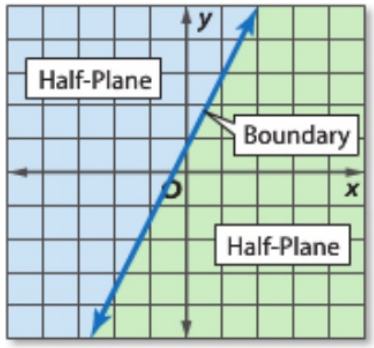
LESSON 5-6

Graphing Inequalities in Two Variables

1 Graph Linear Inequalities

The graph of a linear inequality is the set of points that represent all of the possible solutions of that inequality. An equation defines a **boundary**, which divides the coordinate plane into two **half-planes**.



The boundary may or may not be included in the solution. When it is included, the solution is a **closed half-plane**. When not included, the solution is an **open half-plane**.

KeyConcept Graphing Linear Inequalities

Step 1 Graph the boundary. Use a solid line when the inequality contains $\leq$ or $\geq$. Use a dashed line when the inequality contains $<$ or $>$.

Step 2 Use a test point to determine which half-plane should be shaded.

Step 3 Shade the half-plane that contains the solution.

EXAMPLE 1 Graph an Inequality (< or >)

Graph $2y - 4x > 6$.

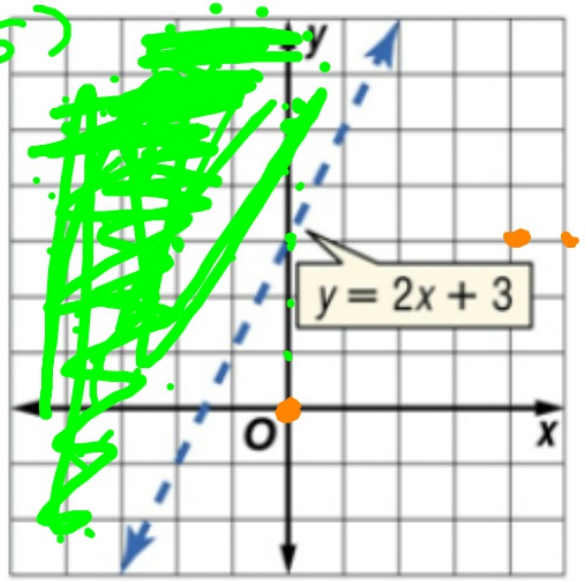
Step 1 Solve for y in terms of x.

$2y - 4x > 6$	Original inequality
$2y - 4x + 4x > 4x + 6$	Add $4x$ to each side.
$2y > 4x + 6$	Simplify.
$\frac{2y}{2} > \frac{4x + 6}{2}$	Divide each side by 2.
$y > 2x + 3$	Simplify.

EXAMPLE 1 Graph an Inequality (< or >)

Step 2 Graph $y = 2x + 3$.

Since $y > 2x + 3$ does not include values when $y = 2x + 3$, the boundary is not included in the solution set. The boundary should be drawn as a dashed line.



Step 3 Select a point in one of the half-planes and test it. Let's use (0, 0).

$y > 2x + 3$	Original inequality
$0 > 2(0) + 3$	$x = 0, y = 0$
$0 > 3$	false

EXAMPLE 1 Graph an Inequality (< or >)

Since the statement is false, the half-plane containing the origin is not part of the solution. Shade the other half-plane.

Check Test a point in the other half-plane, for example, (–3, 1).

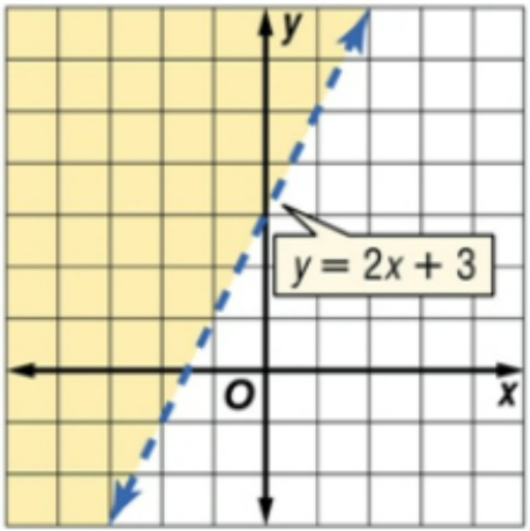
$y > 2x + 3$ Original inequality

$1 > 2(-3) + 3$ $x = -3$, $y = 1$

$1 > -3$ ✓

Since the statement is true, the half-plane containing (–3, 1) should be shaded. The graph of the solution is correct.

