



New Vocabulary

- least common multiple (LCM)
- least common denominator (LCD)

Now

- Add and subtract rational expressions with like denominators.
- Add and subtract rational expressions with unlike denominators.

 KeyConcept Add or Subtract Rational Expressions with Like Denominators

Let a , b , and c be polynomials with $c \neq 0$.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example 1 Add Rational Expressions with Like Denominators

Find $\frac{5n}{n+3} + \frac{15}{n+3}$.

$$\frac{5n}{n+3} + \frac{15}{n+3} = \frac{5n+15}{n+3}$$

The common denominator is $n+3$.

$$= \frac{5(n+3)}{n+3}$$

Factor the numerator.

$$= \frac{5(n+3)}{n+3}$$

Divide by the common factor, $n+3$.

$$= \frac{5}{1}$$

Simplify.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Find each sum or difference.

1. $\frac{3}{7n} + \frac{2}{7n}$ **5**

2. $\frac{x+8}{2} + \frac{x}{2}$ **x+4**

3. $\frac{14r}{9-r} - \frac{2r}{r-9}$

4. $\frac{7}{5t} - \frac{3+t}{5t}$ **4-t**

 KeyConcept Add or Subtract Rational Expressions with Like Denominators

Let a , b , and c be polynomials with $c \neq 0$.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example 2 Subtract Rational Expressions with Like Denominators

Find $\frac{3m-5}{m+4} - \frac{4m+2}{m+4}$.

$$\begin{aligned}\frac{3m-5}{m+4} - \frac{4m+2}{m+4} &= \frac{(3m-5) - (4m+2)}{m+4} \\ &= \frac{(3m-5) + [-(4m+2)]}{m+4}\end{aligned}$$

The common denominator is $m+4$.

The additive inverse of $(4m+2)$ is $-(4m+2)$.

Remember to distribute! $= \frac{3m-5-4m-2}{m+4}$

$$\begin{aligned}&= \frac{-m-7}{m+4}\end{aligned}$$

Distributive Property

Simplify.

Check Your Understanding



= Step-by-Step Solutions begin on page R13.

Examples 1–3 Find each sum or difference. 3. $\frac{16r}{9-r}$

1. $\frac{3}{7n} + \frac{2}{7n}$ $\frac{5}{7n}$

2. $\frac{x+8}{2} + \frac{x}{2}$ $x+4$

3. $\frac{14r}{9-r} - \frac{2r}{r-9}$

4. $\frac{7}{5t} - \frac{3+t}{5t}$ $\frac{4-t}{5t}$

① $\frac{3}{7n} + \frac{2}{7n}$ ② $\frac{x+8}{2} + \frac{x}{2} = \frac{2x+8}{2}$
 $= 2(x+4)$

$$= \frac{vn - un}{n - 4} \text{ or } -\frac{un}{n - 4}$$

Subtract the numerators and simplify.

Check Your Understanding



= Step-by-Step Solutions begin on page R13.

Examples 1–3 Find each sum or difference. 3. $\frac{16r}{9-r}$

1. $\frac{3}{7n} + \frac{2}{7n}$ $\frac{5}{7n}$

2. $\frac{x+8}{2} + \frac{x}{2}$ $x+4$

3. $\frac{14r}{9-r} - \frac{2r}{r-9}$

4. $\frac{7}{5t} - \frac{3+t}{5t}$ $\frac{4-t}{5t}$

③ $\frac{14r}{9-r} + \frac{2r}{9-r}$

$= \frac{16r}{9-r}$

$\frac{2r}{-(-r+9)}$



Example 4 LCMs of Polynomials

Find the LCM of each pair of polynomials.

a. $6x$ and $4x^3$

Step 1 Find the prime factors of each expression.

$$6x = 2 \cdot 3 \cdot x$$

$$4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$$

Step 2 Use each prime factor, 2, 3, and x , the greatest number of times it appears in either of the factorizations.

$$6x = 2 \cdot 3 \cdot x$$

$$4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$$

$$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \text{ or } 12x^3$$

b. $n^2 + 5n + 4$ and $(n + 1)^2$

$$n^2 + 5n + 4 = (n + 1)(n + 4) \quad \text{Factor each expression.}$$

$$(n + 1)^2 = (n + 1)(n + 1)$$

$(n + 1)$ is a factor twice in the second expression. $(n + 4)$ is a factor once.

$$\text{LCM} = (n + 1)(n + 1)(n + 4) \text{ or } (n + 1)^2(n + 4)$$

(again)

$$\begin{array}{c} (x+2) \\ (x+2) \end{array} \quad | \quad \begin{array}{c} (x-1) \cancel{(x+2)} \\ (x-1) \cancel{(x+2)} \end{array} \quad | \quad \begin{array}{c} (x-1) (x+2) \\ (x-1) (x+2) \end{array}$$

Example 4 Find the LCM of each pair of polynomials.

