LESSON 11–4 Multiplying and Dividing Rational Expressions

KeyConcept Multiplying Rational Expressions

Words Let a, b, c, and d be polynomials with $b \neq 0$ and $d \neq 0$. Then, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

Example $\frac{x}{2x-3} \cdot \frac{4x^2}{5} = \frac{4x^3}{5(2x-3)}$

KeyConcept Dividing Rational Expressions

Symbols Let a, b, c, and d be polynomials with $b \neq 0$, $c \neq 0$, and $d \neq 0$. Then, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

Example $\frac{x-3}{x} \div \frac{2x^2}{5} = \frac{x-3}{x} \cdot \frac{5}{2x^2} = \frac{5(x-3)}{2x^3}$

Now

- Multiply rational expressions.
- Divide rational expressions.

Example 1 Multiply Expressions Involving Monomials

Find each product.

a. $\frac{r^2x}{9t^3} \cdot \frac{3t^4}{r}$

Divide by the common factors before multiplying.

$$\frac{r^2x}{9t^3} \cdot \frac{3t^4}{r} = \frac{\overset{r}{\cancel{r^2}}x}{\underset{3}{\cancel{9}}\underset{1}{\cancel{t^3}}} \cdot \frac{\overset{1}{\cancel{3}}\overset{t}{\cancel{t^4}}}{\underset{1}{\cancel{r}}}$$ Divide by the common factors 3, r, and t^3.

$$= \frac{rxt}{3}$$ Simplify.

b. $\frac{a+4}{a^2} \cdot \frac{a}{a^2+2a-8}$

$$\frac{a+4}{a^2} \cdot \frac{a}{a^2+2a-8} = \frac{a+4}{a^2} \cdot \frac{a}{(a+4)(a-2)}$$ Factor the denominator.

$$= \frac{\overset{1}{\cancel{a+4}}}{\underset{a}{\cancel{a^2}}} \cdot \frac{\overset{1}{\cancel{a}}}{\underset{1}{\cancel{(a+4)}}(a-2)}$$ The GCF is $a(a+4)$.

$$= \frac{1}{a(a-2)} \text{ or } \frac{1}{a^2-2a}$$ Simplify.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Find each product.**

1. $\frac{2x^3}{7x} \cdot \frac{14}{x}$ **4x**

2. $\frac{3ab}{4c^4} \cdot \frac{16c^2}{9b}$ $\frac{4a}{3c^2}$

3. $\frac{t}{(t-5)(t+5)} \cdot \frac{t+5}{6}$ $\frac{t}{6(t-5)}$

4. $\frac{8}{r+1} \cdot \frac{r^2-1}{2}$ $4(r-1)$

Example 3 Divide by a Rational Expression

Find $\frac{4}{15n^3} \div \frac{12}{25n}$.

$$\frac{4}{15n^3} \div \frac{12}{25n} = \frac{4}{15n^3} \cdot \frac{25n}{12}$$ Multiply by $\frac{25n}{12}$, the reciprocal of $\frac{12}{25n}$.

$$= \frac{\overset{1}{\cancel{4}}}{\underset{3n^2}{\cancel{15n^3}}} \cdot \frac{\overset{5}{\cancel{25n}}}{\underset{3}{\cancel{12}}}$$ Divide by common factors 4, 5, and n.

$$= \frac{5}{9n^2}$$ Simplify.

(2) $\frac{2}{3x^3} \cdot \frac{x}{4} = \frac{2}{3x}$

Examples 3–5 Find each quotient.

7. $\frac{8}{3x^2} \div \frac{4}{x}$ $\frac{2}{3x}$

8. $\frac{c^5}{2} \div \frac{c^3}{6d^2}$ $3c^2d^2$

9. $\frac{b^2 + 6b + 5}{6b + 6} \div (b + 5)$ $\frac{1}{6}$

10. $\frac{2x + 8}{x + 3} \div \frac{x + 4}{x^2 + 6x + 9}$ $2(x + 3)$

(9) $\frac{(b+1)(b+5)}{6(b+1)} \cdot \frac{1}{b+5} = \frac{(b+1)(b+5)}{6(b+1)(b+5)} = \frac{1}{6}$

Examples 3–5 Find each quotient.

7. $\frac{8}{3x^2} \div \frac{4}{x}$ $\frac{2}{3x}$

8. $\frac{c^5}{2} \div \frac{c^3}{6d^2}$ $3c^2d^2$

9. $\frac{b^2 + 6b + 5}{6b + 6} \div (b + 5)$ $\frac{1}{6}$

10. $\frac{2x + 8}{x + 3} \div \frac{x + 4}{x^2 + 6x + 9}$ $2(x + 3)$ $2x+6$ ✓

8 $\frac{c^2}{2} \cdot \frac{3}{c^3} \cdot \frac{6d^2}{1} = 3c^{-1}d^2$

10 $\frac{2x+8}{x+3} \cdot \frac{x^2+6x+9}{x+4}$

$\frac{2(x+4)}{(x+3)} \cdot \frac{(x+3)(x+3)}{(x+4)} = 2(x+3)$

Example 4 Divide by Rational Expressions and Polynomials

Find each quotient.

