New Vocabulary

- least common multiple (LCM)
- least common denominator (LCD)

Now

- Add and subtract rational expressions with like denominators.
- Add and subtract rational expressions with unlike denominators.

KeyConcept Add or Subtract Rational Expressions with Like Denominators

Let a, b, and c be polynomials with $c \neq 0$.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example 1 Add Rational Expressions with Like Denominators

PT

Find $\frac{5n}{n+3} + \frac{15}{n+3}$.

$\frac{5n}{n+3} + \frac{15}{n+3} = \frac{5n+15}{n+3}$ The common denominator is $n+3$.

$= \frac{5(n+3)}{n+3}$ Factor the numerator.

$= \frac{5(\cancel{n+3})}{\cancel{n+3}}$ Divide by the common factor, $n+3$.

$= \frac{5}{1}$ or 5 Simplify.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Find each sum or difference.

1. $\frac{3}{7n} + \frac{2}{7n}$ $\frac{5}{7n}$

2. $\frac{x+8}{2} + \frac{x}{2}$ $x+4$

3. $\frac{14r}{9-r} - \frac{2r}{r-9}$

4. $\frac{7}{5t} - \frac{3+t}{5t}$ $\frac{4-t}{5t}$

KeyConcept Add or Subtract Rational Expressions with Like Denominators

Let a, b, and c be polynomials with $c \neq 0$.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example 2 Subtract Rational Expressions with Like Denominators

Find $\frac{3m-5}{m+4} - \frac{4m+2}{m+4}$.

$$\frac{3m-5}{m+4} - \frac{4m+2}{m+4} = \frac{(3m-5)-(4m+2)}{m+4}$$ The common denominator is $m+4$.

$$= \frac{(3m-5)+[-(4m+2)]}{m+4}$$ The additive inverse of $(4m+2)$ is $-(4m+2)$.

$$= \frac{3m-5-4m-2}{m+4}$$ Distributive Property

$$= \frac{-m-7}{m+4}$$ Simplify.

Remember to distribute!

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Find each sum or difference.

1. $\frac{3}{7n} + \frac{2}{7n}$ $\frac{5}{7n}$

2. $\frac{x+8}{2} + \frac{x}{2}$ $x+4$

3. $\frac{14r}{9-r} - \frac{2r}{r-9}$ $\frac{16r}{9-r}$

4. $\frac{7}{5t} - \frac{3+t}{5t}$ $\frac{4-t}{5t}$

① $\frac{3}{7n} + \frac{2}{7n}$ ② $\frac{x+8}{2} + \frac{x}{2} = \frac{2x+8}{2}$

$= \frac{2(x+4)}{2}$

$= \frac{3n - 6n}{n-4}$ or $-\frac{3n}{n-4}$ Subtract the numerators and simplify.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Find each sum or difference. 3. $\frac{16r}{9-r}$

1. $\frac{3}{7n} + \frac{2}{7n}$ $\frac{5}{7n}$

2. $\frac{x+8}{2} + \frac{x}{2}$ $x+4$

3. $\frac{14r}{9-r} - \frac{2r}{r-9}$

4. $\frac{7}{5t} - \frac{3+t}{5t}$ $\frac{4-t}{5t}$

$\frac{2r}{-(-r+9)}$

(3) $\frac{14r}{9-r} + \frac{2r}{9-r}$

$= \frac{16r}{9-r}$

Example 4 LCMs of Polynomials

Find the LCM of each pair of polynomials.

a. $6x$ and $4x^3$

Step 1 Find the prime factors of each expression.

$6x = 2 \cdot 3 \cdot x$ $\qquad 4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$

Step 2 Use each prime factor, 2, 3, and x, the greatest number of times it appears in either of the factorizations.

$6x = 2 \cdot 3 \cdot x$ $\qquad 4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$

LCM $= 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$ or $12x^3$

b. $n^2 + 5n + 4$ and $(n + 1)^2$

$n^2 + 5n + 4 = (n + 1)(n + 4)$ Factor each expression.

