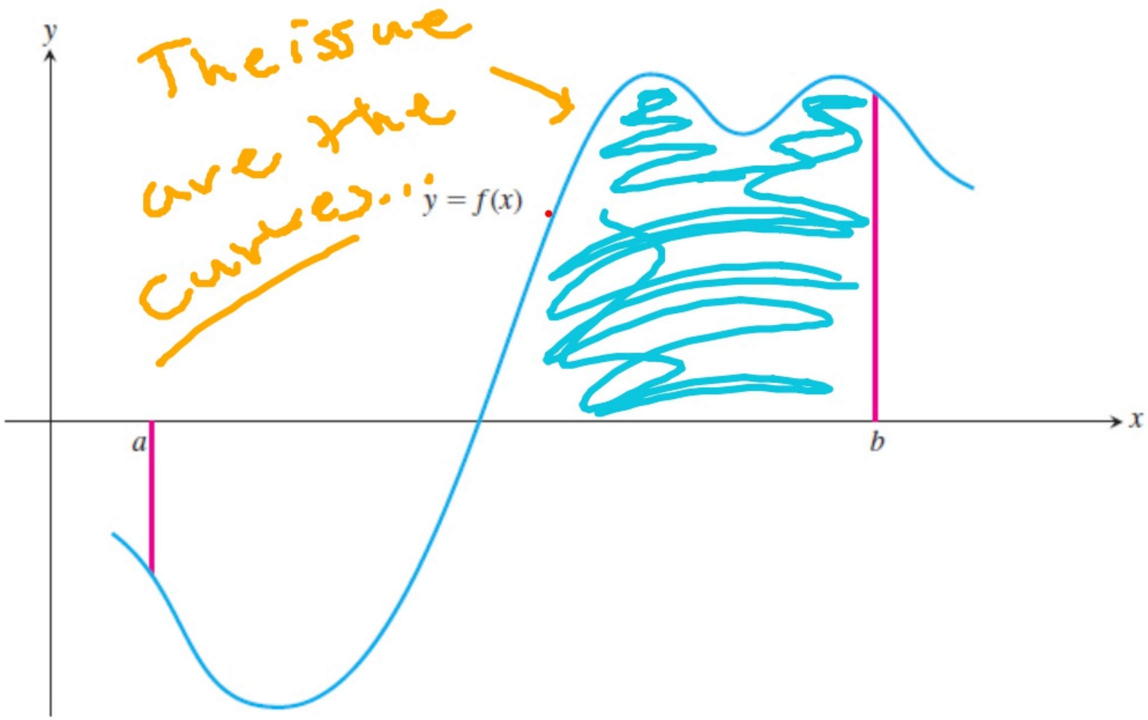
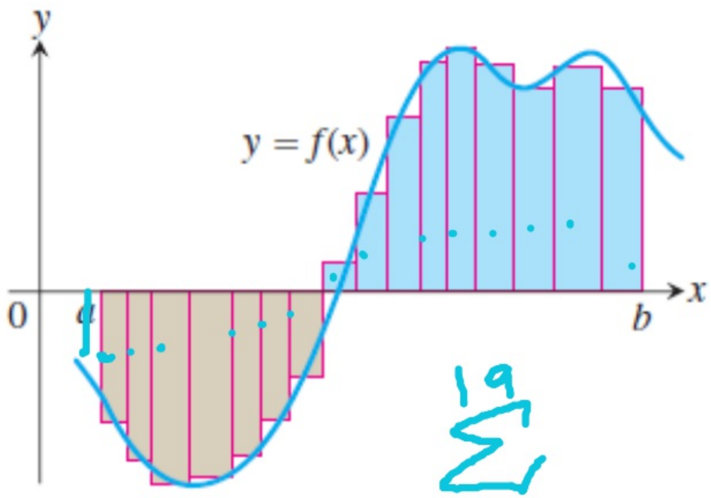
5.2 Definite Integrals

Q: What is the area enclosed between the graph and the x axis in the closed interval [a,b] ?





(a)

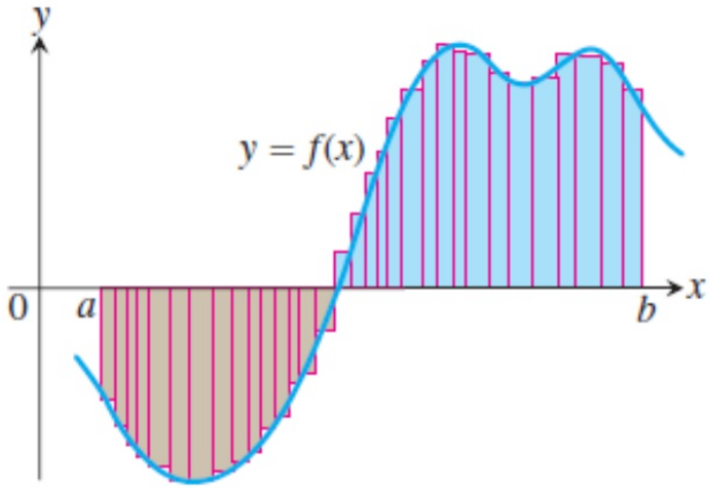
$\sum_{i=1}^{19}$

A: We could break it up into rectangles, then add up all the rectangles...