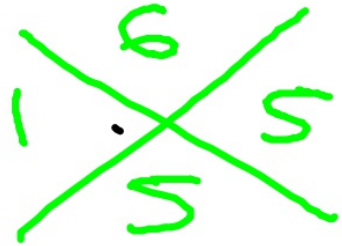
a. $\dfrac{2x+6}{x^2} \div (x+3)$

$$\frac{2x+6}{x^2} \div (x+3) = \frac{2x+6}{x^2} \div \frac{x+3}{1}$$ Write the binomial as a fraction.

$$= \frac{2x+6}{x^2} \cdot \frac{1}{x+3}$$ Multiply by the reciprocal of $x+3$.

$$= \frac{2(x+3)}{x^2} \cdot \frac{1}{x+3}$$ Factor $4x+6$.

$$= \frac{2(\cancel{x+3})}{x^2} \cdot \frac{1}{\cancel{x+3}} \text{ or } \frac{2}{x^2}$$ Divide out the common factor and simplify.



(9) $\dfrac{(b+1)(b+5)}{6(b+1)}$ x

Examples 3–5 **Find each quotient.**

7. $\dfrac{8}{3x^2} \div \dfrac{4}{x}$

8. $\dfrac{c^5}{2} \div \dfrac{c^3}{6d^2}$ $3c^2d^2$

9. $\dfrac{b^2+6b+5}{6b+6} \div (b+5)$ $\frac{1}{6}$

10. $\dfrac{2x+8}{x+3} \div \dfrac{x+4}{x^2+6x+9}$ $2(x+3)$

Example 4 Divide by Rational Expressions and Polynomials

Find each quotient.

b. $\dfrac{a-2}{4a+4} \div \dfrac{a+5}{a+1}$

$$\frac{a-2}{4a+4} \div \frac{a+5}{a+1} = \frac{a-2}{4a+4} \cdot \frac{a+1}{a+5}$$

Multiply by the reciprocal.

$$= \frac{a-2}{4(a+1)} \cdot \frac{\overset{1}{\cancel{a+1}}}{a+5}$$

Factor $4a+4$.

$$= \frac{a-2}{4(a+5)}$$

The GCF is $a+1$ and simplify.

Examples 3–5 **Find each quotient.**

7. $\dfrac{8}{3x^2} \div \dfrac{4}{x}$ $\dfrac{2}{3x}$

8. $\dfrac{c^5}{2} \div \dfrac{c^3}{6d^2}$ $3c^2d^2$

9. $\dfrac{b^2+6b+5}{6b+6} \div (b+5)$ $\dfrac{1}{6}$

10. $\dfrac{2x+8}{x+3} \div \dfrac{x+4}{x^2+6x+9}$ $2(x+3)$

Example 5 Expression Involving Polynomials

Find $\dfrac{y-3}{y^2-10y+16} \div \dfrac{y^2-9}{y-8}$.

$$\frac{y-3}{y^2-10y+16} \div \frac{y^2-9}{y-8}$$

$$= \frac{y-3}{y^2-10y+16} \cdot \frac{y-8}{y^2-9}$$

Multiply by the reciprocal, $\frac{y-8}{y^2-9}$.

$$= \frac{y-3}{(y-2)(y-8)} \cdot \frac{y-8}{(y-3)(y+3)}$$

Factor $y^2-10y+16$ and y^2-9.

$$= \frac{\cancel{y-3}^{\,1}}{(y-2)\cancel{(y-8)}^{\,1}} \cdot \frac{\cancel{y-8}^{\,1}}{\cancel{(y-3)}^{\,1}(y+3)}$$

The GCF is $(y-3)(y-8)$.

$$= \frac{1}{(y-2)(y+3)}$$

Simplify.

Examples 3–5 Find each quotient.

7. $\dfrac{8}{3x^2} \div \dfrac{4}{x}$ $\dfrac{2}{3x}$

8. $\dfrac{c^5}{2} \div \dfrac{c^3}{6d^2}$ $3c^2d^2$

9. $\dfrac{b^2+6b+5}{6b+6} \div (b+5)$ $\dfrac{1}{6}$

10. $\dfrac{2x+8}{x+3} \div \dfrac{x+4}{x^2+6x+9}$ $2(x+3)$

Practice and Problem Solving

Extra Practice is on page R11.

Example 1 Find each product.