5. $3t, 8t^2$ **24t²**

6. $5m + 15, 2m + 6$ **10(m + 3)**

7. $(x^2 - 8x + 7), (x^2 + x - 2)$ **(x - 7)(x - 1)(x + 2)**

(x-1)(x-7) **(x-1)(x+2)** **LCD?**
(x-7)(x-1)(x+2)

8

$$\frac{6}{n^4} + \frac{2}{n^2} \cdot \frac{n^2}{n^2} = \frac{6 + 2n^2}{n^4}$$

9 $\frac{3(3y)}{(4x)5y} + \frac{(2x4x)}{(5y)4x}$

$$\frac{15y}{20xy} + \frac{8x}{20xy}$$

Examples 5–7 Find each sum or difference.

8. $\frac{6}{n^4} + \frac{2}{n^2}$ $\frac{6 + 2n^2}{n^4}$

9. $\frac{3}{4x} + \frac{2}{5y}$ $\frac{15y + 8x}{20xy}$

10. $\frac{4}{5n} - \frac{1}{10n^3}$ $\frac{8n^2 - 1}{10n^3}$

11. $\frac{8}{3c} - \frac{-5}{6d}$

12. $\frac{a}{a+4} + \frac{6}{a+2}$

13. $\frac{x}{x-3} - \frac{3}{x+2}$

10 $\frac{4 \cdot 2n^2}{5n \cdot 2n^2} - \frac{1}{10n^3} = \frac{8n^2}{10n^3} - \frac{1}{10n^3}$



12. $\frac{a^2 + 8a + 24}{(a+4)(a+2)}$

Examples 5–7 Find each sum or difference.

8. $\frac{6}{n^4} + \frac{2}{n^2} \frac{6+2n^2}{n^4}$

9. $\frac{3}{4x} + \frac{2}{5y} \frac{15y+8x}{20xy}$

10. $\frac{4}{5n} - \frac{1}{10n^3} \frac{8n^2-1}{10n^3}$

11. $\frac{8}{3c} - \frac{-5}{6d} \frac{16d+5c}{6cd}$

12. $\frac{a}{a+4} + \frac{6}{a+2}$

13. $\frac{x}{x-3} - \frac{3}{x+2}$

$$\textcircled{11} \quad \frac{2 \cdot 8 \cdot d}{2 \cdot 3c \cdot d} + \frac{5 \cdot c}{6d \cdot c} = \frac{16d+5c}{6cd}$$

$$\textcircled{12} \quad \frac{a(a+2)}{(a+4)(a+2)} + \frac{6(a+4)}{(a+2)(a+4)}$$

$$\frac{a^2+2a+6a+24}{(a+4)(a+2)}$$

simplify

$$= \frac{a^2+8a+24}{(a+4)(a+2)}$$

$$9. \frac{3}{4x} + \frac{2}{5y} \quad \frac{15y+8x}{20xy}$$

LCD: 20xy

$$\begin{aligned} & \frac{5y \cdot 3}{5y \cdot 4x} + \frac{2 \cdot 4x}{5y \cdot 4x} \\ & \frac{15y}{20xy} + \frac{8x}{20xy} = \frac{15y + 8x}{20xy} \end{aligned}$$

$$13. \frac{x}{x-3} - \frac{3}{x+2}$$

LCM: $(x-3)(x+2)$

$$\begin{aligned} & \frac{x}{(x-3)(x+2)} - \frac{3}{(x+2)(x-3)} \\ & \left(\frac{x^2 + 2x + 0}{x^2 - x - 6} \right) - \left(\frac{3x - 1}{x^2 - x - 6} \right) \\ & = \frac{x^2 - x + 0}{x^2 - x - 6} \end{aligned}$$

Diagram illustrating the subtraction of the fractions:

- The first fraction $\frac{x}{(x-3)(x+2)}$ is shown with its denominator $(x-3)(x+2)$ circled in orange. A curved arrow above the fraction points to the LCM $(x-3)(x+2)$.
- The second fraction $\frac{3}{(x+2)(x-3)}$ is shown with its denominator $(x+2)(x-3)$ circled in orange. A curved arrow above the fraction points to the LCM $(x-3)(x+2)$.
- The numerators x and 3 are circled in blue.
- A yellow circle highlights the minus sign between the two fractions.
- A pink bracket groups the first fraction's numerator and denominator, and another pink bracket groups the second fraction's numerator and denominator.
- A yellow circle highlights the term -1 in the second fraction's numerator.
- A pink bracket groups the numerators $x^2 + 2x + 0$ and $3x - 1$.
- A pink bracket groups the denominators $x^2 - x - 6$ and $x^2 - x - 6$.
- An oval encloses the result $= \frac{x^2 - x + 0}{x^2 - x - 6}$.
- A diagram on the right shows the LCM $(x-3)(x+2)$ as a rectangle divided into four quadrants:
 - Top-left quadrant: x
 - Top-right quadrant: $x^2 | 2x$
 - Bottom-left quadrant: -3
 - Bottom-right quadrant: $-3x | -6$

Practice and Problem Solving

Extra Practice Is on page R11.