"height" "base"

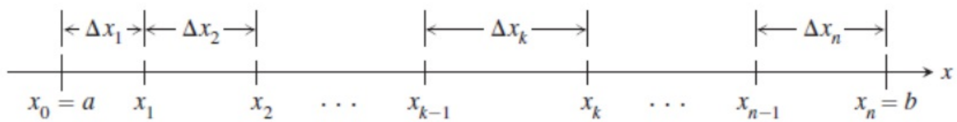
$$S_n = \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k.$$

Riemann sum for f on the interval $[a, b]$.



(b)

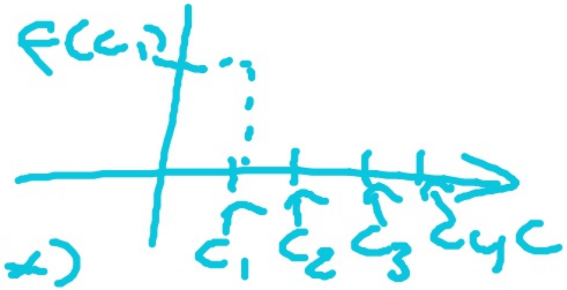
The basic idea is, the smaller the partitions, the more accurate the area...



This agruement can be molded into a limit, and is given a new notation altogether;

$$\lim_{n\to\infty}\sum_{k=1}^{n} \underbrace{f(c_k)}_{\text{“height”}}\,\underbrace{\Delta x}_{\text{“base”}} = \int_a^b f(x)\,dx.$$

$$\sum_{i=1}^{4} f(x)$$



$$\int_a^b f(x)\,dx$$

Upper limit of integration

Integral sign

Lower limit of integration

The function is the **integrand.**

x is the **variable of integration.**

Integral of f from a to b

When you find the value of the integral, you have **evaluated the integral.**

In Exercises 1–6, each c_k is chosen from the kth subinterval of a regular partition of the indicated interval into n subintervals of length Δx. Express the limit as a definite integral.

1. $\lim_{n\to\infty} \sum_{k=1}^{n} c_k^2 \Delta x$, $[0, 2]$ $\int_0^2 x^2\, dx$

2. $\lim_{n\to\infty} \sum_{k=1}^{n} (c_k^2 - 3c_k)\Delta x$, $[-7, 5]$ $\int_{-7}^{5} (x^2 - 3x)\, dx$

3. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{c_k} \Delta x$, $[1, 4]$ $\int_1^4 \frac{1}{x}\, dx$

4. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1 - c_k} \Delta x$, $[2, 3]$ $\int_2^3 \frac{1}{1-x}\, dx$

DEFINITION **Area Under a Curve (as a Definite Integral)**

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve** $y = f(x)$ **from** a **to** b is the integral of f from a to b,

$$A = \int_a^b \underbrace{f(x)\,dx}_{dy \cdot dx}.$$

last semester...

$$\frac{dy}{dx}$$

Note for the eager ones in here;

We will get into the antiderivative, but we will first approach this geometrically.

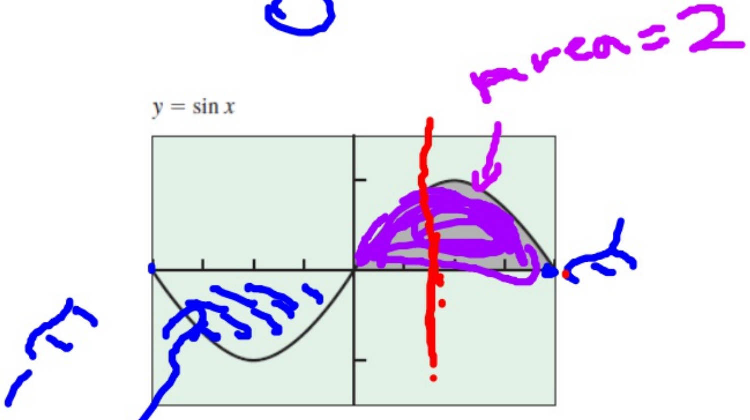
Also, we will be VERY careful when finding the area

$$\int_a^b f(x)\,dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis}).$$

EXPLORATION 1 Finding Integrals by Signed Areas

It is a fact (which we will revisit) that $\int_0^{\pi} \sin x \, dx = 2$ (Figure 5.20). With that information, what you know about integrals and areas, what you know about graphing curves, and sometimes a bit of intuition, determine the values of the following integrals. Give as convincing an argument as you can for each value, based on the graph of the function.

