

2-6 Special Functions

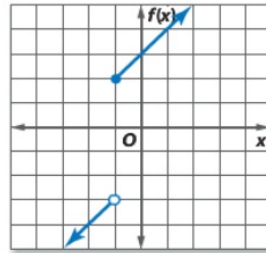
Example 1 Piecewise-Defined Function

Graph $f(x) = \begin{cases} x - 2 & \text{if } x < -1 \\ x + 3 & \text{if } x \geq -1 \end{cases}$. Identify the domain and range.

Step 1 Graph $f(x) = x - 2$ for $x < -1$.

$$\begin{aligned} f(x) &= x - 2 \\ &= (-1) - 2 \\ &= -3 \end{aligned}$$

Because -1 does not satisfy the inequality, begin with a circle at $(-1, -3)$.

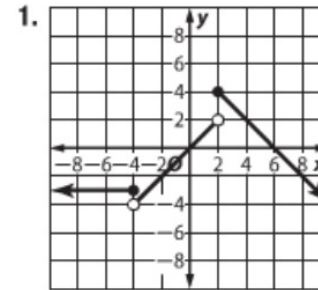


Step 2 Graph $f(x) = x + 3$ for $x \geq -1$.

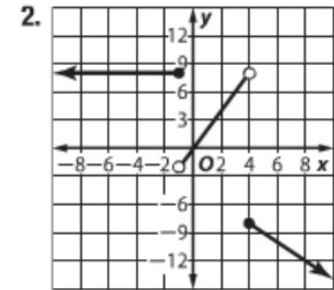
$$\begin{aligned} f(x) &= x + 3 \\ &= (-1) + 3 \\ &= 2 \end{aligned}$$

Because -1 satisfies the inequality, begin with a dot at $(-1, 2)$.

Lesson 2-6



$$\begin{aligned} D &= \{\text{all real numbers}\}; \\ R &= \{y \mid y \leq 4\} \end{aligned}$$



$$\begin{aligned} D &= \{\text{all real numbers}\}; \\ R &= \{y \mid 8 \geq y > -2 \text{ or } y \leq -8\} \end{aligned}$$

Example 1 Graph each function. Identify the domain and range. **1, 2. See Chapter 2 Answer Appendix.**

$$1. g(x) = \begin{cases} -3 & \text{if } x \leq -4 \\ x & \text{if } -4 < x < 2 \\ -x + 6 & \text{if } x \geq 2 \end{cases}$$

$$2. f(x) = \begin{cases} 8 & \text{if } x \leq -1 \\ 2x & \text{if } -1 < x < 4 \\ -4 - x & \text{if } x \geq 4 \end{cases}$$

Example 2 Write a Piecewise-Defined Function

Write the piecewise-defined function shown in the graph.

Examine and write a function for each portion of the graph.

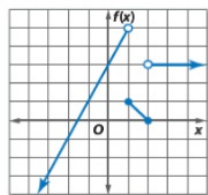
The left portion of the graph is the graph of $f(x) = 2x + 3$. There is a circle at $(1, 5)$, so the linear function is defined for $\{x|x < 1\}$.

The center portion of the graph is the graph of $f(x) = -x + 2$. There are dots at $(1, 1)$ and $(2, 0)$, so the linear function is defined for $\{x|1 \leq x \leq 2\}$.

The right portion of the graph is the constant function $f(x) = 3$. There is a circle at $(2, 3)$, so the constant function is defined for $\{x|x > 2\}$.

Write the piecewise-defined function.

$$f(x) = \begin{cases} 2x + 3 & \text{if } x < 1 \\ -x + 2 & \text{if } 1 \leq x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$$