Answer:



EXAMPLE 2 Graph an Inequality (≤ or ≥)

Graph $x + 4y \geq 2$.

Step 1 Solve for y in terms of x.

$x + 4y \geq 2$	Original inequality
$4y \geq -x + 2$	Subtract x from both sides and simplify.
$y \geq -\frac{1}{4}x + \frac{1}{2}$	Divide each side by 4.

EXAMPLE 2 Graph an Inequality (≤ or ≥)

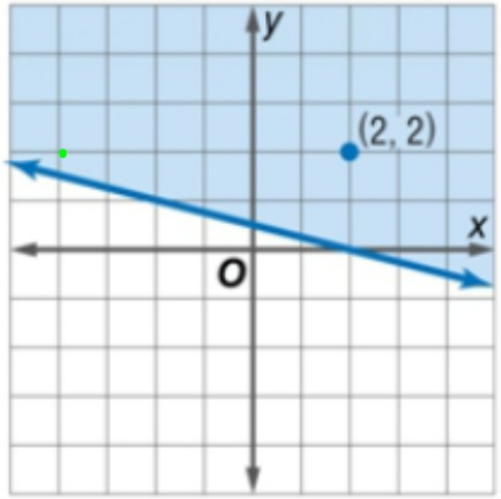
Graph $y \geq -\frac{1}{4}x + \frac{1}{2}$. Because the inequality symbol is ≥, graph the boundary with a solid line.

Step 2 Select a test point. Let's use (2, 2). Substitute the values into the original inequality.

$x + 4y \geq 2$	Original inequality
$2 + 4(2) \geq 2$	$x = 2$ and $y = 2$
$10 \geq 2$	Simplify.

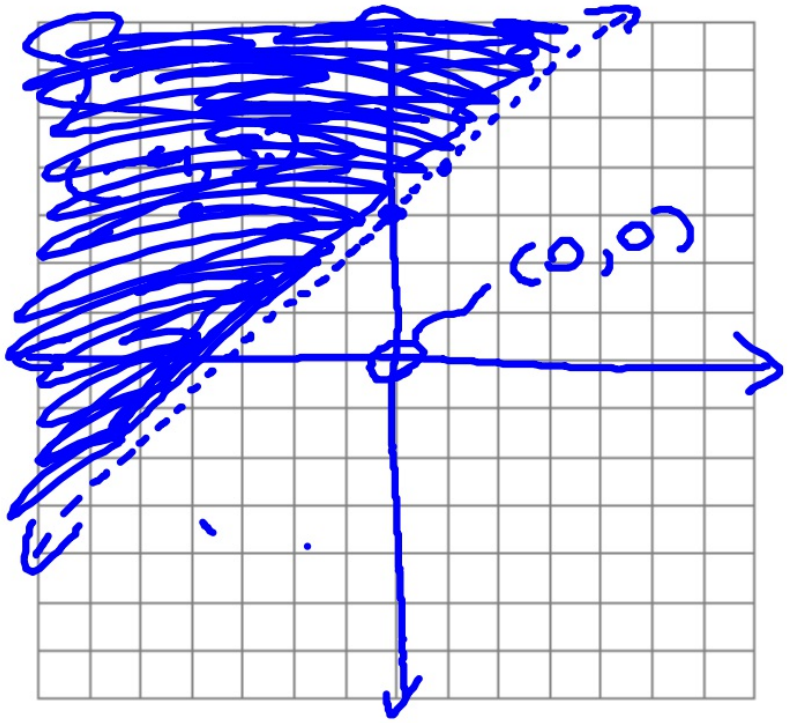
Step 3 Since the statement is true, shade the same half-plane.

