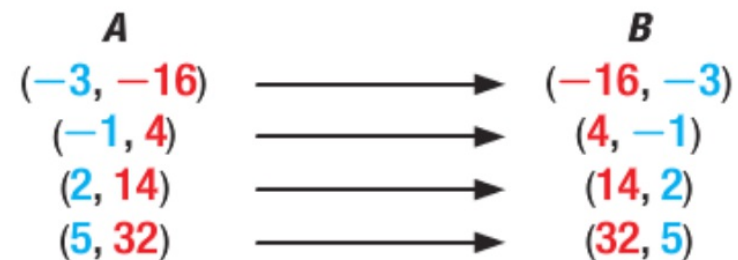


1 Inverse Relations An **inverse relation** is the set of ordered pairs obtained by exchanging the x -coordinates with the y -coordinates of each ordered pair in a relation. If $(5, 3)$ is an ordered pair of a relation, then $(3, 5)$ is an ordered pair of the inverse relation.

 **Key Concept** Inverse Relations

Words If one relation contains the element (a, b) , then the inverse relation will contain the element (b, a) .

Example A and B are inverse relations.



Example 1 Inverse Relations

Find the inverse of each relation.

a. $\{(4, -10), (7, -19), (-5, 17), (-3, 11)\}$

To find the inverse, exchange the coordinates of the ordered pairs.

$$(4, -10) \rightarrow (-10, 4) \quad (-5, 17) \rightarrow (17, -5)$$

$$(7, -19) \rightarrow (-19, 7) \quad (-3, 11) \rightarrow (11, -3)$$

The inverse is $\{(-10, 4), (-19, 7), (17, -5), (11, -3)\}$.

b.

x	-4	-1	5	9
y	-13	-8.5	0.5	6.5

Write the coordinates as ordered pairs. Then exchange the coordinates of each pair.

$$(-4, -13) \rightarrow (-13, -4) \quad (5, 0.5) \rightarrow (0.5, 5)$$

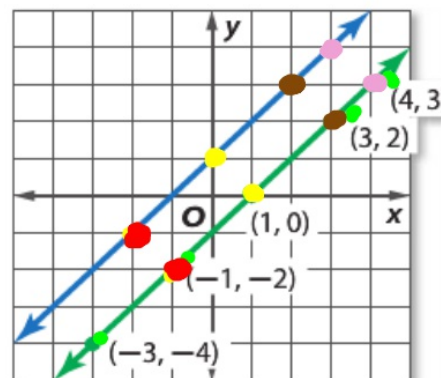
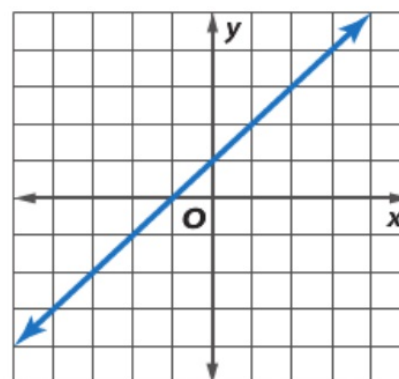
$$(-1, -8.5) \rightarrow (-8.5, -1) \quad (9, 6.5) \rightarrow (6.5, 9)$$

The inverse is $\{(-13, -4), (-8.5, -1), (0.5, 5), (6.5, 9)\}$.

Example 2 Graph Inverse Relations

Graph the inverse of the relation.

The graph of the relation passes through the points at $(-4, -3)$, $(-2, -1)$, $(0, 1)$, $(2, 3)$, and $(3, 4)$. To find points through which the graph of the inverse passes, exchange the coordinates of the ordered pairs. The graph of the inverse passes through the points at $(-3, -4)$, $(-1, -2)$, $(1, 0)$, $(3, 2)$, and $(4, 3)$. Graph these points and then draw the line that passes through them.