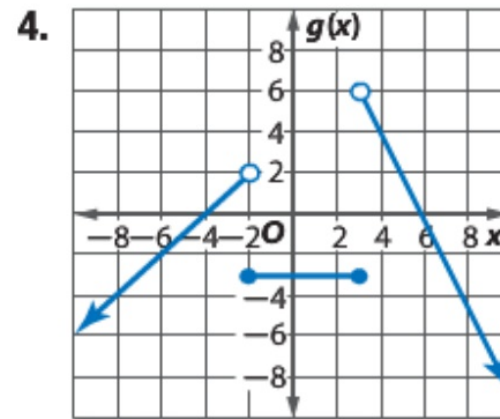
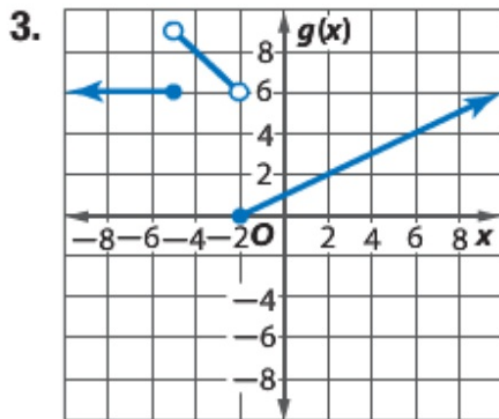


$$3. \ g(x) = \begin{cases} x + 4 & \text{if } x < -2 \\ -3 & \text{if } -2 \leq x \leq 3 \\ -2x + 12 & \text{if } x > 3 \end{cases}$$

$$4. \ g(x) = \begin{cases} 6 & \text{if } x \leq -5 \\ -x + 4 & \text{if } -5 < x < -2 \\ \frac{1}{2}x + 1 & \text{if } x \geq -2 \end{cases}$$

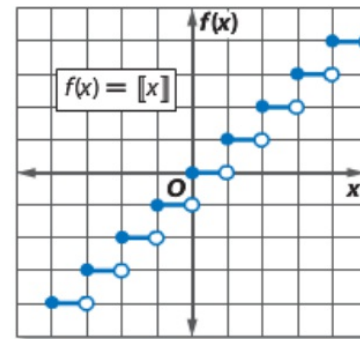
Example 2

Write the piecewise-defined function shown in each graph. **3, 4. See Chapter 3.**



2 Step Functions and Absolute Value Functions Unlike a piecewise-defined function, a **piecewise-linear function** contains a single expression. A common piecewise-linear function is the step function. The graph of a **step function** consists of line segments.

› The **greatest integer function**, written $f(x) = \llbracket x \rrbracket$, is one kind of step function. The symbol $\llbracket x \rrbracket$ means *the greatest integer less than or equal to x* . For example, $\llbracket 3.25 \rrbracket = 3$ and $\llbracket -4.6 \rrbracket = -5$.



Real-World Example 3 Use a Step Function

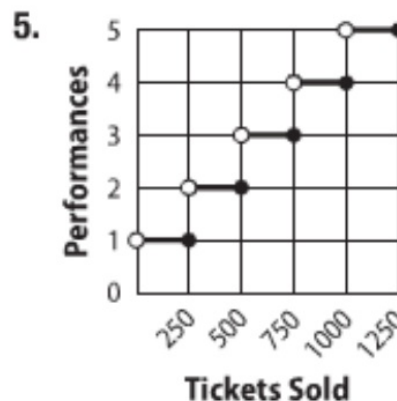
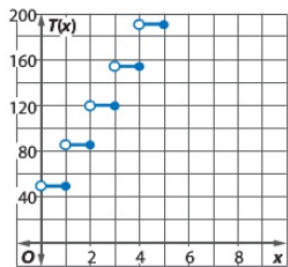
BUSINESS An automotive repair center charges \$50 for any part of the first hour of labor, and \$35 for any part of each additional hour. Draw a graph that represents this situation.

Understand The total labor charge is \$50 for the first hour plus \$35 for each additional fraction of an hour, so the graph will be a step function.

Plan If the time spent on labor is greater than 0 hours, but less than or equal to 1 hour, then the labor charge is \$50. If the time is greater than 1 hour but less than 2 hours, then the labor charge is \$85, and so on.