$(n + 1)^2 = (n + 1)(n + 1)$

$(n + 1)$ is a factor twice in the second expression. $(n + 4)$ is a factor once.

LCM $= (n + 1)(n + 1)(n + 4)$ or $(n + 1)^2(n + 4)$

Example 4 **Find the LCM of each pair of polynomials.**

5. $3t, 8t^2$ $24t^2$

6. $5m + 15, 2m + 6$ $10(m + 3)$

7. $(x^2 - 8x + 7), (x^2 + x - 2)$ $(x - 7)(x - 1)(x + 2)$

(8) $\frac{6}{n^4} + \frac{2}{n^2} \cdot \frac{n^2}{n^2}$

$\frac{6}{n^4} + \frac{2n^2}{n^4}$

$= \frac{6+2n^2}{n^4}$

(9) $\frac{3(5y)}{(4x)5y} + \frac{(2)(4x)}{(5y)4x}$

$\frac{15y}{20xy} + \frac{8x}{20xy}$

12. $\frac{a^2+8a+24}{(a+4)(a+2)}$ 

Examples 5–7 Find each sum or difference.

8. $\frac{6}{n^4} + \frac{2}{n^2}$ $\frac{6+2n^2}{n^4}$

9. $\frac{3}{4x} + \frac{2}{5y}$ $\frac{15y+8x}{20xy}$

10. $\frac{4}{5n} - \frac{1}{10n^3}$ $\frac{8n^2-1}{10n^3}$

11. $\frac{8}{3c} - \frac{-5}{6d}$ 

12. $\frac{a}{a+4} + \frac{6}{a+2}$

13. $\frac{x}{x-3} - \frac{3}{x+2}$

(10) $\frac{4 \cdot 2n^2}{5n \cdot 2n^2} - \frac{1}{10n^3} = \frac{8n^2}{10n^3} - \frac{1}{10n^3}$

Examples 5–7 Find each sum or difference.

8. $\frac{6}{n^4} + \frac{2}{n^2}$ $\frac{6 + 2n^2}{n^4}$

9. $\frac{3}{4x} + \frac{2}{5y}$ $\frac{15y + 8x}{20xy}$

10. $\frac{4}{5n} - \frac{1}{10n^3}$ $\frac{8n^2 - 1}{10n^3}$

11. $\frac{8}{3c} - \frac{-5}{6d}$ $\frac{16d + 5c}{6cd}$

12. $\frac{a}{a+4} + \frac{6}{a+2}$

13. $\frac{x}{x-3} - \frac{3}{x+2}$

(11) $\frac{2 \cdot 8 \cdot d}{2 \cdot 3c \cdot d} + \frac{5 \cdot c}{6d \cdot c} = \frac{16d + 5c}{6cd}$

(12) $\frac{a(a+2)}{(a+4)(a+2)} + \frac{6(a+4)}{(a+2)(a+4)}$

$\frac{a^2 + 2a + 6a + 24}{(a+4)(a+2)}$

simplify

$= \frac{a^2 + 8a + 24}{(a+4)(a+2)}$

(:)