Example 3 Find Inverse Linear Functions



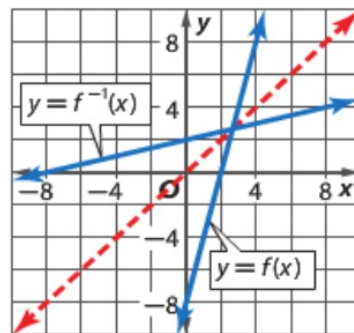
Find the inverse of each function.

a. $f(x) = 4x - 8$

- Step 1** $f(x) = 4x - 8$ Original equation
 $y = 4x - 8$ Replace $f(x)$ with y .
- Step 2** $x = 4y - 8$ Interchange y and x .
- Step 3** $x + 8 = 4y$ Add 8 to each side.
 $\frac{x + 8}{4} = y$ Divide each side by 4.
- Step 4** $\frac{x + 8}{4} = f^{-1}(x)$ Replace y with $f^{-1}(x)$.

The inverse of $f(x) = 4x - 8$ is $f^{-1}(x) = \frac{x + 8}{4}$ or $f^{-1}(x) = \frac{1}{4}x + 2$.

CHECK Graph both functions and the line $y = x$ on the same coordinate plane. $f^{-1}(x)$ appears to be the reflection of $f(x)$ in the line $y = x$. ✓



b. $f(x) = -\frac{1}{2}x + 11$

- Step 1** Original equation

Example 1 Find the inverse of each relation.

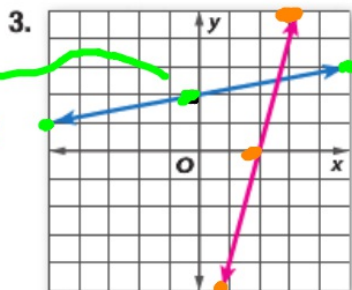
1. $\{(4, -15), (-8, -18), (-2, -16.5), (3, -15.25)\}$ $\{(-15, 4), (-18, -8), (-16.5, -2), (-15.25, 3)\}$

2.

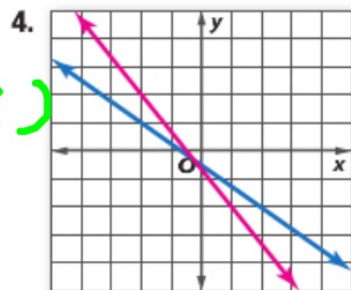
x	-3	0	1	6
y	11.8	3.7	1	-12.5

$\{(11.8, -3), (3.7, 0), (1, 1), (-12.5, 6)\}$

Example 2 Graph the inverse of each relation.



$(0, 2)$
 $(-5, 1)$



$(3, 5)$
 $(2, 0)$
 $(1, -5)$

Example 3 Find the inverse of each function.

5. $f(x) = -2x + 7$ $f^{-1}(x) = -\frac{1}{2}x + \frac{7}{2}$

6. $f(x) = \frac{2}{3}x + 6$ $f^{-1}(x) = \frac{3}{2}x - 9$

Example 4

7. **CCSS REASONING** Dwayne and his brother purchase season tickets to the Cleveland Crusaders games. The ticket package requires a one-time purchase of a personal seat license costing \$1200 for two seats. A ticket to each game costs \$70. The cost $C(x)$ in dollars for Dwayne for the first season is $C(x) = 600 + 70x$, where x is the number of games Dwayne attends.

- a. Find the inverse function. $C^{-1}(x) = \frac{1}{70}x - \frac{60}{7}$ **b. x is Dwayne's total cost, and $C^{-1}(x)$ is the number of games Dwayne attended.**
- b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?
- c. How many games did Dwayne attend if his total cost for the season was \$950? **5**

Example 3 Find the inverse of each function.

5. $f(x) = -2x + 7$ $f^{-1}(x) = -\frac{1}{2}x + \frac{7}{2}$

6. $f(x) = \frac{2}{3}x + 6$ $f^{-1}(x) = \frac{3}{2}x - 9$

$$f(x) = -2x + 7$$

$$y = -2x + 7$$

$$\frac{x}{-2} = \frac{-2y + 7}{-2}$$

$$\frac{x - 7}{-2} = \frac{-2y}{-2}$$

$$\frac{x - 7}{-2} = y$$

$$y = -\frac{1}{2}x + \frac{7}{2}$$
$$f^{-1}(x) = -\frac{1}{2}x + \frac{7}{2}$$

Example 1 Find the inverse of each relation.

