Examples

6-5 (

a. $\sqrt{32x^8}$

$\sqrt{32x^8} = \sqrt{4^2 \cdot 2 \cdot (x^4)^2}$ Factor into squares.

$= \sqrt{4^2} \cdot \sqrt{(x^4)^2} \cdot \sqrt{2}$ Product Property of Radicals

$= 4x^4\sqrt{2}$ Simplify.

b. $\sqrt[4]{16a^{24}b^{13}}$

$\sqrt[4]{16a^{24}b^{13}} = \sqrt[4]{2^4 \cdot (a^6)^4(b^3)^4 \cdot b}$ Factor into squares.

$= \sqrt[4]{2^4} \cdot \sqrt[4]{(a^6)^4} \cdot \sqrt[4]{(b^3)^4} \cdot \sqrt[4]{b}$ Product Property of Radicals

$= 2a^6|b^3|\sqrt[4]{b}$ Simplify.

In this case, the absolute value symbols are not necessary because in order for $\sqrt[4]{16a^{24}b^{13}}$ to be defined, b must be nonnegative.

Thus, $\sqrt[4]{16a^{24}b^{13}} = 2a^6b^3\sqrt[4]{b}$.

KeyConcept Product Property of Radicals

Words	For any real numbers a and b and any integer $n > 1$, $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, if n is even and a and b are both nonnegative or if n is odd.
Examples	$\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$ or 4 and $\sqrt[3]{3} \cdot \sqrt[3]{9} = \sqrt[3]{27}$ or 3

Examples 1–5 CCSS **PRECISION** Simplify.

1. $\sqrt{36ab^4c^5}$

   $6b^2c^2\sqrt{ac}$

2. $\sqrt{144x^7y^5}$

   $12x^3y^2\sqrt{xy}$

To eliminate radicals from a denominator or fractions from a radicand, you can use a process called **rationalizing the denominator**. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

If the denominator is:	Multiply the numerator and denominator by:	Examples
$\sqrt{b}$	$\sqrt{b}$	$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$
$\sqrt[n]{b^x}$	$\sqrt[n]{b^{n-x}}$	$\frac{5}{\sqrt[3]{2}} = \frac{5}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$ or $\frac{5\sqrt[3]{4}}{2}$

Example 2 Simplify Expressions with the Quotient Property

Simplify.

a. $\sqrt{\frac{x^6}{y^7}}$

b. $\sqrt[4]{\frac{6}{5x}}$

To eliminate radicals from a denominator or fractions from a radicand, you can use a process called **rationalizing the denominator**. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

If the denominator is:	Multiply the numerator and denominator by:	Examples
$\sqrt{b}$	$\sqrt{b}$	$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$
$\sqrt[n]{b^x}$	$\sqrt[n]{b^{n-x}}$	$\frac{5}{\sqrt[3]{2}} = \frac{5}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$ or $\frac{5\sqrt[3]{4}}{2}$

3. $\frac{\sqrt{c^5}}{\sqrt{d^9}}$ $\frac{c^2\sqrt{cd}}{d^5}$

4. $\sqrt[4]{\frac{5x}{8y}}$ $\frac{\sqrt[4]{10xy^3}}{2y}$

Concept Summary: Simplifying Radical Expressions

A radical expression is in simplified form when the following conditions are met.

- The index n is as small as possible.
- The radicand contains no factors (other than 1) that are nth powers of an integer or polynomial.
- The radicand contains no fractions.
- No radicals appear in a denominator.

Example 3 Multiply Radicals

Simplify $5\sqrt[3]{-12ab^4} \cdot 3\sqrt[3]{18a^2b^2}$.

(5) $15\sqrt{16x^2} = 15(4x)$

(7) $6\sqrt[3]{216x^3y^3} = 6(6xy)$

5. $5\sqrt{2x} \cdot 3\sqrt{8x}$ **60*x***

6. $4\sqrt{5a^5} \cdot \sqrt{125a^3}$

7. $3\sqrt[3]{36xy} \cdot 2\sqrt[3]{6x^2y^2}$ **36*xy***

8. $\sqrt[4]{3x^3y^2} \cdot \sqrt[4]{27xy^2}$

Radicals can be added and subtracted in the same manner as monomials. In order to add or subtract, the radicals must be like terms. Radicals are **like radical expressions** if *both* the index and the radicand are identical.

