24

$y = (x^2-1)^{-1}$

$y' = -(x^2-1)^{-2} \cdot 2x$

$y' = \dfrac{-2x}{(x^2-1)^2} = 0$

$x = 0 \ldots$

value 0

$-1, 1$

24. $y = \dfrac{1}{x^2 - 1}$ Local max at $(0, -1)$

und. @ $x = -1, 1$

V.A @ $-1, 1$

test $x = -.5$ $x = .5$

y'

-1 $\quad \frac{+}{+}$ $\quad 0$ $\quad \frac{-}{+}$ $\quad 1$

$+$ $\quad -$

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4.2 The Mean Value Theorem

THEOREM 3 Mean Value Theorem for Derivatives

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$\underset{m}{f'(c)} = \frac{\overset{y_2 - y_1}{f(b) - f(a)}}{\underset{x_2 - x_1}{b - a}}.$$

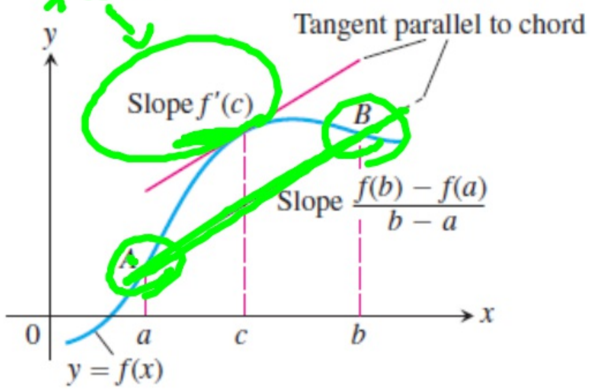
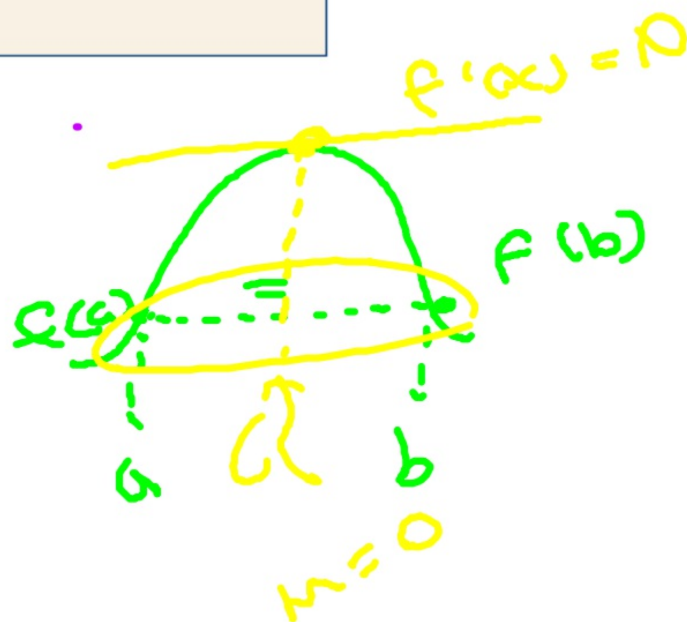


Figure 4.10 Figure for the Mean Value Theorem.

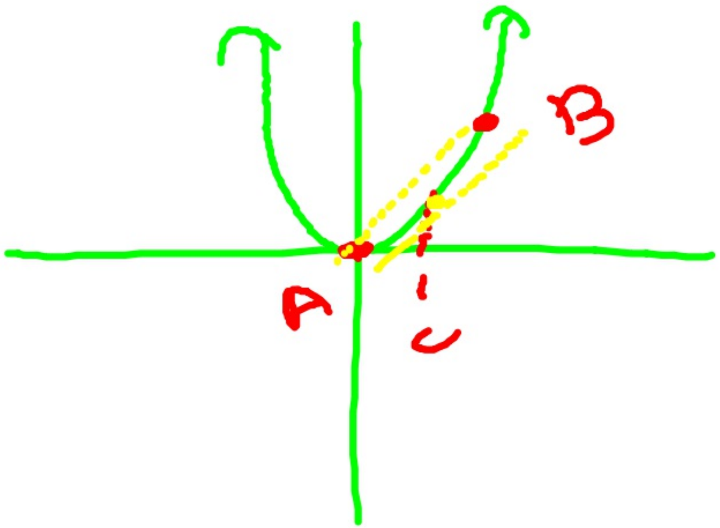


$$c = 1.$$

nterpret The tangent line to $f(x) = x^2$ at $x = 1$ has slope 2 and is parallel to t
hord joining $A(0, 0)$ and $B(2, 4)$ (Figure 4.12).