Answer:



Examples 1–2 Graph each inequality. **1–6. See Ch. 5 Answer Appendix.**

1. $y > x + 3$
2. $y \geq -8$
3. $x + y > 1$
4. $y \leq x - 6$
5. $y < 2x - 4$
6. $x - y \leq 4$



$y > \frac{1}{1}x + 3$

$m = \frac{1}{1}$

$b = 3$

Test (0,0);

$0 > 3$ false

Test (-4,3)

$3 > -4 + 3$

$3 > -1$

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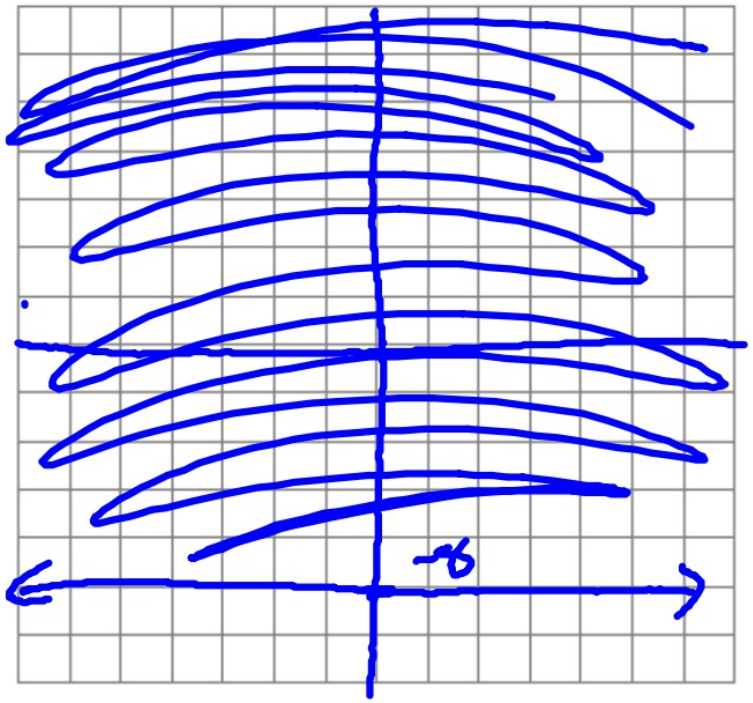

$y = -8$ $y \geq -8$

x	y
-2	-8
0	-8
4	-8

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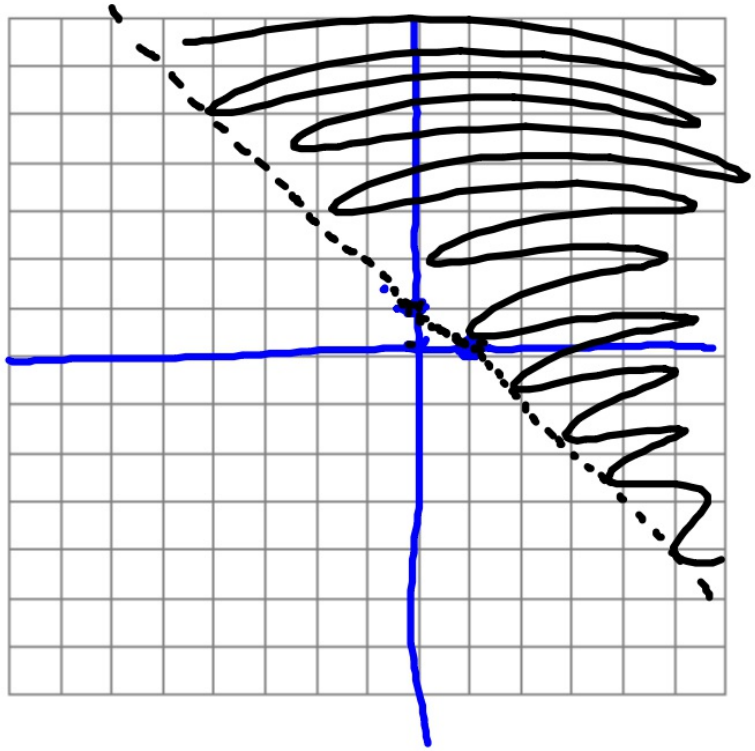

(3) $x + y = 1$

$-x \quad -x$

$y = -x + 1$

$m = \frac{-1}{1}$

Test (0,0)