Like: $\sqrt{3b}$ and $4\sqrt{3b}$

Unlike: $\sqrt{3b}$ and $\sqrt[3]{3b}$

Unlike: $\sqrt{2b}$ and $\sqrt{3b}$

Example 4 Add and Subtract Radicals

Simplify $\sqrt{98} - 2\sqrt{32}$.

9. $5\sqrt{32} + \sqrt{27} + 2\sqrt{75}$

10. $4\sqrt{40} + 3\sqrt{28} - \sqrt{200}$

Example 5 Multiply Radicals

Simplify $(4\sqrt{3} + 5\sqrt{2})(3\sqrt{2} - 6)$.

F O I L

$$(4\sqrt{3} + 5\sqrt{2})(3\sqrt{2} - 6) = 4\sqrt{3} \cdot 3\sqrt{2} + 4\sqrt{3} \cdot (-6) + 5\sqrt{2} \cdot 3\sqrt{2} + 5\sqrt{2} \cdot (-6)$$

$$= 12\sqrt{3 \cdot 2} - 24\sqrt{3} + 15\sqrt{2^2} - 30\sqrt{2}$$ Product Property

$$= 12\sqrt{6} - 24\sqrt{3} + 30 - 30\sqrt{2}$$ Simplify.

11. $(4 + 2\sqrt{5})(3\sqrt{3} + 4\sqrt{5})$

12. $(8\sqrt{3} - 2\sqrt{2})(8\sqrt{3} + 2\sqrt{2})$

Real-World Example 6 Use a Conjugate to Rationalize a Denominator

ARCHITECTURE **Refer to the beginning of the lesson. Use a conjugate to rationalize the denominator and simplify $\frac{2}{\sqrt{5}-1}$.**

$$\frac{2}{\sqrt{5}-1} = \frac{2}{\sqrt{5}-1} \cdot \frac{\sqrt{5}+1}{\sqrt{5}+1}$$ $\sqrt{5}+1$ is the conjugate of $\sqrt{5}-1$.

$$= \frac{2\sqrt{5}+2(1)}{(\sqrt{5})^2 + 1(\sqrt{5}) - 1(\sqrt{5}) - 1(1)}$$ Multiply.

$$= \frac{2\sqrt{5}+2}{5+\sqrt{5}-\sqrt{5}-1}$$ Simplify.

$$= \frac{2\sqrt{5}+2}{4}$$ Subtract.

$$= \frac{\sqrt{5}+1}{2}$$ Simplify.

13. $\frac{5}{\sqrt{2}+3}$

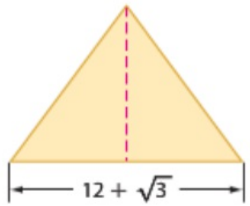
14. $\frac{8}{\sqrt{6}-5}$

15. $\frac{4+\sqrt{2}}{\sqrt{2}-3}$

16. $\frac{6-\sqrt{3}}{\sqrt{3}+4}$

Example 6

17. **GEOMETRY** Find the altitude of the triangle if the area is $189 + 4\sqrt{3}$ square centimeters. $32 - 2\sqrt{3}$ cm



18. $6a^4b^2\sqrt{2b}$
19. $3a^7b\sqrt{ab}$
20. $2a^8b^4\sqrt{6c}$
21. $3|a^3|bc^2\sqrt{2bc}$

Practice and Problem Solving

Extra Practice is on page R6.

Examples 1–4 Simplify. 27. $32a^5b^3\sqrt{b}$ 29. $25x^6y^3\sqrt{2xy}$ 30. $9\sqrt{10} + 8\sqrt{5} + 9\sqrt{2}$

18. $\sqrt{72a^8b^5}$ 19. $\sqrt{9a^{15}b^3}$ 20. $\sqrt{24a^{16}b^8c}$ 21. $\sqrt{18a^6b^3c^5}$