Now try Exer

continued



THEOREM 3 **Mean Value Theorem for Derivatives**

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f(b) - f(a)$$

EXAMPLE 2 Exploring the Mean Value Theorem

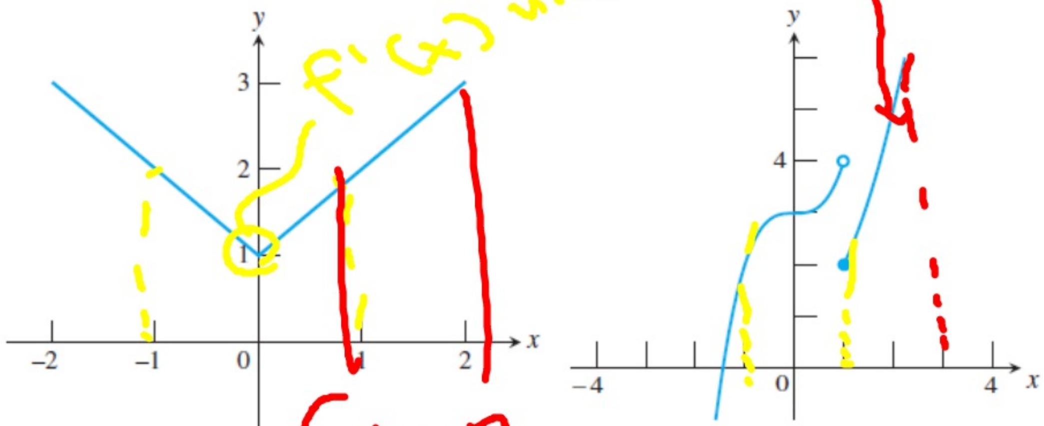
Explain why each of the following functions fails to satisfy the conditions of the Mean Value Theorem on the interval $[-1, 1]$.

$[1, 2\tfrac{1}{2}]$

(a) $f(x) = \sqrt{x^2 + 1}$ **(b)** $f(x) = \begin{cases} x^3 + 3 & \text{for } x < 1 \\ x^2 + 1 & \text{for } x \geq 1 \end{cases}$

SOLUTION

$f'(x)$ und



$[1, 2]$

(a) (b)

THEOREM 3 Mean Value Theorem for Derivatives

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In Exercises 1–8, (**a**) state whether or not the function satisfies the hypotheses of the Mean Value Theorem on the given interval, and (**b**) if it does, find each value of c in the interval (a, b) that satisfies the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

1. $f(x) = x^2 + 2x - 1$ on $[0, 1]$ ($a = 0$, $b = 1$)

2. $f(x) = x^{2/3}$ on $[0, 1]$

3. $f(x) = x^{1/3}$ on $[-1,1]$ No. There is a vertical tangent at $x = 0$.

4. $f(x) = |x - 1|$ on $[0, 4]$ No. There is a corner at $x = 1$.

5. $f(x) = \sin^{-1} x$ on $[-1, 1]$

6. $f(x) = \ln(x - 1)$ on $[2, 4]$

8. $f(x) = \begin{cases} \sin^{-1} x, & -1 \le x < 1 \\ x/2+1, & 1 \le x \le 3 \end{cases}$ on $[-1, 3]$