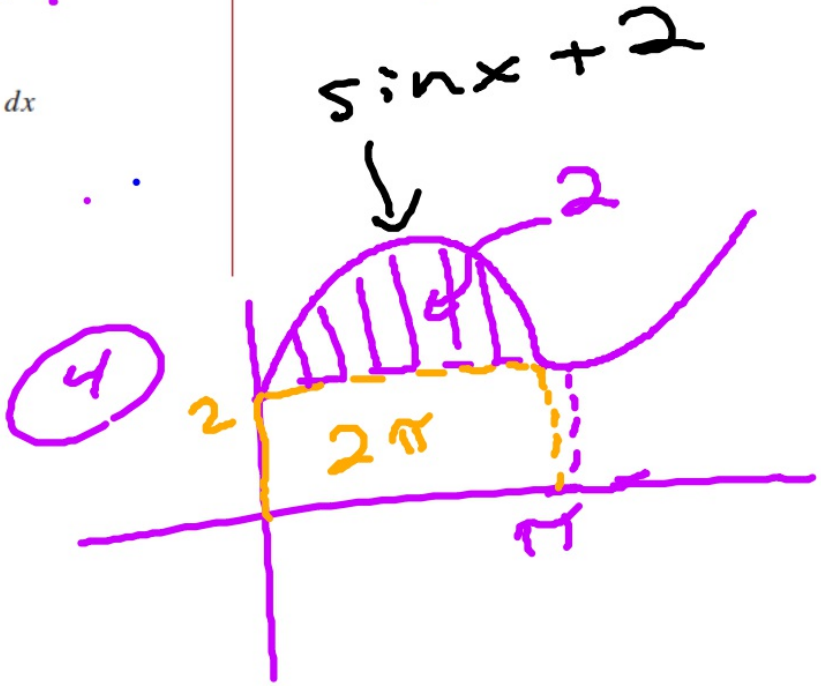
1. $\int_{\pi}^{2\pi} \sin x \, dx$ $= -2$
2. $\int_{0}^{2\pi} \sin x \, dx$ $= 0$
3. $\int_{0}^{\pi/2} \sin x \, dx$ $= 1$
4. $\int_{0}^{\pi} (2 + \sin x) \, dx$ $2\pi + 2$
5. $\int_{0}^{\pi} 2 \sin x \, dx$ 4
6. $\int_{2}^{\pi+2} \sin (x - 2) \, dx$
7. $\int_{-\pi}^{\pi} \sin u \, du$ $= 0$
8. $\int_{0}^{2\pi} \sin (x/2) \, dx$
9. $\int_{0}^{\pi} \cos x \, dx$



[−π, π] by [−1.5, 1.5]

Figure 5.20

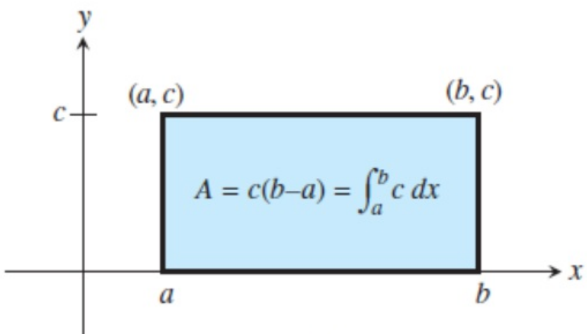
$\int_0^{\pi} \sin x \, dx = 2$. (Exploration 1)



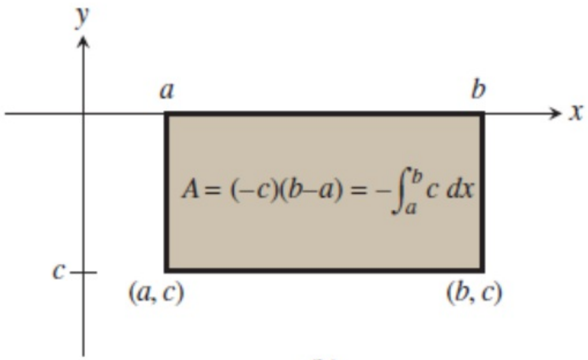
THEOREM 2 The Integral of a Constant

If $f(x) = c$, where c is a constant, on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = \int_a^b c\,dx = c(b - a).$$



(a)



(b)

In Exercises 7–12, evaluate the integral.

