New Vocabulary

- rational equation
- extraneous solution

Real-World Example 1 Use Cross Products to Solve Equations

DOLPHINS Refer to the information above. Solve $\frac{3}{x+5} = \frac{2}{x}$ to find the speed of a coastal dolphin. Check the solution.

$\frac{3}{x+5} = \frac{2}{x}$ Original equation

$3x = 2(x+5)$ Find the cross products.

$3x = 2x + 10$ Distributive Property

$x = 10$ Subtract $2x$ from each side.

So, a coastal dolphin can swim 10 miles per hour.

CHECK

$\frac{3}{x+5} = \frac{2}{x}$ Original equation

$\frac{3}{10+5} \stackrel{?}{=} \frac{2}{10}$ Replace x with 10.

$\frac{3}{15} \stackrel{?}{=} \frac{1}{5}$ Simplify.

$\frac{1}{5} = \frac{1}{5}$ ✓ Simplify.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Solve each equation. State any extraneous solutions. 6. $-\frac{4}{3}$; extraneous: 1

1. $\frac{2}{x+1} = \frac{4}{x}$

2. $\frac{t+3}{5} = \frac{2t+3}{9}$ 12

3. $\frac{a+3}{a} - \frac{6}{5a} = \frac{1}{a}$

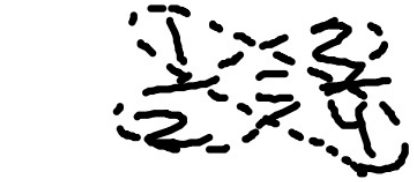
4. $4 - \frac{p}{p-1} = \frac{2}{p-1}$ 2

5. $\frac{2t}{t+1} + \frac{4}{t-1} = 2$

6. $\frac{x+3}{x^2-1} - \frac{2x}{x-1} = 1$

Example 2 Use the LCD to Solve Rational Equations

Solve $\frac{4}{\quad} + \frac{5y}{\quad} = 5$. Check the solution.



2·2=1·4

cross-multiply

$2x = 4(x+1)$

$2x = 4x + 4$

$-4x \quad -4x$

$-2x = 4$

$x = -2$

$4 = y$ Simplify.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Solve each equation. State any extraneous solutions. **6.** $-\frac{4}{3}$; extraneous: 1

1. $\frac{2}{x+1} = \frac{4}{x}$ **−2**
2. $\frac{t+3}{5} = \frac{2t+3}{9}$ **12**
3. $\frac{a+3}{a} - \frac{6}{5a} = \frac{1}{a}$
4. $4 - \frac{p}{p-1} = \frac{2}{p-1}$ **2**
5. $\frac{2t}{t+1} + \frac{4}{t-1} = 2$
6. $\frac{x+3}{x^2-1} - \frac{2x}{x-1} = 1$

1–3 Solve each equation. State any extraneous solutions. **6.** $-\frac{4}{3}$; extraneous: 1

1. $\frac{2}{x+1} = \frac{4}{x}$ **−2**

2. $\frac{t+3}{5} = \frac{2t+3}{9}$ **12**

3. $\frac{a+3}{a} - \frac{6}{5a} = \frac{1}{a}$ $-\frac{4}{5}$

4. $4 - \frac{p}{p-1} = \frac{2}{p-1}$ **2**

5. $\frac{2t}{t+1} + \frac{4}{t-1} = 2$ **−3**

6. $\frac{x+3}{x^2-1} - \frac{2x}{x-1} = 1$

(3) LCD: $5a$

$5a\left(\frac{a+3}{a} - \frac{6}{5a}\right) = 5a\left(\frac{1}{a}\right)$ ← $\frac{5a}{a}$

$5(a+3) - 6 = 5$

$5a + 15 - 6 = 5$

$5a + 9 = 5$

$5a = -4$

$a = \frac{-4}{5}$

Examples 1–3 Solve each equation. State any extraneous solutions. 3; extraneous: 1

1. $\frac{2}{x+1} = \frac{4}{x}$ **−2**

2. $\frac{t+3}{5} = \frac{2t+3}{9}$ **12**

3. $\frac{a+3}{a} - \frac{6}{5a} = \frac{1}{a}$ $-\frac{4}{5}$

4. $4 - \frac{p}{p-1} = \frac{2}{p-1}$ **2**

5. $\frac{2t}{t+1} + \frac{4}{t-1} = 2$ **−3**

6. $\frac{x+3}{x^2-1} - \frac{2x}{x-1} = 1$

(4) $\frac{4}{1}\frac{(p-1)}{(p-1)} - \frac{p}{(p-1)}\frac{(p-1)}{1} = \frac{2}{(p-1)}(p-1)$

$4(p-1) - p = 2$

$4p - 4 - p = 2$

$3p - 4 = 2$

$3p = 6$

$p = 2$

Example 3 Extraneous Solutions

Solve $\frac{2n}{n-5} + \frac{4n-30}{n-5} = 5$. State any extraneous solutions.

$$\frac{2n}{n-5} + \frac{4n-30}{n-5} = 5$$ Original equation

$$(n-5)\left(\frac{2n}{n-5} + \frac{4n-30}{n-5}\right) = (n-5)5$$ Multiply each side by the LCD, $n-5$.

$$\left(\frac{\cancel{n-5}^{1}}{1} \cdot \frac{2n}{\cancel{n-5}_{1}}\right) + \left(\frac{\cancel{n-5}^{1}}{1} \cdot \frac{4n-30}{\cancel{n-5}_{1}}\right) = (n-5)5$$ Distributive Property

$$2n + 4n - 30 = 5n - 25$$ Simplify.

$$6n - 30 = 5n - 25$$ Add like terms.

$$6n - 5n - 30 = 5n - 5n - 25$$ Subtract $5n$ from each side.

$$n - 30 = -25$$ Simplify.

$$n - 30 + 30 = -25 + 30$$ Add 30 to each side.

$$n = 5$$ Simplify.

Since $n = 5$ results in a zero in the denominator of the original equation, it is an extraneous solution. So, the equation has no solution.

check out #19...

19. $\frac{3n}{n-1} + \frac{6n-9}{n-1} = 6$ **no solution; extraneous: 1**

Practice and Problem Solving

Extra Practice is on page R11.

Examples 1–3 Solve each equation. State any extraneous solutions.

9. $\frac{8}{n} = \frac{3}{n-5}$ **8**

10. $\frac{6}{t+2} = \frac{4}{t}$ **4**

11. $\frac{3g+2}{12} = \frac{g}{2}$ $\frac{2}{3}$

12. $\frac{5h}{4} + \frac{1}{2} = \frac{3h}{8}$ $-\frac{4}{7}$

13. $\frac{2}{3w} = \frac{2}{15} + \frac{12}{5w}$ **−13**

14. $\frac{c-4}{c+1} = \frac{c}{c-1}$ $\frac{2}{3}$

15. $\frac{x-1}{x+1} - \frac{2x}{x-1} = -1$ **0**

16. $\frac{y+4}{y-2} + \frac{6}{y-2} = \frac{1}{y+3}$ **−4, −8**

17. $\frac{a}{a+3} + \frac{a^2}{a+3} = 2$ **−2, 3**

18. $\frac{12}{a+3} + \frac{6}{a^2-9} = \frac{8}{a+3}$ $\frac{3}{2}$

19. $\frac{3n}{n-1} + \frac{6n-9}{n-1} = 6$ **no solution; extraneous: 1**

20. $\frac{n^2-n-6}{n^2-n} - \frac{n-5}{n-1} = \frac{n-3}{n^2-n}$ **no solution; extraneous: 1**