Examples 1–3 Find each sum or difference.

15. $\frac{a}{4} + \frac{3a}{4}$ **a**

18. $\frac{11}{4r} - \frac{-1}{4r}$ **$\frac{3}{r}$**

22. **$\frac{5x-1}{11x-3}$**

25. **$\frac{-w+5}{8w}$**

16. $\frac{1}{6m} + \frac{5m}{6m}$ **$\frac{1+5m}{6m}$**

19. $\frac{8b}{ab} + \frac{3a}{ab}$ **$\frac{8b+3a}{ab}$**

21. $\frac{3c-7}{2c-1} + \frac{2c+1}{1-2c}$ **$\frac{c-8}{2c-1}$**

24. $\frac{5x+2}{2x+5} - \frac{x-8}{2x+5}$ **2**

22. $\frac{15x}{33x-9} + \frac{3}{9-33x}$

25. $\frac{w+2}{8w} - \frac{2w-3}{8w}$

17. $\frac{5y}{6} - \frac{y}{6}$ **$\frac{2y}{3}$**

20. $\frac{t+2}{3} + \frac{t+5}{3}$ **$\frac{2t+7}{3}$**

23. $\frac{n+6}{10} - \frac{n+1}{10}$ **$\frac{1}{2}$**

26. $\frac{3a+1}{a-1} - \frac{a+4}{a-1}$ **$\frac{2a-3}{a-1}$**

Example 4 Find the LCM of each pair of polynomials.

27. x^3y, x^2y^2 **x^3y^2**

28. $5ab, 10b$ **$10ab$**

30. $2n - 10, 4n - 20$

$4(n-5)$

31. $(x^2 + 9x + 18), x + 3$

$(x+6)(x+3)$

Examples 5, 7 Find each sum or difference.

33. $\frac{5}{4x} + \frac{1}{10x}$ **$\frac{27}{20x}$**

36. $\frac{6g}{g+5} - \frac{g-2}{2g}$

39. $\frac{-2}{7r} + \frac{4}{t}$ **$\frac{-2t+28r}{7rt}$**

42. $\frac{4}{a} - \frac{1}{3a}$ **$\frac{11}{3a}$**

45. $\frac{w-3}{w^2-w-20} + \frac{w}{w+4}$ **$\frac{w^2-4w-3}{(w+4)(w-5)}$**

47. $\frac{2x}{x^2+8x+15} - \frac{x+3}{x+5}$ **$\frac{-x^2-4x-9}{(x+3)(x+5)}$**

34. $\frac{6}{r} + \frac{2}{r^2}$ **$\frac{6r+2}{r^2}$**

37. $\frac{7}{4k+8} - \frac{k}{k+2}$ **$\frac{7-4k}{4(k+2)}$**

40. $\frac{n}{n-2} + \frac{n}{n+1}$

43. $\frac{6}{5t^2} - \frac{2}{3t}$ **$\frac{18-10t}{15t^2}$**

46. $\frac{n}{2n+10} + \frac{1}{n^2-25}$ **$\frac{n^2-5n+2}{2(n+5)(n-5)}$**

48. $\frac{r-3}{r^2+6r+9} - \frac{r-9}{r^2-9}$ **$\frac{36}{(r-3)(r+3)^2}$**



29. $(3r-1), (r+2)$ **$(3r-1)(r+2)$**

32. $(k^2-2k-8), (k+2)^2$
 $(k-4)(k+2)^2$

35. $\frac{3}{2a} + \frac{1}{5b}$ **$\frac{15b+2a}{10ab}$**

36. $\frac{11g^2-3g+10}{2g(g+5)}$

38. $\frac{5}{2d+2} - \frac{d}{d+5}$

39. $\frac{d}{d+5} + \frac{7}{d-1}$ **$\frac{-2d^2+3d+25}{(2d+2)(d+5)}$**

40. $\frac{2n^2-n}{(n-2)(n+1)}$

41. $\frac{d^2+6d+35}{(d+5)(d-1)}$