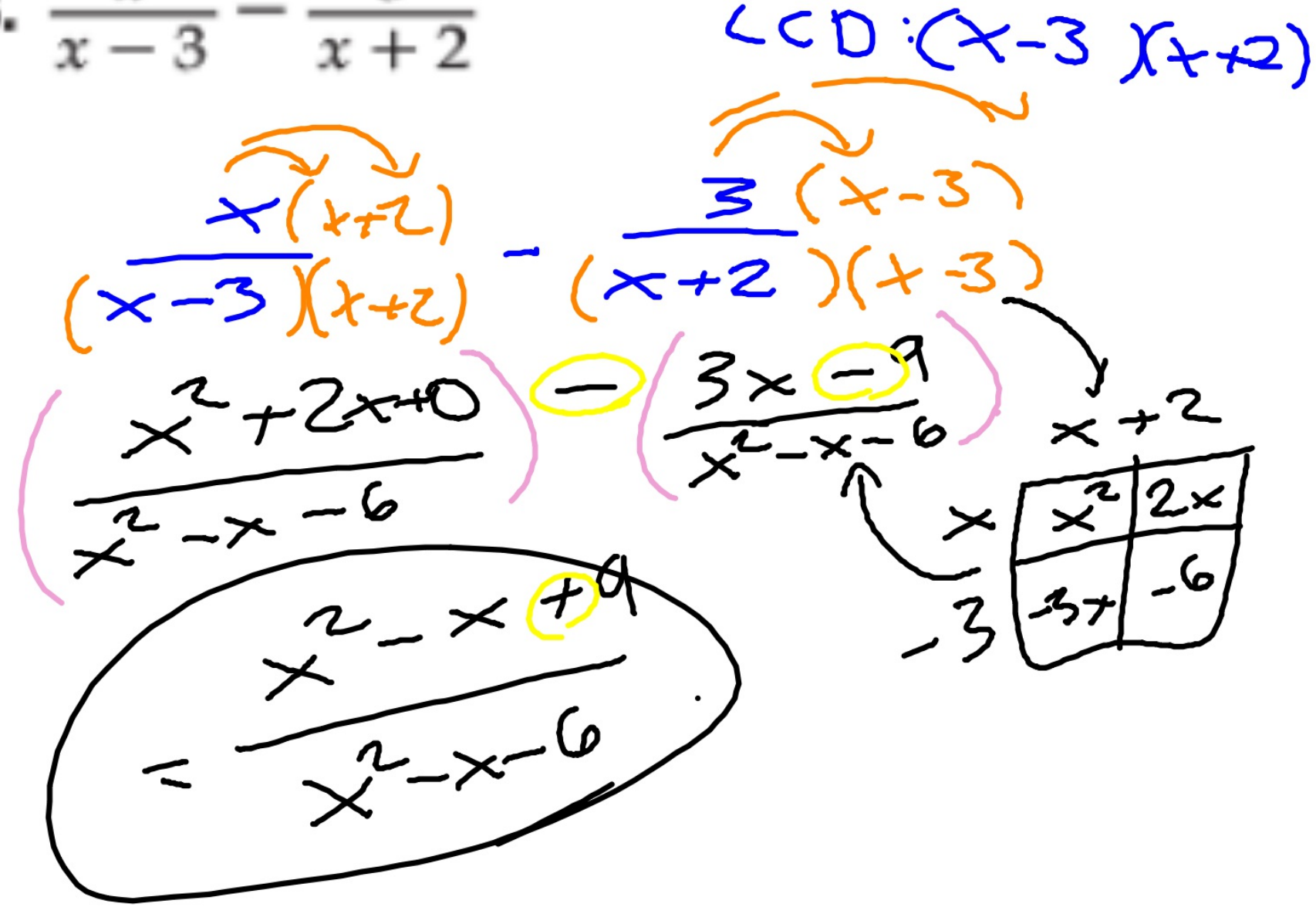
9. $\frac{3}{4x} + \frac{2}{5y}$ $\frac{15y + 8x}{20xy}$

LCD: $20xy$

$5y \cdot \frac{3}{4x} + \frac{2 \cdot 4x}{5y \cdot 4x}$

$5y \cdot \frac{15y}{20xy} + \frac{8x}{20xy} = \frac{15y + 8x}{20xy}$

13. $\dfrac{x}{x-3} - \dfrac{3}{x+2}$



Practice and Problem Solving

Extra Practice is on page R11.

Examples 1–3 Find each sum or difference.