7. $\displaystyle\int_{-2}^{1} 5\,dx$ 15

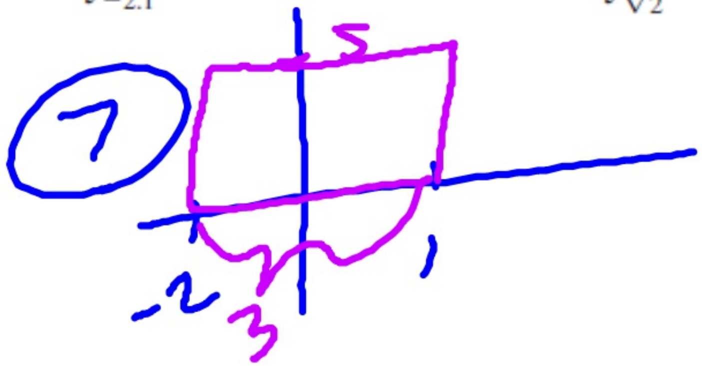
8. $\displaystyle\int_{3}^{7} (-20)\,dx$

9. $\displaystyle\int_{0}^{3} (-160)\,dt$ -480

10. $\displaystyle\int_{-4}^{-1} \frac{\pi}{2}\,d\theta$ $\frac{3\pi}{2}$

11. $\displaystyle\int_{-2.1}^{3.4} 0.5\,ds$

12. $\displaystyle\int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2}\,dr$



In Exercises 13–22, use the graph of the integrand and areas to evaluate the integral.

13. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$ 21

14. $\int_{1/2}^{3/2} (-2x + 4)\, dx$ 2

15. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$ $\frac{9\pi}{2}$

16. $\int_{-4}^{0} \sqrt{16 - x^2}\, dx$ 4π

17. $\int_{-2}^{1} |x|\, dx$ $\frac{5}{2}$

18. $\int_{-1}^{1} (1 - |x|)\, dx$ 1

19. $\int_{-1}^{1} (2 - |x|)\, dx$ 3

20. $\int_{-1}^{1} (1 + \sqrt{1 - x^2})\, dx$ $2 + \frac{\pi}{2}$

21. $\int_{\pi}^{2\pi} \theta\, d\theta$ $\frac{3\pi^2}{2}$

22. $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$ 24

In Exercises 13–22, use the graph of the integrand and areas to evaluate the integral.

13. $\int_{-2}^{4}\left(\frac{x}{2}+3\right)dx$ 21

14. $\int_{1/2}^{3/2}(-2x+4)\,dx$ 2

15. $\int_{-3}^{3}\sqrt{9-x^2}\,dx$ $\frac{9\pi}{2}$

16. $\int_{-4}^{0}\sqrt{16-x^2}\,dx$ 4π

17. $\int_{-2}^{1}|x|\,dx$ $\frac{5}{2}$

18. $\int_{-1}^{1}(1-|x|)\,dx$ 1

19. $\int_{-1}^{1}(2-|x|)\,dx$ 3

20. $\int_{-1}^{1}\left(1+\sqrt{1-x^2}\right)dx$ $2+\frac{\pi}{2}$

21. $\int_{\pi}^{2\pi}\theta\,d\theta$ $\frac{3\pi^2}{2}$

22. $\int_{\sqrt{2}}^{5\sqrt{2}} r\,dr$ 24

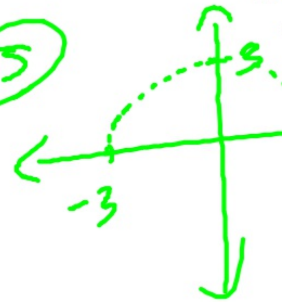
17

$|-2| = 2$

$|1| = 1$

find the area of the 2 triangles

15



semi circle

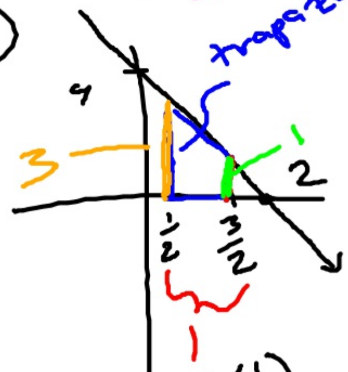
$$\int_{-3}^{3}\sqrt{9-x^2} = \frac{\pi r^2}{2}$$

$r = 3$

$$\frac{\pi 3^2}{2} = \frac{9\pi}{2}$$

14

trapezoid



$$A = \frac{1}{2}(1+3)(1) = \frac{4}{2}(1) = 2(1) = 2$$