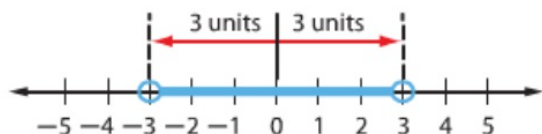**Solve each inequality. Graph the solution set on a number line.**

a. $|x| < 3$

$|x| < 3$ means that the distance between x and 0 on a number line is less than 3 units. To make $|x| < 3$ true, substitute numbers for x that are fewer than 3 units from 0.

3 units | 3 units

−5 −4 −3 −2 −1 0 1 2 3 4 5

> Notice that the graph of $|x| < 3$ is the same as the graph of $x > -3$ and $x < 3$.

All of the numbers between −3 and 3 are less than 3 units from 0. The solution set is $\{x \mid -3 < x < 3\}$ or $(-3, 3)$.

b. $|x| > 5$

$|x| > 5$ means that the distance between x and 0 on a number line is more than 5 units. To make $|x| > 5$ true, substitute numbers for x that are more than 5 units from 0.
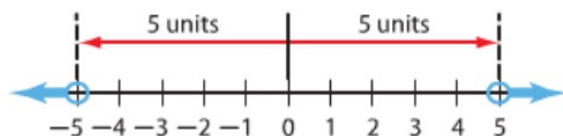
5 units   5 units

−5 −4 −3 −2 −1 0 1 2 3 4 5

> Notice that the graph of $|x| > 5$ is the same as the graph of $x < -5$ or $x > 5$.

Example 4 Solve a Multi-Step Absolute Value Inequality

Solve $|6y - 5| \geq 13$. Graph the solution set on a number line.

$|6y - 5| \geq 13$ is equivalent to $6y - 5 \geq 13$ or $6y - 5 \leq -13$. Solve the inequality.

$6y - 5 \geq 13$ or	$6y - 5 \leq -13$	Rewrite the inequality.
$6y \geq 18$	$6y \leq -8$	Add 5 to each side.
$y \geq 3$	$y \leq -\dfrac{8}{6}$ or $-\dfrac{4}{3}$	Divide each side by 6.

The solution set is $\left\{ y \mid y \leq -\dfrac{4}{3} \text{ or } y \geq 3 \right\}$ or $\left(-\infty, -\dfrac{4}{3} \right] \cup [3, \infty)$.

5. $|c| \geq 8$

7. $|z| < 6$

9. $|3v + 5| > 14$

6. $|q| \geq -1$

8. $|x| \leq -4$

10. $|4t - 3| \leq 7$

5. $\{ c \mid c \geq 8 \text{ or } c \leq -8 \}$

6. {all real numbers}

7. $\{ z \mid -6 < z < 6 \}$

8. $\varnothing$

9. $\left\{ v \mid v > 3 \text{ or } v < -\dfrac{19}{3} \right\}$

10. $\{ t \mid -1 \leq t \leq 2.5 \}$

⑨ $|3v + 5| > 14$

$3v + 5 > 14$
$\quad -5 \quad -5$
$\overline{3v = 9}$
$v > 3$

$3v + 5 < -14$
$\quad -5 \quad -5$
$\overline{3v = -19}$
$v < \dfrac{-19}{3}$

$-\dfrac{19}{3} \quad 0 \quad 3$

MONEY Amanda is apartment hunting in a specific area. She discovers that the average monthly rent for a 2-bedroom apartment is $600 a month, but the actual price could differ from the average as much as $225 a month.

a. Write an absolute value inequality to describe this situation.

Let r = average monthly rent. $|600 - r| \leq 225$

b. Solve the inequality to find the range of monthly rent.

Rewrite the absolute value inequality as a compound inequality. Then solve for r.

$$-225 \leq 600 - r \qquad \leq 225$$

$$-225 - 600 \leq 600 - r - 600 \leq 225 - 600$$

$$-825 \leq \qquad -r \qquad \leq -375$$

$$825 \geq \qquad r \qquad \geq 375$$

The solution set is $\{r \mid 375 \leq r \leq 825\}$ or $[375, 825]$. Thus, monthly rent could fall between $375 and $825, inclusive.