8. $\{(-5, 13), (6, 10.8), (3, 11.4), (-10, 14)\}$ $\{(13, -5), (10.8, 6), (11.4, 3), (14, -10)\}$

9. $\{(-4, -49), (8, 35), (-1, -28), (4, 7)\}$ $\{(-49, -4), (35, 8), (-28, -1), (7, 4)\}$

10.

x	y
-8	-36.4
-2	-15.4
1	-4.9
5	9.1
11	30.1

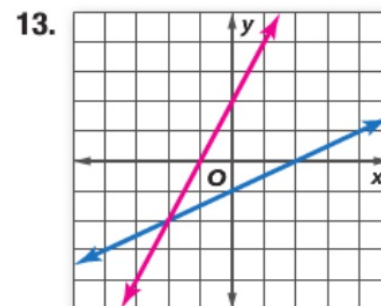
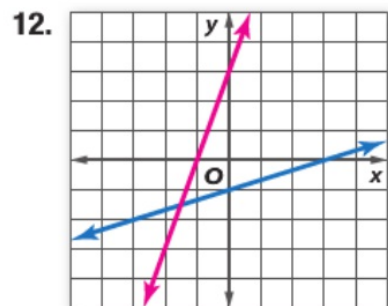
$\{(-36.4, -8), (-15.4, -2), (-4.9, 1), (9.1, 5), (30.1, 11)\}$

11.

x	y
-3	7.4
-1	4
1	0.6
3	-2.8
5	-6.2

$\{(7.4, -3), (4, -1), (0.6, 1), (-2.8, 3), (-6.2, 5)\}$

Example 2 Graph the inverse of each relation.



Example 3

Find the inverse of each function.

14. $f(x) = 25 + 4x$ $f^{-1}(x) = \frac{1}{4}x - \frac{25}{4}$

15. $f(x) = 17 - \frac{1}{3}x$ $f^{-1}(x) = -3x + 51$

16. $f(x) = 4(x + 17)$ $f^{-1}(x) = \frac{1}{4}x - 17$

17. $f(x) = 12 - 6x$ $f^{-1}(x) = -\frac{1}{6}x + 2$

18. $f(x) = \frac{2}{5}x + 10$ $f^{-1}(x) = \frac{5}{2}x - 25$

19. $f(x) = -16 - \frac{4}{3}x$ $f^{-1}(x) = -\frac{3}{4}x - 12$

Example 4

20. **DOWNLOADS** An online music subscription service allows members to download songs for \$0.99 each after paying a monthly service charge of \$3.99. The total monthly cost $C(x)$ of the service in dollars is $C(x) = 3.99 + 0.99x$, where x is the number of songs downloaded.

b. x is the total monthly cost of the service, and $C^{-1}(x)$ is the number of songs downloaded.

- Find the inverse function. $C^{-1}(x) = \frac{x - 3.99}{0.99}$
- What do x and $C^{-1}(x)$ represent in the context of the inverse function?
- How many songs were downloaded if a member's monthly bill is \$27.75? **24**

21. **LANDSCAPING** At the start of the mowing season, Chuck collects a one-time maintenance fee of \$10 from his customers. He charges the Fosters \$35 for each cut. The total amount collected from the Fosters in dollars for the season is $C(x) = 10 + 35x$, where x is the number of times Chuck mows the Fosters' lawn.

- Find the inverse function. $C^{-1}(x) = \frac{1}{35}x - \frac{2}{7}$
- What do x and $C^{-1}(x)$ represent in the context of the inverse function?
- How many times did Chuck mow the Fosters' lawn if he collected a total of \$780 from them? **22**

21b. x is the total amount collected from the Fosters, and $C^{-1}(x)$ is the number of times Chuck mowed the Fosters' lawn.

B Write the inverse of each equation.