$0 + 0 > 1$ false

Examples 1–2 Graph each inequality. **1–6. See Ch. 5 Answer Appendix.**

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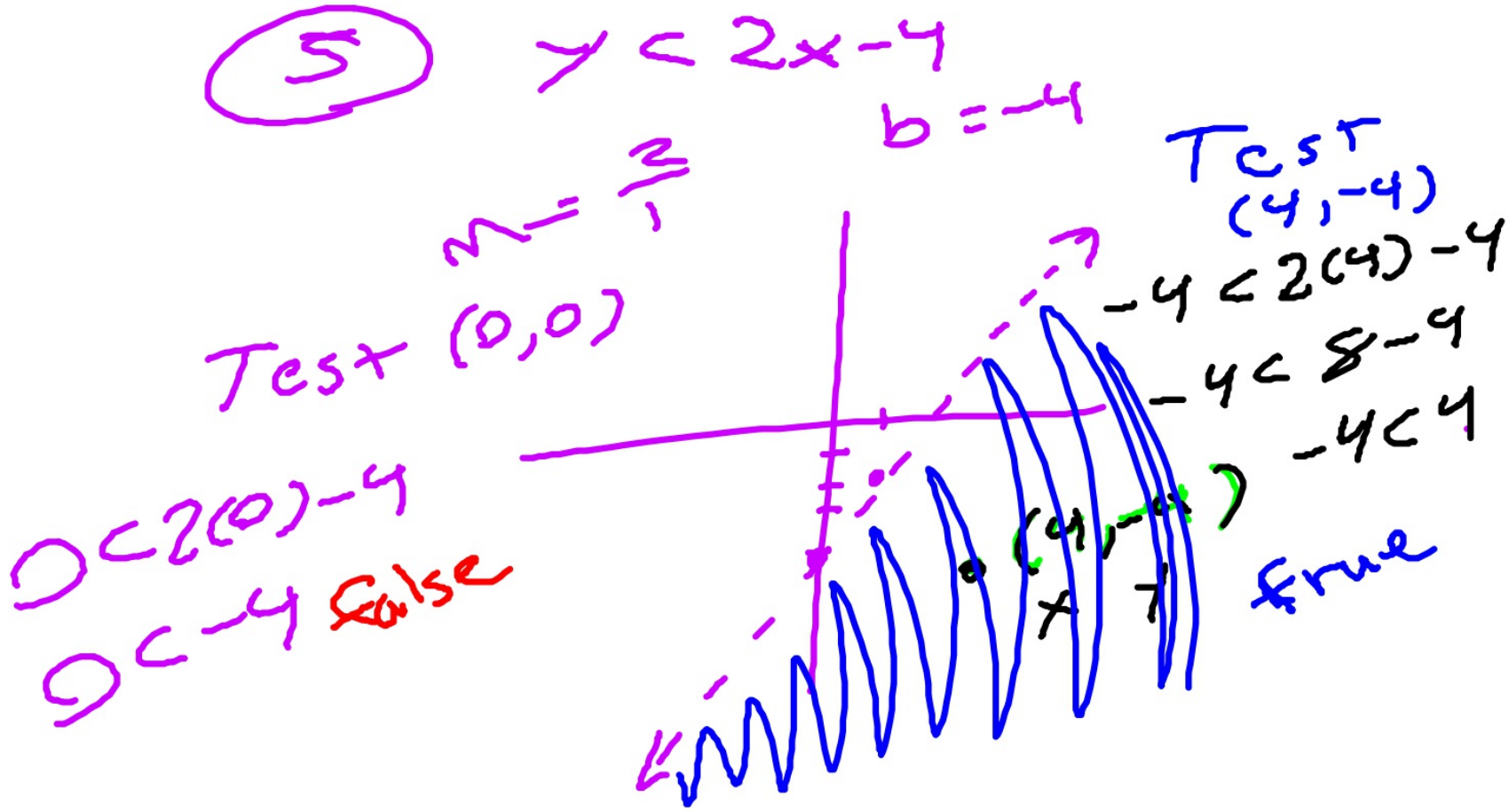