11. $\frac{10n^2}{4} \cdot \frac{2}{n}$ **5n**

12. $\frac{12c^3}{21b} \cdot \frac{14b^2}{6c}$ **$\frac{4bc^2}{3}$**

13. $\frac{x^5y}{2z^3} \cdot \frac{18z^4}{xy}$ **$9x^4z$**

14. $\frac{5c^3d}{c^4d} \cdot \frac{f^2d^3c}{10cf^4}$ **$\frac{d^3}{2cf^2}$**

15. $\frac{9}{t-2} \cdot \frac{(t+2)(t-2)}{3}$ **$3(t+2)$**

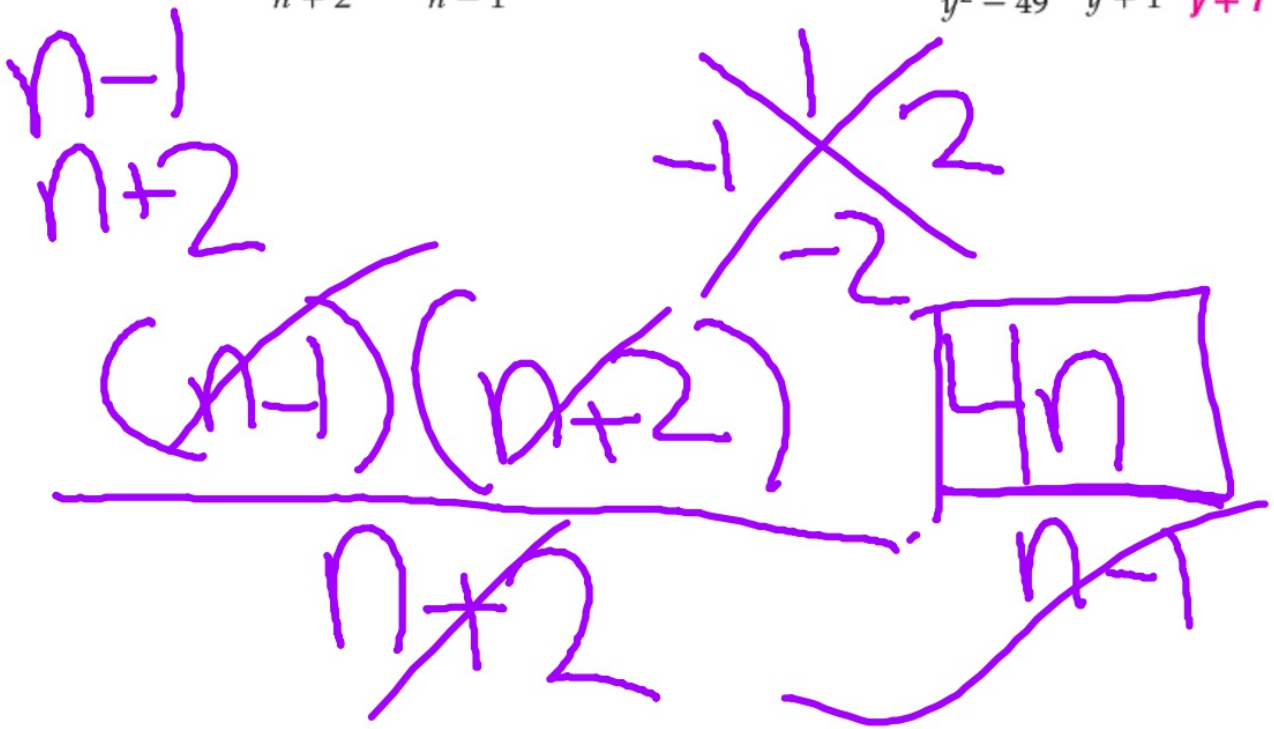
16. $\frac{(a+4)(a-5)}{a^2} \cdot \frac{6a}{a+4}$ **$\frac{6(a-5)}{a}$**

17. $\frac{(k+6)(k-1)}{k+2} \cdot \frac{(k+1)(k+2)}{(k+1)(k-1)}$ **$k+6$**

18. $\frac{(r-8)(r+3)}{r} \cdot \frac{2r}{(r+8)(r+3)}$ **$\frac{2(r-8)}{r+8}$**

19. $\frac{n^2+n-2}{n+2} \cdot \frac{4n}{n-1}$ **$4n$**

20. $\frac{y^2-1}{y^2-49} \cdot \frac{y-7}{y+1}$ **$\frac{y-1}{y+7}$**



Examples 3–5 Find each quotient.

23. $\frac{x^5}{y} \div \frac{x}{y^2}$

24. $\frac{3r^4}{k^2} \div \frac{18r^3}{k}$ $\frac{r}{6k}$

25. $\frac{21b^3}{4c^2} \div \frac{7}{6c^2}$

26. $\frac{f^4g^2h}{x^2y} \div f^3g$ $\frac{fgh}{x^2y}$

27. $\frac{6b-12}{b+5} \div (12b+18)$

28. $\frac{k+3}{k+2} \div \frac{k}{5k+10}$

29. $\frac{5x^2}{x^2-5x+4} \div \frac{10x}{x-1}$

30. $\frac{n^2+7n+12}{16n^2} \div \frac{n+3}{2n}$ $\frac{n+4}{8n}$

31. $\frac{r+2}{r+1} \div \frac{4}{r^2+3r+2}$

32. $\frac{3a}{a^2+2a+1} \div \frac{a-1}{a+1}$ $\frac{3a}{(a+1)(a-1)}$

28. $\frac{5(k+3)}{k}$