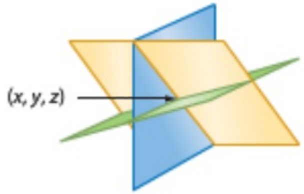
# 3-4 Systems of Equations in Three Variables

**1 Systems in Three Variables** Like systems of equations in two variables, systems in three variables can have one solution, infinite solutions, or no solution. A solution of such a system is an **ordered triple** $(x, y, z)$.

The graph of an equation in three variables is a three-dimensional graph in the shape of a plane. The graphs of a system of equations in three variables form a system of planes.

**One Solution**

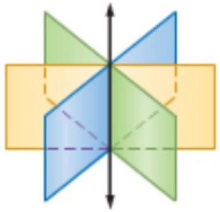
The three individual planes intersect at a specific point.




**Infinitely Many Solutions**

The planes intersect in a line.

Every coordinate on the line represents a solution of the system.



The planes intersect in the same plane.

Every equation is equivalent.
Every coordinate in the plane represents a solution of the system.

**No Solution** There are no points in common with all three planes.

## Example 1 A System with One Solution

**Solve the system of equations.**

$$3x - 2y + 4z = 35$$
$$-4x + y - 5z = -36$$
$$5x - 3y + 3z = 31$$

The coefficient of 1 in the second equation makes $y$ a good choice for elimination.

**Step 1** Eliminate one variable by using two pairs of equations.

$3x - 2y + 4z = 35$
$-4x + y - 5z = -36$

Multiply by 2.

| | |
|---|---|
| $3x - 2y + 4z = 35$ | Equation 1 |
| $(+)\ -8x + 2y - 10z = -72$ | Equation 2 × 2 |
| $-5x \quad - 6z = -37$ | |

$-4x + y - 5z = -36$
$5x - 3y + 3z = 31$

Multiply by 3.

| | |
|---|---|
| $-12x + 3y - 15z = -108$ | Equation 2 × 3 |
| $(+)\ 5x - 3y + 3z = 31$ | Equation 3 |
| $-7x \quad - 12z = -77$ | |

The $y$-terms in each equation have been eliminated. We now have a system of two equations and two variables, $x$ and $z$.

**Step 2** Solve the system of two equations.

**1.** $(-2, -3, 5)$
**2.** $(4, -6, 1)$
**3.** $(-4, 3, 6)$

1. $-3a - 4b + 2c = 28$
   $a + 3b - 4c = -31$
   $2a + 3c = 11$

2. $3y - 5z = -23$
   $4x + 2y + 3z = 7$
   $-2x - y - z = -3$

3. $3x + 6y - 2z = -6$
   $2x + y + 4z = 19$
   $-5x - 2y + 8z = 62$

Use substitution to solve for $z$.

| | |
|---|---|
| $-5x - 6z = -37$ | Equation with two variables |
| $-5(-1) - 6z = -37$ | Substitution |
| $5 - 6z = -37$ | Multiply. |
| $-6z = -42$ | Subtract 5 from each side. |
| $z = 7$ | Divide each side by −6. |

The result is $x = -1$ and $z = 7$.

**Step 3** Substitute the two values into one of the original equations to find $y$.

| | |
|---|---|
| $-4x + y - 5z = -36$ | Equation 2 |
| $-4(-1) + y - 5(7) = -36$ | Substitution |
| $4 + y - 35 = -36$ | Multiply. |
| $y = -5$ | Add 31 to each side. |

## Example 2 No Solution and Infinite Solutions

**Solve each system of equations.**

**a.** $5x + 4y - 5z = -10$
$-4x - 10y - 8z = -16$
$6x + 15y + 12z = 24$

Eliminate $x$ in the second two equations.

$-4x - 10y - 8z = -16$ **Multiply by 3.** $-12x - 30y - 24z = -48$

$6x + 15y + 12z = 24$ **Multiply by 2.** $(+)\ 12x + 30y + 24z = 48$

$0 = 0$

The equation $0 = 0$ is always true. This indicates that the last two equations represent the same plane. Check to see if this plane intersects the first plane.

$5x + 4y - 5z = -10$ **Multiply by 4.** $20x + 16y - 20z = -40$

$-4x - 10y - 8z = -16$ **Multiply by 5.** $(+)\ -20x - 50y - 40z = -80$

$-34y - 60z = -120$

The planes intersect in a line. So, there are an infinite number of solutions.

**b.** $-6a + 9b - 12c = 21$
$-2a + 3b - 4c = 7$
$10a - 15b + 20c = -30$

Eliminate $a$ in the first two equations.

$-6a + 9b - 12c = 21$ $\quad -6a + 9b - 12c = 21$

$-2a + 3b - 4c = 7$ **Multiply by −3.** $(+)\ 6a - 9b + 12c = -21$

$0 = 0$

The equation $0 = 0$ is always true. This indicates that the first two equations represent the same plane. Check to see if this plane intersects the last plane.

$-2a + 3b - 4c = 7$ **Multiply by 5.** $-10a + 15b - 20c = 35$

$10a - 15b + 20c = -30$ $\quad (+)\ 10a - 15b + 20c = -30$

$0 = 5$

The equation $0 = 5$ is never true. So, there is no solution of this system.