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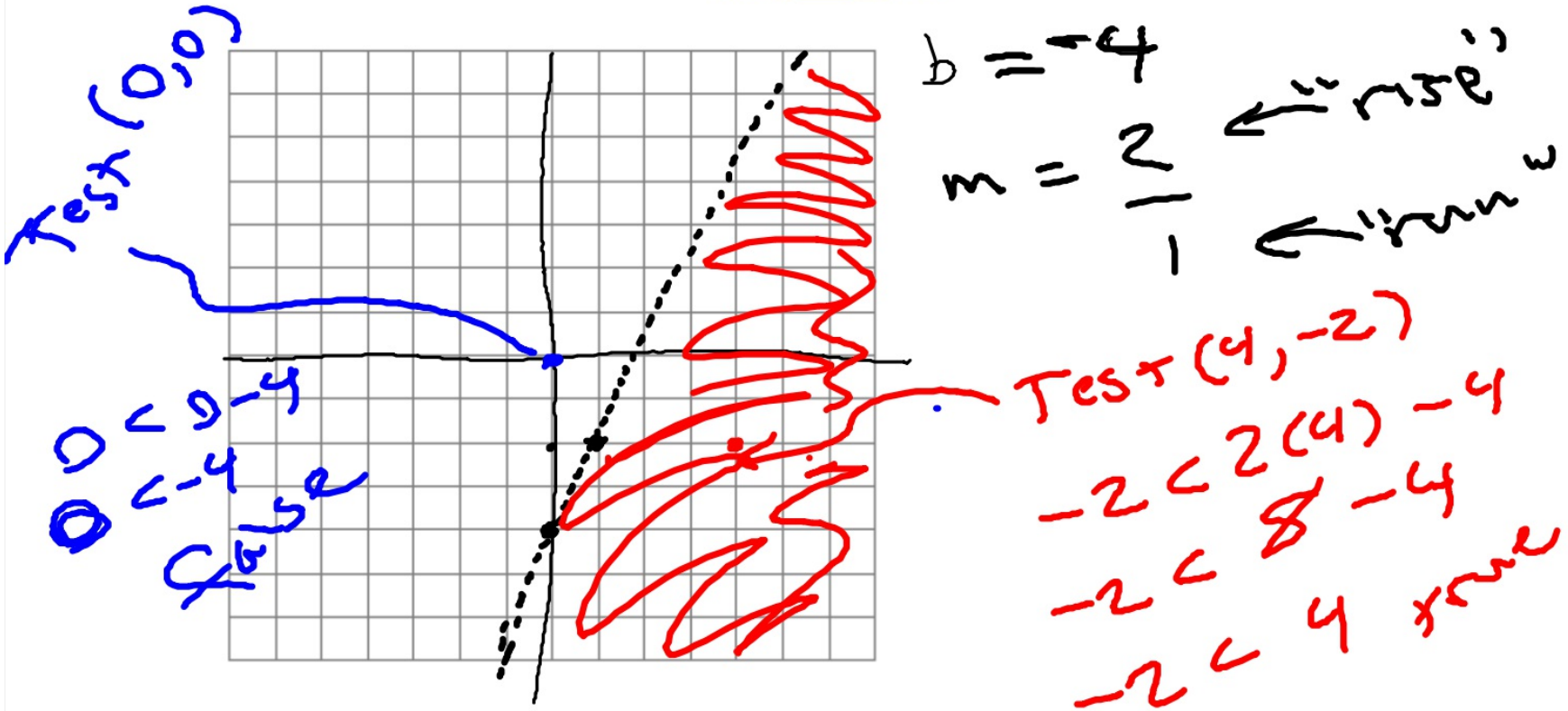