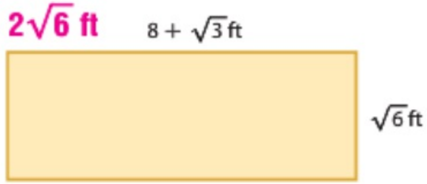
22. $\frac{\sqrt{5a^5}}{\sqrt{b^{13}}}$ $\frac{a^2\sqrt{5ab}}{b^7}$ 23. $\sqrt{\frac{7x}{10y^3}}$ $\frac{\sqrt{70xy}}{10y^2}$ 24. $\frac{\sqrt[3]{6x^2}}{\sqrt[3]{5y}}$ $\frac{\sqrt[3]{150x^2y^2}}{5y}$ 25. $\sqrt[4]{\frac{7x^3}{4b^2}}$ $\frac{\sqrt[4]{28b^2x^3}}{2|b|}$

26. $3\sqrt{5y} \cdot 8\sqrt{10yz}$ $120y\sqrt{2z}$ 27. $2\sqrt{32a^3b^5} \cdot \sqrt{8a^7b^2}$ 28. $6\sqrt{3ab} \cdot 4\sqrt{24ab^3}$ $144ab^2\sqrt{2}$

29. $5\sqrt{x^8y^3} \cdot 5\sqrt{2x^5y^4}$ 30. $3\sqrt{90} + 4\sqrt{20} + \sqrt{162}$ 31. $9\sqrt{12} + 5\sqrt{32} - \sqrt{72}$ $18\sqrt{3} + 14\sqrt{2}$

32. $4\sqrt{28} - 8\sqrt{810} + \sqrt{44}$ $8\sqrt{7} - 72\sqrt{10} + 2\sqrt{11}$ 33. $3\sqrt{54} + 6\sqrt{288} - \sqrt{147}$ $9\sqrt{6} + 72\sqrt{2} - 7\sqrt{3}$

34. **GEOMETRY** Find the perimeter of the rectangle. $16 + 2\sqrt{3} + 2\sqrt{6}$ ft



35. **GEOMETRY** Find the area of the rectangle. $8\sqrt{6} + 3\sqrt{2}$ ft²

36. **GEOMETRY** Find the exact surface area of a sphere with radius of $4 + \sqrt{5}$ inches. $(84 + 32\sqrt{5})\pi$ in²

Examples 5–6 Simplify. 37. $56\sqrt{3} + 42\sqrt{6} - 36\sqrt{2} - 54$ 40. $36\sqrt{2} + 36\sqrt{6} + 20\sqrt{3} + 60$

37. $(7\sqrt{2} - 3\sqrt{3})(4\sqrt{6} + 3\sqrt{12})$ 38. $(8\sqrt{5} - 6\sqrt{3})(8\sqrt{5} + 6\sqrt{3})$ 212

42. $\frac{\sqrt{10} + \sqrt{6}}{2}$

39. $(12\sqrt{10} - 6\sqrt{5})(12\sqrt{10} + 6\sqrt{5})$ 1260 40. $(6\sqrt{3} + 5\sqrt{2})(2\sqrt{6} + 3\sqrt{8})$

43. $\frac{20 - 7\sqrt{3}}{11}$

41. $\frac{6}{\sqrt{3} - \sqrt{2}}$ $6\sqrt{3} + 6\sqrt{2}$ 42. $\frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}}$ 43. $\frac{9 - 2\sqrt{3}}{\sqrt{3} + 6}$ 44. $\frac{2\sqrt{2} + 2\sqrt{5}}{\sqrt{5} + \sqrt{2}}$ 2

22. $\dfrac{\sqrt{5a^5}}{\sqrt{b^{13}}}$ $\dfrac{a^2\sqrt{5ab}}{b^7}$

23. $\sqrt{\dfrac{7x}{10y^3}}$ $\dfrac{\sqrt{70xy}}{10y^2}$

24. $\dfrac{\sqrt[3]{6x^2}}{\sqrt[3]{5y}}$ $\dfrac{\sqrt[3]{150x^2y^2}}{5y}$

25. $\sqrt[4]{\dfrac{7x^3}{4b^2}}$ $\dfrac{\sqrt[4]{28b^2x^3}}{2|b|}$

25

$$\frac{\sqrt[4]{7x^3}}{\sqrt[4]{4b^2}} \cdot \frac{\sqrt[4]{4b^2}}{\sqrt[4]{4b^2}}$$

$2 \cdot 2 \cdot b \cdot b$

$$\frac{\sqrt[4]{28b^2x^3}}{\sqrt[4]{16b^4}}$$