**4.** $-4r - s + 3t = -9$
$3r + 2s - t = 3$
$r + 3s - 5t = 29$

**5.** $3x + 5y - z = 12$
$-2x - 3y + 5z = 14$
$4x + 7y + 3z = 38$

**6.** $2a - 3b + 5c = 58$
$-5a + b - 4c = -51$
$-6a - 8b + c = 22$

**4. (−2, 2, −5)**
**5. infinite solutions**
**6. (3, −4, 8)**

1. (−2, −3, 5)
2. (4, −6, 1)
3. (−4, 3, 6)

**1.** $-3a - 4b + 2c = 28$
$a + 3b - 4c = -31$
$2a + 3c = 11$

**2.** $3y - 5z = -23$
$4x + 2y + 3z = 7$
$-2x - y - z = -3$

**3.** $3x + 6y - 2z = -6$
$2x + y + 4z = 19$
$-5x - 2y + 8z = 62$

$-3a - 4b + 2c = 28$
$a + 3b - 4c = -31$ ⇒ $\times 3$

$-3a - 4b + 2c = 28$
$+\ 3a + 9b - 12c = -93$
$5b - 10c = -65$

93
$-28$
65

$a + 3b - 4c = -31$ ⇒ $\times -2$
$2a + 3c = 11$

$-2a - 6b + 8c = 62$
$2a + 3c = 11$
$-6b + 11c = 73$

**Guided**Practice

6000 at 10%; 18,000 at 8%; 26,000 at 12%

**3.** Ms. Garza invested $50,000 in three different accounts. She invested three times as much money in an account that paid 8% interest than an account that paid 10% interest. The third account earned 12% interest. If she earned a total of $5160 in interest in a year, how much did she invest in each account?

Example 3

7. **DOWNLOADING** Heather downloaded some television shows. A sitcom uses 0.3 gigabyte of memory; a drama, 0.6 gigabyte; and a talk show, 0.6 gigabyte. She downloaded 7 programs totaling 3.6 gigabytes. There were twice as many episodes of the drama as the sitcom.

**a.** Write a system of equations for the number of episodes of each type of show.

**b.** How many episodes of each show did she download?

## Practice and Problem Solving

Extra Practice is on page R3.

**Examples 1–2** Solve each system of equations.

8. $-5x + y - 4z = 60$
$2x + 4y + 3z = -12$
$6x - 3y - 2z = -52$
**(−8, 4, −4)**

9. $4a + 5b - 6c = 2$
$-3a - 2b + 7c = -15$
$-a + 4b + 2c = -13$
**(−3, −2, −4)**

10. $-2x + 5y + 3z = -25$
$-4x - 3y - 8z = -39$
$6x + 8y - 5z = 14$
**(8, −3, 2)**

11. $4r + 6s - t = -18$
$3r + 2s - 4t = -24$
$-5r + 3s + 2t = 15$
**(−2, −1, 4)**

12. $-2x + 15y + z = 44$
$4x + 3y + 3z = 18$
$-3x + 6y - z = 8$
**no solution**

13. $4x + 2y + 6z = 13$
$-12x + 3y - 5z = 8$
$-4x + 7y + 7z = 34$
**infinite solutions**

14. $8x + 3y + 6z = 43$
$-3x + 5y + 2z = 32$
$5x - 2y + 5z = 24$
**(−1, 3, 7)**

15. $-6x - 5y + 4z = 53$
$5x + 3y + 2z = -11$
$8x - 6y + 5z = 4$
**(−4, −1, 6)**

16. $-9a + 3b - 2c = 61$
$8a + 7b + 5c = -138$
$5a - 5b + 8c = -45$
**(−8, −7, −5)**

17. $2x - y + z = 1$
$x + 2y - 4z = 3$
$4x + 3y - 7z = -8$
**no solution**

18. $x + 2y = 12$
$3y - 4z = 25$
$x + 6y + z = 20$
**(6, 3, −4)**

19. $r - 3s + t = 4$
$3r - 6s + 9t = 5$
$4r - 9s + 10t = 9$
**infinite solutions**

**Example 3**

20. CCSS **SENSE-MAKING** A friend e-mails you the results of a recent high school swim meet. The e-mail states that 24 individuals placed, earning a combined total of 53 points. First place earned 3 points, second place earned 2 points, and third place earned 1 point. There were as many first-place finishers as second- and third-place finishers combined.

a. Write a system of three equations that represents how many people finished in each place. **$x + y + z = 24$, $3x + 2y + z = 53$, $x = y + z$**

b. How many swimmers finished in first place, in second place, and in third place?

c. Suppose the e-mail had said that the athletes scored a combined total of 47 points. Explain why this statement is false and the solution is unreasonable.

**20b. 7 swimmers placed third, 5 swimmers placed second, and 12 swimmers placed first.**

**20c. The statement is false because when you solve for second place, you get a negative as an answer and you cannot have a negative person.**

B

21. **AMUSEMENT PARKS** Nick goes to the amusement park to ride roller coasters, bumper cars, and water slides. The wait for the roller coasters is 1 hour, the wait for the bumper