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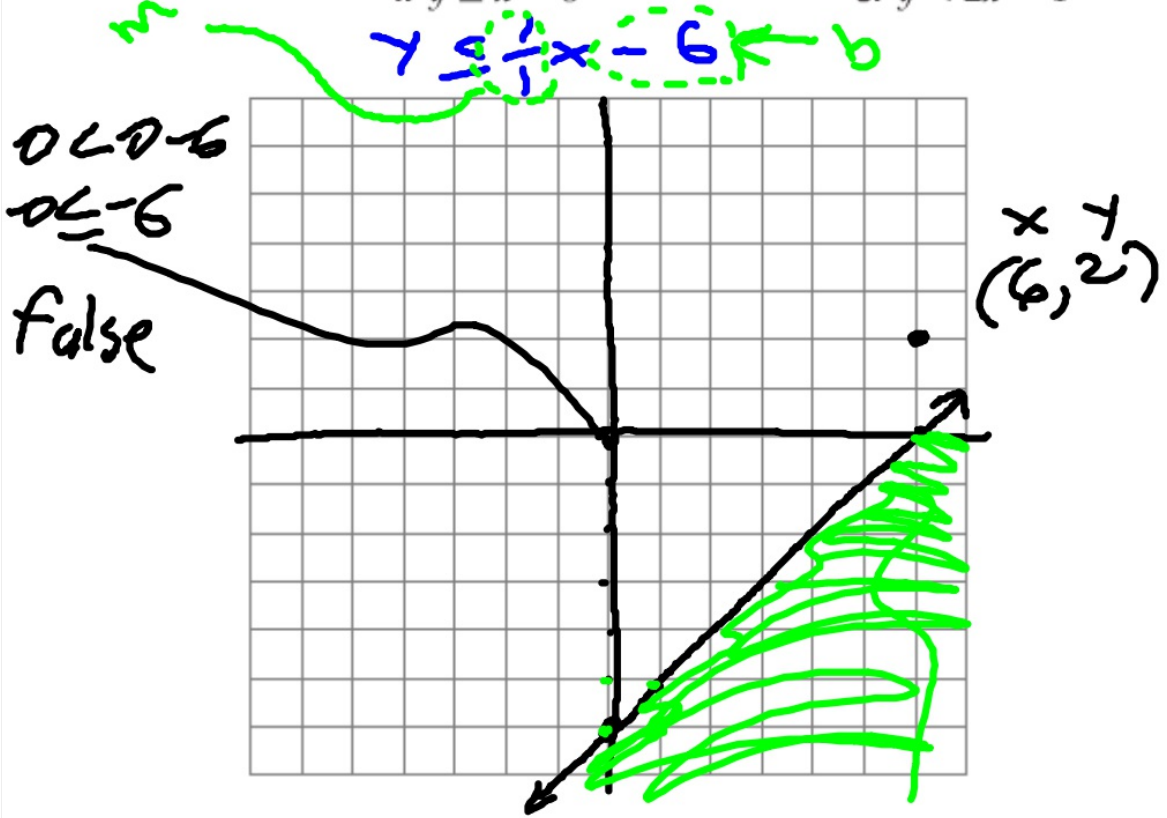