Solve Use the pattern of times and costs to make a table, where x is the number of hours of labor and $T(x)$ is the total labor charge. Then graph.

x	$T(x)$
$0 < x \leq 1$	\$50
$1 < x \leq 2$	\$85
$2 < x \leq 3$	\$120
$3 < x \leq 4$	\$155
$4 < x \leq 5$	\$190



Example 3

5 CCSS REASONING Springfield High School's theater can hold 250 students. The drama club is performing a play in the theater. Draw a graph of a step function that shows the relationship between the number of tickets sold x and the minimum number of plays y that the drama club must perform. **See Chapter 2 Answer Appendix.**

Graph each function. Identify the domain and range. **6, 7. See Chapter 2 Answer Appendix.**

6. $g(x) = -2\llbracket x \rrbracket$

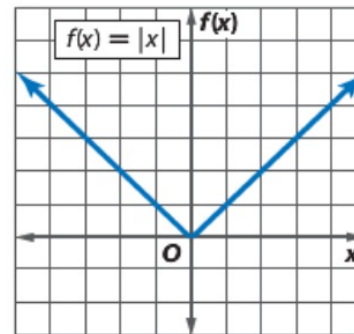
7. $h(x) = \llbracket x - 5 \rrbracket$

KeyConcept Parent Function of Absolute Value Functions

Parent function: $f(x) = |x|$, defined as

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

Type of graph: V-shaped
Domain: all real numbers
Range: all nonnegative real numbers
Intercepts: $x = 0, f(x) = 0$
Not defined: $f(x) < 0$



Example 4 Absolute Value Functions

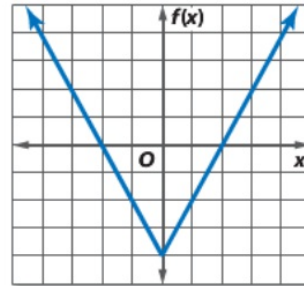
Graph $f(x) = |2x| - 4$. Identify the domain and range.

Create a table of values.

x	$ 2x - 4$
-3	2
-2	0
-1	-2
0	-4
1	-2
2	0
3	2

Graph the points and connect them.

The domain is the set of all real numbers. The range is $\{f(x) | f(x) \geq -4\}$.



Example 4 Graph each function. Identify the domain and range.

8. $g(x) = |-3x|$

9. $f(x) = 2|x|$

10. $h(x) = |x + 4|$

11. $s(x) = |-2x| + 6$

Example 1Graph each function. Identify the domain and range. **12–15. See margin.**

$$12. f(x) = \begin{cases} -3x & \text{if } x \leq -4 \\ x & \text{if } 0 < x \leq 3 \\ 8 & \text{if } x > 3 \end{cases}$$

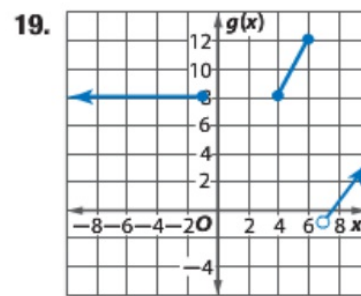
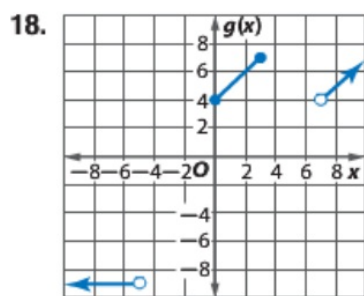
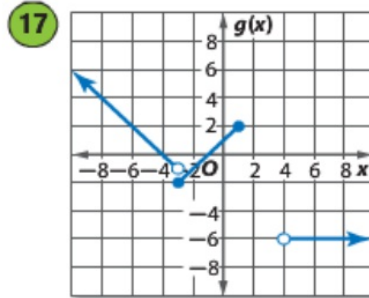
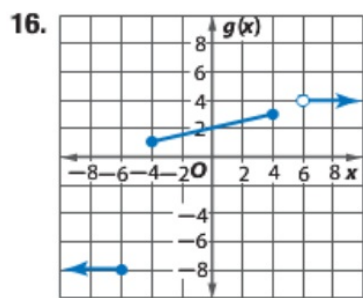
$$13. f(x) = \begin{cases} 2x & \text{if } x \leq -6 \\ 5 & \text{if } -6 < x \leq 2 \\ -2x + 1 & \text{if } x > 4 \end{cases}$$

$$14. g(x) = \begin{cases} 2x + 2 & \text{if } x < -6 \\ x & \text{if } -6 \leq x \leq 2 \\ -3 & \text{if } x > 2 \end{cases}$$

