

Example 4 Use Zeros to Write a Polynomial Function

Write a polynomial function of least degree with integral coefficients, the zeros of which include -1 and $5 - i$.

Understand If $5 - i$ is a zero, then $5 + i$ is also a zero according to the Complex Conjugates Theorem. So, $x + 1$, $x - (5 - i)$, and $x - (5 + i)$ are factors of the polynomial.

Plan Write the polynomial function as a product of its factors.

$$P(x) = (x + 1)[x - (5 - i)][x - (5 + i)]$$

Solve Multiply the factors to find the polynomial function.

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|--|-----------------------|
| $P(x) = (x + 1)[x - (5 - i)][x - (5 + i)]$ | Write the equation. |
| $= (x + 1)[(x - 5) + i][(x - 5) - i]$ | Regroup terms. |
| $= (x + 1)[(x - 5)^2 - i^2]$ | Difference of squares |
| $= (x + 1)[x^2 - 10x + 25 - (-1)]$ | Square terms. |
| $= (x + 1)(x^2 - 10x + 26)$ | Simplify. |
| $= x^3 - 10x^2 + 26x + x^2 - 10x + 26$ | Multiply. |
| $= x^3 - 9x^2 + 16x + 26$ | Combine like terms. |

Check Because there are 3 zeros, the degree of the polynomial function must be 3, so $P(x) = x^3 - 9x^2 + 16x + 26$ is a polynomial function of least degree with integral coefficients and zeros of -1 , $5 - i$, and $5 + i$.

Example 4

Write a polynomial function of least degree with integral coefficients that have the given zeros.

13. 4, -1, 6 $f(x) = x^3 - 9x^2 + 14x + 24$

14. 3, -1, 1, 2 $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

15. -2, 5, -3i
 $f(x) = x^4 - 3x^3 - x^2 - 27x - 90$

16. -4, 4 + i $f(x) = x^3 - 4x^2 - 15x + 68$

13

$f(x) = (x-4)(x+1)(x-6)$

$(x^2 - 3x - 4)(x - 6)$

$x^2 - 3x - 4$

| | | | |
|------|---------|---------|-------|
| x | x^3 | $-3x^2$ | $-4x$ |
| -6 | $-6x^2$ | $+18x$ | $+24$ |

15

$(x+2)(x-5)(x+3i)(x-3i)$

$(x^2 - 3x - 10)(x^2 - 9i^2)$

$(x^2 - 3x - 10)(x^2 + 9)$

$x^2 - 3x - 10$

| | | | |
|-----|--------|---------|----------|
| x | x^4 | $-3x^3$ | $-10x^2$ |
| 9 | $9x^2$ | $-27x$ | -90 |

$i^2 = -1$

Solve each equation. State the number and type of roots. **17–26. See margin.**

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|----------------------------|---------------------------|
| 17. $2x^2 + x - 6 = 0$ | 18. $4x^2 + 1 = 0$ |
| 19. $x^3 + 1 = 0$ | 20. $2x^2 - 5x + 14 = 0$ |
| 21. $-3x^2 - 5x + 8 = 0$ | 22. $8x^3 - 27 = 0$ |
| 23. $16x^4 - 625 = 0$ | 24. $x^3 - 6x^2 + 7x = 0$ |
| 25. $x^5 - 8x^3 + 16x = 0$ | 26. $x^5 + 2x^3 + x = 0$ |

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

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| 27. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$ 0 or 2; 0 or 2; 0, 2, or 4 | 28. $f(x) = 2x^3 - 7x^2 - 2x + 12$ 0 or 2; 1; 0 or 2 |
| 29. $f(x) = -3x^5 + 5x^4 + 4x^2 - 8$ 0 or 2; 1; 2 or 4 | 30. $f(x) = x^4 - 2x^2 - 5x + 19$ 0 or 2; 0 or 2; 0, 2, or 4 |
| 31. $f(x) = 4x^6 - 5x^4 - x^2 + 24$ 0 or 2; 0 or 2; 2, 4, or 6 | 32. $f(x) = -x^5 + 14x^3 + 18x - 36$ 0 or 2; 1; 2 or 4 |

Find all of the zeros of each function.

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|---|---|
| 33. $f(x) = x^3 + 7x^2 + 4x - 12$ -6, -2, 1 | 34. $f(x) = x^3 + x^2 - 17x + 15$ -5, 1, 3 |
| 35. $f(x) = x^4 - 3x^3 - 3x^2 - 75x - 700$ -4, 7, -5i, 5i | 36. $f(x) = x^4 + 6x^3 + 73x^2 + 384x + 576$ -3, -3, -8i, 8i |
| 37. $f(x) = x^4 - 8x^3 + 20x^2 - 32x + 64$ | 38. $f(x) = x^5 - 8x^3 - 9x$ -3, 0, 3, -i, i |

Write a polynomial function of least degree with integral coefficients that have the given zeros. **39–44. See margin.**

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|----------------|------------------|--------------------|-------------------|
| 39. 5, -2, -1 | 40. -4, -3, 5 | 41. -1, -1, 2i | 37. 4, 4, -2i, 2i |
| 42. -3, 1, -3i | 43. 0, -5, 3 + i | 44. -2, -3, 4 - 3i | |

43

$$(x+0)(x+5) \left[x - (3+i) \right] \left[x - (3-i) \right]$$

$$x(x+5) ((x-3)-i) ((x-3)+i)$$

$$x(x+5) \left[(x-3)^2 - i^2 \right]$$

$$(x^2+5x) (x^2-6x+9-(-1))$$

$$(x^2+5x) (x^2-6x+10)$$

... then distribute :)

ANSWER KEY

17. $-2, \frac{3}{2}$; 2 real
 18. $-\frac{1}{2}i, \frac{1}{2}i$; 2 imaginary
 19. $-1, \frac{1 \pm \sqrt{3}}{2}$; 1 real, 2 imaginary
 20. $\frac{5 \pm \sqrt{87}}{4}$; 2 imaginary
 21. $-\frac{8}{3}, 1$; 2 real
 22. $\frac{3}{2}, \frac{-3 \pm 3\sqrt{3}}{4}$; 1 real, 2 imaginary
 23. $-\frac{5}{2}, \frac{5}{2}, -\frac{5}{2}i, \frac{5}{2}i$; 2 real, 2 imaginary
 24. 0, $3 + \sqrt{2}$, $3 - \sqrt{2}$; 3 real
 25. -2, -2, 0, 2, 2; 5 real
 26. 0, -i, -i, i, i; 1 real, 4 imaginary
 39. $f(x) = x^3 - 2x^2 - 13x - 10$
 40. $f(x) = x^3 + 2x^2 - 23x - 60$
 41. $f(x) = x^4 + 2x^3 + 5x^2 + 8x + 4$
 42. $f(x) = x^4 + 2x^3 + 6x^2 + 18x - 27$
 43. $f(x) = x^4 - x^3 - 20x^2 + 50x$
 44. $f(x) = x^4 - 3x^3 - 9x^2 + 77x + 150$