$2 \leq 6 - 6$

$2 \leq 0$ false

Examples 1–2 Graph each inequality. **12–23. See Ch. 5 Answer Appendix.**

12. $y < x - 3$

13. $y > x + 12$

14. $y \geq 3x - 1$

15. $y \leq -4x + 12$

16. $6x + 3y > 12$

17. $2x + 2y < 18$

18. $5x + y > 10$

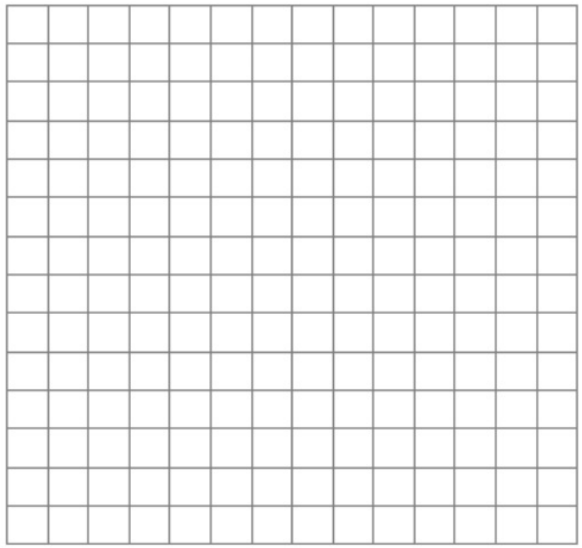
19. $2x + y < -3$

20. $-2x + y \geq -4$

21. $8x + y \leq 6$

22. $10x + 2y \leq 14$

23. $-24x + 8y \geq -48$



Examples 1–2 Graph each inequality. **12–23. See Ch. 5 Answer Appendix.**

$y = mx + b$

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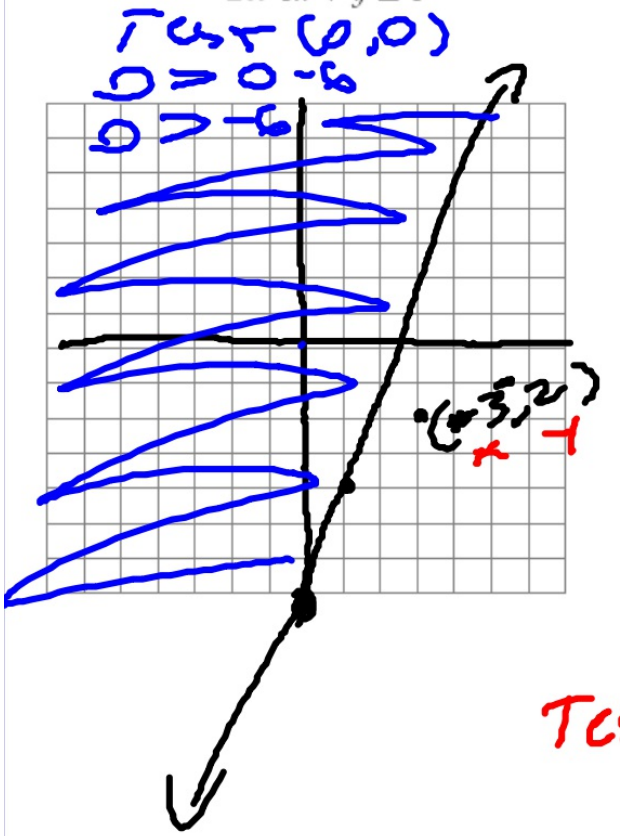

23 $-24x + 8y \geq -48$

$+24x \quad +24x$

$\frac{8y}{8} \geq \frac{24x}{8} - \frac{48}{8}$

$y > 3x - 6$

$m = \frac{3}{1}$

Test (3,-2)

$-2 > 3(3) - 6$

$-2 > 9 - 6$

$-2 > 3$ false

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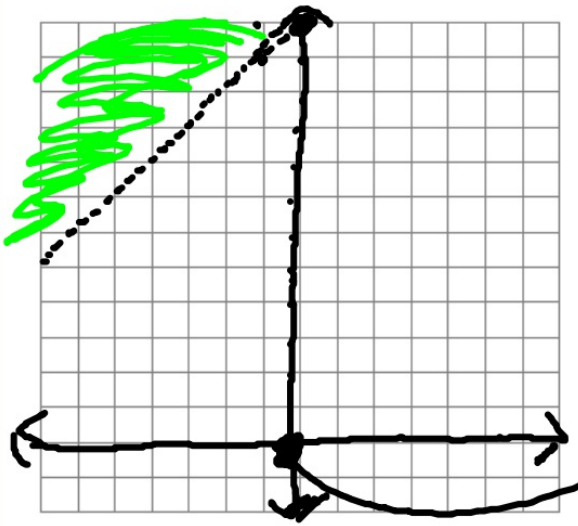

13 $y > \frac{-1}{-1}x + 12$

Test (0,0)

$0 > 0 + 12$

$0 > 12$. False

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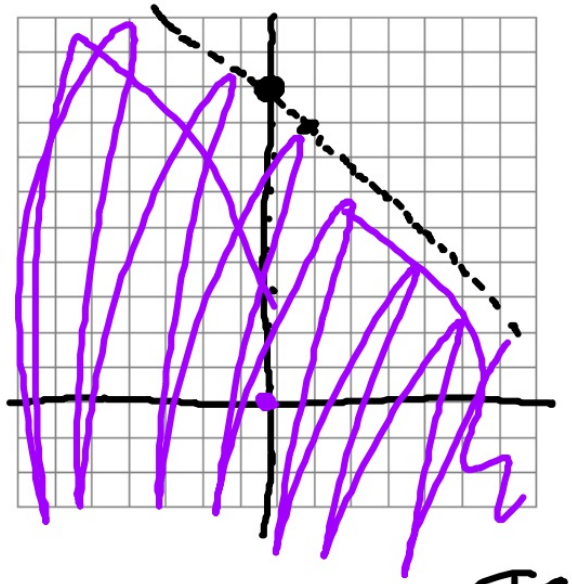

y = mx + b

0 < 18

(17) $2x + 2y < 18$

$\frac{2x}{2} + \frac{2y}{2} < \frac{18}{2}$

$x + y < 9$

solve for y!!!

$-x \quad -x$

$y < -x + 9$

$m = -1$

$b = 9$

Test (0,0)

$0 < 0 + 9$

$0 < 9$