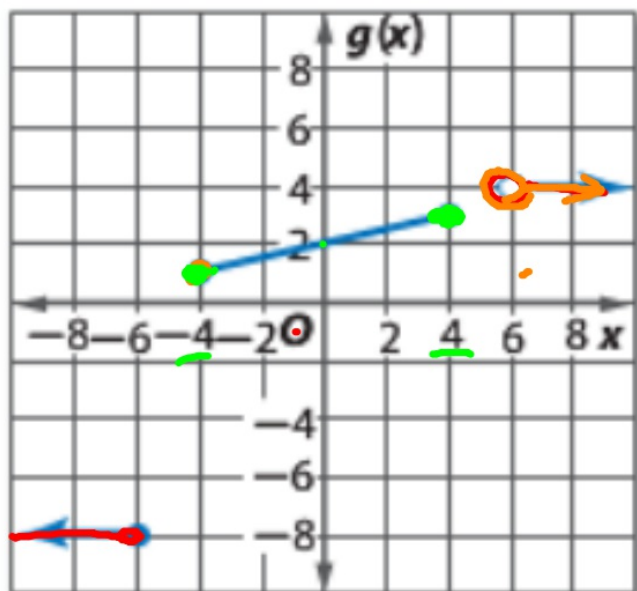
$$15. g(x) = \begin{cases} -2 & \text{if } x < -4 \\ x - 3 & \text{if } -1 \leq x \leq 5 \\ 2x - 15 & \text{if } x > 7 \end{cases}$$

Example 2

Write the piecewise-defined function shown in each graph.

16–19. See Chapter 2 Answer Appendix.

16.

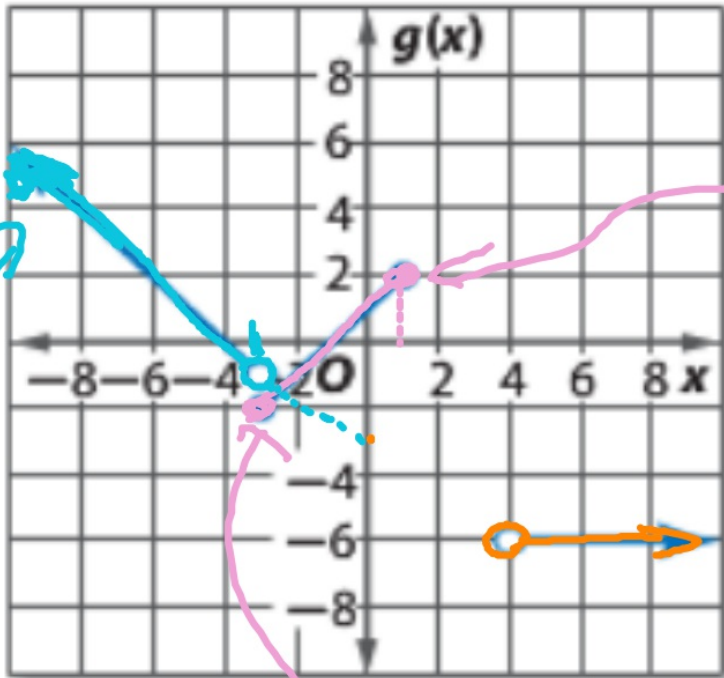


15

$$g(x) = \begin{cases} -8, & x \leq -4 \\ \frac{1}{4}x + 2, & -4 \leq x \leq 4 \\ 4, & x > 6 \end{cases}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{2}{8} = \frac{1}{4}$$