$f(c) = c^2 + 2c - 1$

$f'(c) = 2c + 2$

$f(b)$: $f(1)$

$f(b) = 1 + 2 - 1 = 2$

$f(a)$: $f(a) = f(0) = 0 + 0 - 1 = -1$

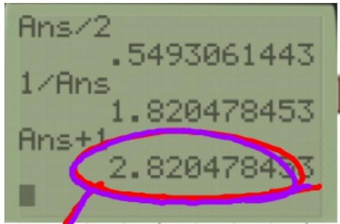
$$2c + 2 = \frac{2 - (-1)}{1 - 0}$$

$$2c + 2 = 3$$

$$-2 \quad -2$$

$$2c = 1$$

$$c = 1/2$$

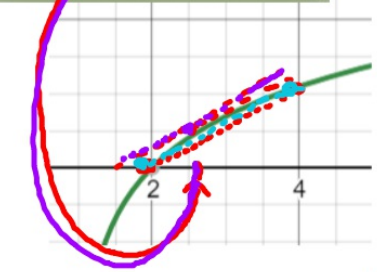


$\ln(x-1)$ on $[2, 4]$

a b

✓ continuous at $[2,4]$

✓ diff. at $(2,4)$



$$\frac{d}{dx}\boxed{\ln(u)} = \frac{1}{u}\cdot u'$$

$f(a) = \ln 1$

$f(b) = \ln 3$

$f(4) =$

$\ln(4-1)$

$u = x - 1$

$u' = 1$

$$f'(c) = \frac{1}{c-1}$$

$$\frac{f(b)-f(a)}{b-a} = \frac{\ln 3 - \ln 1}{2}$$

$$\frac{1}{c-1} = .549$$

$$\frac{1}{.549} = \frac{(c-1)(.549)}{.549}$$

8. $f(x) = \begin{cases} \sin^{-1} x, & -1 \le x < 1 \\ x/2+1, & 1 \le x \le 3 \end{cases}$ on $[-1, 3]$

No. The split function is discontinuous at $x = 1$.

In Exercises 1–8, (**a**) state whether or not the function satisfies the hypotheses of the Mean Value Theorem on the given interval, and (**b**) if it does, find each value of c in the interval (a, b) that satisfies the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

8. $f(x) = \begin{cases} \sin^{-1} x, & -1 \le x < 1 \\ x/2 + 1, & 1 \le x \le 3 \end{cases}$ on $[-1, 3]$

No. The split function is discontinuous at $x = 1$.

1. $f(x) = x^2 + 2x - 1$ on $[0, 1]$
2. $f(x) = x^{2/3}$ on $[0, 1]$
3. $f(x) = x^{1/3}$ on $[-1,1]$
4. $f(x) = |x - 1|$ on $[0, 4]$ No. T
5. $f(x) = \sin^{-1}x$ on $[-1, 1]$
6. $f(x) = \ln(x - 1)$ on $[2, 4]$

a b

① continuous? ✓

differentiable? ✓

$[0,1]$

$f'(x) = 2x + 2$

$f(1) = 1^2 + 2(1) - 1$
$= 1 + 2 - 1$
$= 2$

$f(0) = 0^2 + 2(0) - 1$
$= -1$

$f(1) = 2$

$f(0) = -1$

$$\frac{f(1) - f(0)}{1 - 0}$$

$$2c + 2 = \frac{2 - (-1)}{1 - 0}$$

$$2c+2 = \frac{3}{1}$$

$$2c+2 = 3$$

$$2c = 1$$

$$\boxed{c = \frac{1}{2}}$$

EXAMPLE 3 Applying the Mean Value Theorem

Let $f(x) = \sqrt{1 - x^2}$, $A = (-1, f(-1))$, and $B = (1, f(1))$. Find a tangent to f in the interval $(-1, 1)$ that is parallel to the secant AB.

SOLUTION

The function f (Figure 4.14) is continuous on the interval $[-1, 1]$ and

$$f'(x) = \frac{-x}{\sqrt{1 - x^2}}$$

is defined on the interval $(-1, 1)$. The function is not differentiable at $x = -1$ and $x = 1$, but it does not need to be for the theorem to apply. Since $f(-1) = f(1) = 0$, the tangent we are looking for is horizontal. We find that $f' = 0$ at $x = 0$, where the graph has the horizontal tangent $y = 1$. ***Now try Exercise 9.***

COROLLARY 2 Functions with $f' = 0$ are Constant

If $f'(x) = 0$ at each point of an interval I, then there is a constant C for which $f(x) = C$ for all x in I.

In Exercises 9 and 10, the interval $a \le x \le b$ is given. Let $A = (a, f(a))$ and $B = (b, f(b))$. Write an equation for

(a) the secant line AB.

(b) a tangent line to f in the interval (a, b) that is parallel to AB.

9. $f(x) = x + \dfrac{1}{x}, \quad 0.5 \le x \le 2$ **(a)** $y = \dfrac{5}{2}$ **(b)** $y = 2$

10. $f(x) = \sqrt{x - 1}, \quad 1 \le x \le 3$

$\dfrac{1}{.5}$

$\dfrac{10}{5} = ?$

$f(b) = f(2) = 2 + \dfrac{1}{2}$

$f(a) = f(.5) = .5 + 2$

$f(b) - f(a) =$