15. $\frac{a}{4} + \frac{3a}{4}$ **a**

16. $\frac{1}{6m} + \frac{5m}{6m}$ **$\frac{1+5m}{6m}$**

17. $\frac{5y}{6} - \frac{y}{6}$ **$\frac{2y}{3}$**

18. $\frac{11}{4r} - \frac{-1}{4r}$ **$\frac{3}{r}$**

19. $\frac{8b}{ab} + \frac{3a}{ab}$ **$\frac{8b+3a}{ab}$**

20. $\frac{t+2}{3} + \frac{t+5}{3}$ **$\frac{2t+7}{3}$**

21. $\frac{3c-7}{2c-1} + \frac{2c+1}{1-2c}$ **$\frac{c-8}{2c-1}$**

22. $\frac{15x}{33x-9} + \frac{3}{9-33x}$ **$\frac{5x-1}{11x-3}$**

23. $\frac{n+6}{10} - \frac{n+1}{10}$ **$\frac{1}{2}$**

24. $\frac{5x+2}{2x+5} - \frac{x-8}{2x+5}$ **2**

25. $\frac{w+2}{8w} - \frac{2w-3}{8w}$ **$\frac{-w+5}{8w}$**

26. $\frac{3a+1}{a-1} - \frac{a+4}{a-1}$ **$\frac{2a-3}{a-1}$**

Example 4 Find the LCM of each pair of polynomials.

27. x^3y, x^2y^2 **x^3y^2**

28. $5ab, 10b$ **$10ab$**

29. $(3r-1), (r+2)$ **$(3r-1)(r+2)$**

30. $2n-10, 4n-20$ **$4(n-5)$**

31. $(x^2+9x+18), x+3$ **$(x+6)(x+3)$**

32. $(k^2-2k-8), (k+2)^2$ **$(k-4)(k+2)^2$**

Examples 5, 7 Find each sum or difference.

33. $\frac{5}{4x} + \frac{1}{10x}$ **$\frac{27}{20x}$**

34. $\frac{6}{r} + \frac{2}{r^2}$ **$\frac{6r+2}{r^2}$**

35. $\frac{3}{2a} + \frac{1}{5b}$ **$\frac{15b+2a}{10ab}$**

36. $\frac{6g}{g+5} - \frac{g-2}{2g}$ **$\frac{11g^2-3g+10}{2g(g+5)}$**

37. $\frac{7}{4k+8} - \frac{k}{k+2}$ **$\frac{7-4k}{4(k+2)}$**

38. $\frac{5}{2d+2} - \frac{d}{d+5}$ **$\frac{-2d^2+3d+25}{(2d+2)(d+5)}$**

39. $\frac{-2}{7r} + \frac{4}{t}$ **$\frac{-2t+28r}{7rt}$**

40. $\frac{n}{n-2} + \frac{n}{n+1}$ **$\frac{2n^2-n}{(n-2)(n+1)}$**

41. $\frac{d}{d+5} + \frac{7}{d-1}$ **$\frac{d^2+6d+35}{(d+5)(d-1)}$**

42. $\frac{4}{a} - \frac{1}{3a}$ **$\frac{11}{3a}$**

43. $\frac{6}{5t^2} - \frac{2}{3t}$ **$\frac{18-10t}{15t^2}$**

44. $\frac{7}{4r} - \frac{3}{t}$ **$\frac{7t-12r}{4rt}$**

45. $\frac{w-3}{w^2-w-20} + \frac{w}{w+4}$ **$\frac{w^2-4w-3}{(w+4)(w-5)}$**

46. $\frac{n}{2n+10} + \frac{1}{n^2-25}$ **$\frac{n^2-5n+2}{2(n+5)(n-5)}$**

47. $\frac{2x}{x^2+8x+15} - \frac{x+3}{x+5}$ **$\frac{-x^2-4x-9}{(x+3)(x+5)}$**

48. $\frac{r-3}{r^2+6r+9} - \frac{r-9}{r^2-9}$ **$\frac{36}{(r-3)(r+3)^2}$**