17

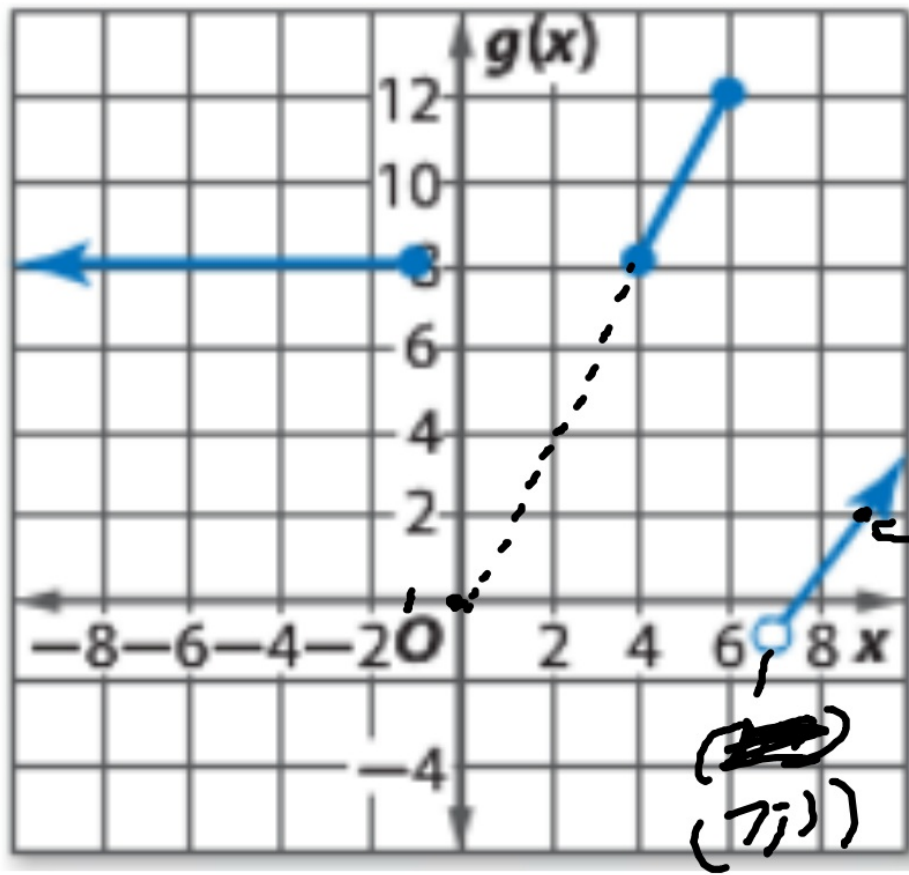


$x = -3$

$$\begin{cases} x-3, & x < -3 \\ \frac{3}{4}x+1, & -3 \leq x \leq 1 \\ -6, & x \geq 4 \end{cases}$$

$(-3, -1)$

$$\frac{\Delta y}{\Delta x} = \frac{3}{4}$$



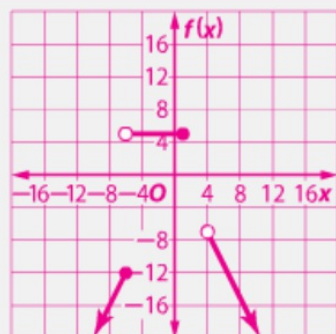
$$\frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$$

$$g(x) = \begin{cases} 8, & x \leq -1 \\ 2x, & 4 \leq x \leq 6 \\ & x > 7 \end{cases}$$

$$19. g(x) = \begin{cases} 8 & \text{if } x \leq -1 \\ 2x & \text{if } 4 \leq x \leq 6 \\ 2x - 15 & \text{if } x > 7 \end{cases}$$

Additional Answers

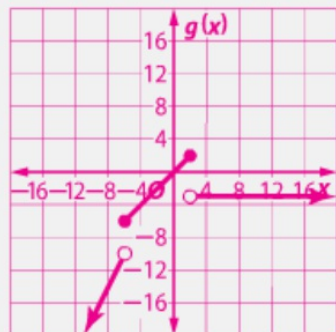
13.



$$D = \{x \mid x \leq 2 \text{ or } x > 4\};$$

$$R = \{f(x) \mid f(x) < -7, \text{ or } f(x) = 5\}$$

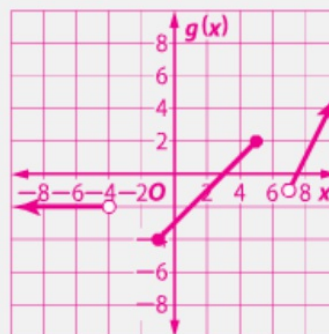
14.



$$D = \{\text{all real numbers}\};$$

$$R = \{g(x) \mid g(x) < -10 \text{ or } -6 \leq g(x) \leq 2\}$$

15.