Example 5   **22.** **CCSS** **MODELING** Forensic scientists use the equation $h = 2.6f + 47.2$ to estimate the height h of a woman given the length in centimeters f of her femur bone.

a. Suppose the equation has a margin of error of ± 3 centimeters. Write an inequality to represent the height of a woman given the length of her femur bone. $|2.6f + 47.2| < 3$

b. If the length of a female skeleton's femur is 50 centimeters, write and solve an absolute value inequality that describes the woman's height in centimeters.
$|h - 177.2| < 3;\ 174.2\ \text{cm} < h < 180.2\ \text{cm}$

Example 5

11. MONEY Khalid is considering several types of paint for his bedroom. He estimates that he will need between 2 and 3 gallons. The table at the right shows the price per gallon for each type of paint Khalid is considering. Write a compound inequality and determine how much he could be spending.
$43.96 \leq c \leq 77.94$; **between $43.96 and $77.94**

Paint Type	Price per Gallon
Flat	$21.98
Satin	$23.98
Semi-Gloss	$24.98
Gloss	$25.98

Practice and Problem Solving

Examples 1–4 Solve each inequality. Graph the solution set on a number line.

12–21. See Chapter 1 Answer Appendix.

12. $8 < 2v - 4 < 16$

13. $-7 \leq 4d - 3 \leq -1$

14. $4r + 3 < -6$ or $3r - 7 > 2$

15. $6y - 3 < -27$ or $-4y + 2 < -26$

16. $|6h| < 12$

17. $|-4k| > 16$

18. $|3x - 4| > 10$

19 $|8t + 3| \leq 4$

20. $|-9n - 3| < 6$

**21.** $|-5j - 4| \geq 12$

Example 5

22. CCSS MODELING Forensic scientists use the equation $h = 2.6f + 47.2$ to estimate the height h of a woman given the length in centimeters f of her femur bone.

a. Suppose the equation has a margin of error of ±3 centimeters. Write an inequality to represent the height of a woman given the length of her femur bone. $|2.6f + 47.2| < 3$

b. If the length of a female skeleton's femur is 50 centimeters, write and solve an absolute value inequality that describes the woman's height in centimeters.
$|h - 177.2| < 3$; **174.2 cm < h < 180.2 cm**

Lesson 1-6

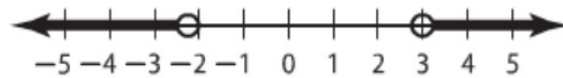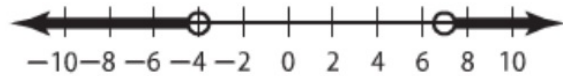**12.** $\{v \mid 6 < v < 10\}$
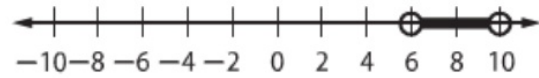
13. $\{d \mid -1 \le d \le 0.5\}$

14. $\left\{r \mid r < -\dfrac{9}{4} \text{ or } r > 3\right\}$

15. $\{y \mid y < -4 \text{ or } y > 7\}$

16. $\{h \mid -2 < h < 2\}$

17. $\{k \mid -4 > k \text{ or } k > 4\}$

18. $\left\{x \mid x > \dfrac{14}{3} \text{ or } x < -2\right\}$
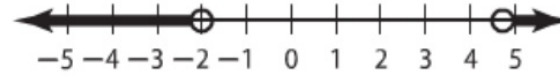
19. $\left\{t \mid -\dfrac{7}{8} \le t \le \dfrac{1}{8}\right\}$

20. $\left\{n \mid -1 < n < \dfrac{1}{3}\right\}$

21. $\left\{j \mid j \ge \dfrac{8}{5} \text{ or } j \le -\dfrac{16}{5}\right\}$