$$D = \{x \mid x < -4, -1 \leq x \leq 5, \text{ or } x > 7\};$$

$$R = \{g(x) \mid g(x) \geq -4\}$$

$$16. g(x) = \begin{cases} -8 & \text{if } x \leq -6 \\ 0.25x + 2 & \text{if } -4 \leq x \leq 4 \\ 4 & \text{if } x > 6 \end{cases}$$

$$17. g(x) = \begin{cases} -x - 4 & \text{if } x < -3 \\ x + 1 & \text{if } -3 \leq x \leq 1 \\ -6 & \text{if } x > 4 \end{cases}$$

$$18. g(x) = \begin{cases} -9 & \text{if } x < -5 \\ x + 4 & \text{if } 0 \leq x \leq 3 \\ x - 3 & \text{if } x > 7 \end{cases}$$

$$19. g(x) = \begin{cases} 8 & \text{if } x \leq -1 \\ 2x & \text{if } 4 \leq x \leq 6 \\ 2x - 15 & \text{if } x > 7 \end{cases}$$

Example 3 Graph each function. Identify the domain and range. **20–23. See Chapter 2 Answer Appendix.**

20. $f(x) = \lceil x \rceil - 6$

21. $h(x) = \lceil 3x \rceil - 8$

22. $f(x) = \lceil 3x + 2 \rceil$

23. $g(x) = 2\lceil 0.5x + 4 \rceil$

Example 4 Graph each function. Identify the domain and range. **24–29. See Chapter 2 Answer Appendix.**

24. $f(x) = |x - 5|$

25. $g(x) = |x + 2|$

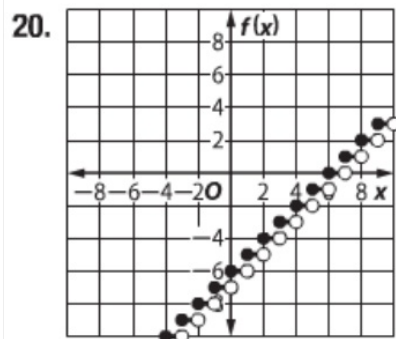
26. $h(x) = |2x| - 8$

27. $k(x) = |-3x| + 3$

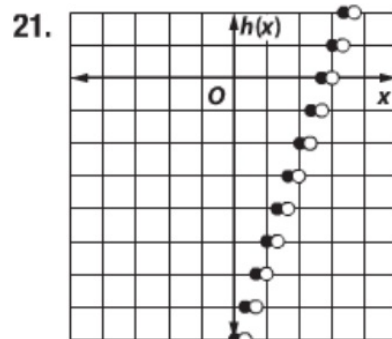
28. $f(x) = 2|x - 4| + 6$

29. $h(x) = -3|0.5x + 1| - 2$

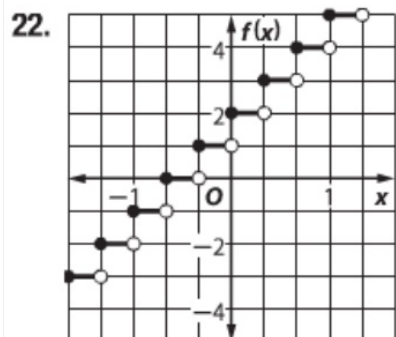
30. **GIVING** Patrick is donating money and volunteering his time to an organization that restores homes for the needy. His employer will match his monetary donations up to \$100.
- Identify the type of function that models the total amount of money received by the charity when Patrick donates x dollars. **piecewise**
 - Write and graph a function for the situation. **See Chapter 2 Answer Appendix.**
31. **CCSS SENSE-MAKING** A car's speedometer reads 60 miles an hour.
- Write an absolute value function for the difference between the car's actual speed a and the reading on the speedometer. **$f(a) = |a - 60|$**
 - What is an appropriate domain for the function? Explain your reasoning. **$\{a | a \geq 0\}$**
 - Use the domain to graph the function. **See Chapter 2 Answer Appendix.**



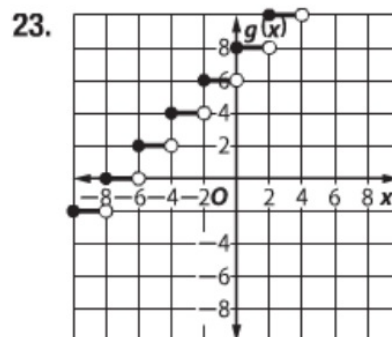
$D = \{\text{all real numbers}\};$
 $R = \{\text{all integers}\}$



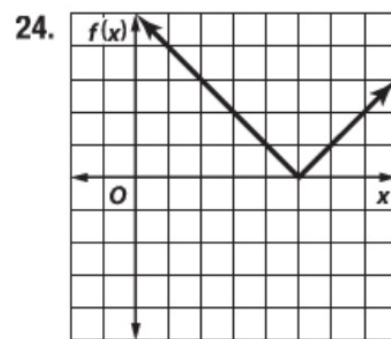
$D = \{\text{all real numbers}\};$
 $R = \{\text{all integers}\}$



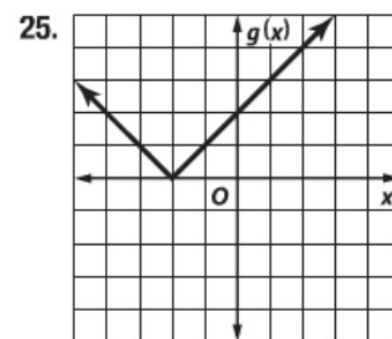
$D = \{\text{all real numbers}\};$
 $R = \{\text{all integers}\}$



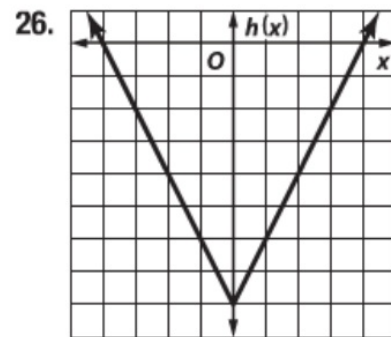
$D = \{\text{all real numbers}\};$
 $R = \{\text{all even integers}\}$



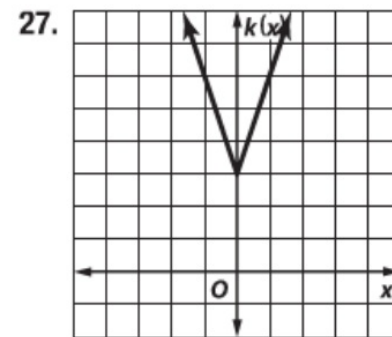
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) \geq 0\}$



$D = \{\text{all real numbers}\};$
 $R = \{g(x) \mid g(x) \geq 0\}$

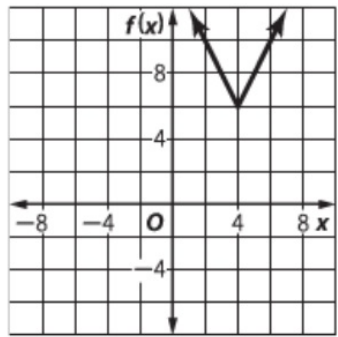


$D = \{\text{all real numbers}\};$
 $R = \{h(x) \mid h(x) \geq -8\}$



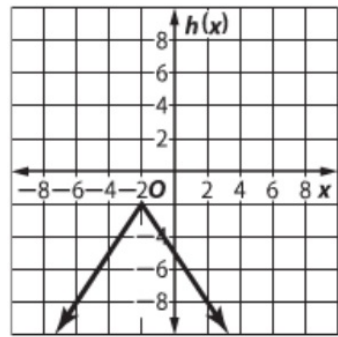
$D = \{\text{all real numbers}\};$
 $R = \{k(x) \mid k(x) \geq 3\}$

28.



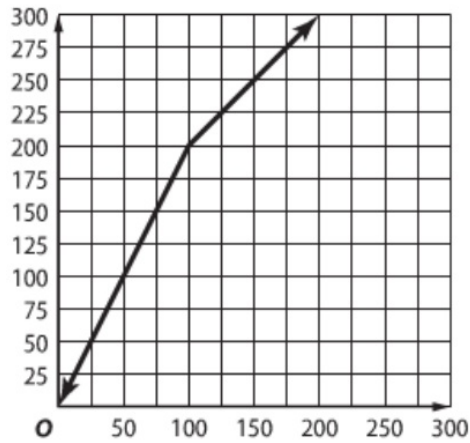
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) \geq 6\}$

29.



$D = \{\text{all real numbers}\};$
 $R = \{h(x) \mid h(x) \leq -2\}$

30b. $f(x) = \begin{cases} 2x & \text{if } 0 < x \leq 100 \\ x + 100 & \text{if } x > 100 \end{cases}$



31c.

