7-1 Graphing Exponential Functions

KeyConcept Parent Function of Exponential Growth Functions

Parent Functions: $f(x) = b^x, b > 1$

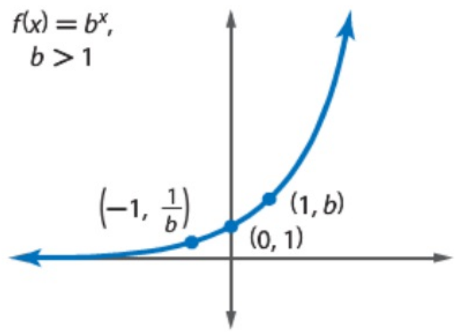
Type of graph: continuous, one-to-one, and increasing

Domain: all real numbers

Range: all positive real numbers

Asymptote: x-axis

Intercept: (0, 1)



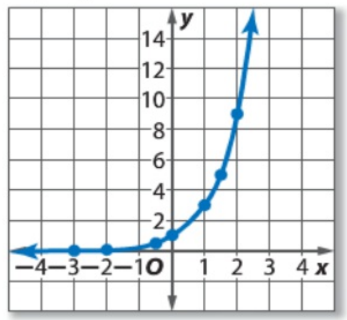
Example 1 Graph Exponential Growth Functions

Graph $y = 3^x$. State the domain and range.

Make a table of values. Then plot the points and sketch the graph.

x	-3	-2	$-\frac{1}{2}$	0
$y = 3^x$	$3^{-3} = \frac{1}{27}$	$3^{-2} = \frac{1}{9}$	$3^{-\frac{1}{2}} = \frac{\sqrt{3}}{3}$	$3^0 = 1$

x	1	$\frac{3}{2}$	2
$y = 3^x$	$3^1 = 3$	$3^{\frac{3}{2}} = \sqrt{27}$	$3^2 = 9$



The domain is all real numbers, and the range is all positive real numbers.

KeyConcept Transformations of Exponential Functions

$$f(x) = ab^{x-h} + k$$

h – Horizontal Translation	*k* – Vertical Translation
h units right if *h* is positive |*h*| units left if *h* is negative	*k* units up if *k* is positive |*k*| units down if *k* is negative

a – Orientation and Shape

If $a < 0$, the graph is reflected in the *x*-axis.

If $|a| > 1$, the graph is stretched vertically.
If $0 < |a| < 1$, the graph is compressed vertically.

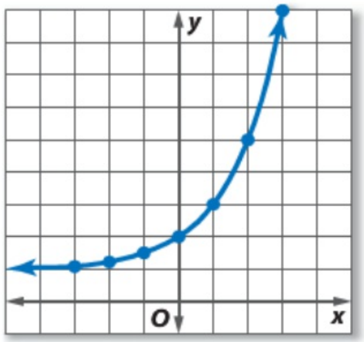
Example 2 Graph Transformations

Graph each function. State the domain and range.

a. $y = 2^x + 1$

The equation represents a translation of the graph of $y = 2^x$ one unit up.

x	$y = 2^x + 1$
−3	$2^{-3} + 1 = 1.125$
−2	$2^{-2} + 1 = 1.25$
−1	$2^{-1} + 1 = 1.5$
0	$2^0 + 1 = 2$
1	$2^1 + 1 = 3$
2	$2^2 + 1 = 5$
3	$2^3 + 1 = 9$



Domain = {all real numbers}; Range = $\{y \mid y > 1\}$

KeyConcept Transformations of Exponential Functions

$f(x) = ab^{x-h} + k$

h – Horizontal Translation	*k* – Vertical Translation
h units right if *h* is positive $\|h\|$ units left if *h* is negative	*k* units up if *k* is positive $\|k\|$ units down if *k* is negative

a – Orientation and Shape

If $a < 0$, the graph is reflected in the *x*-axis.

If $|a| > 1$, the graph is stretched vertically.
If $0 < |a| < 1$, the graph is compressed vertically.

b. $y = -\frac{1}{2} \cdot 5^{x-2}$

KeyConcept Transformations of Exponential Functions

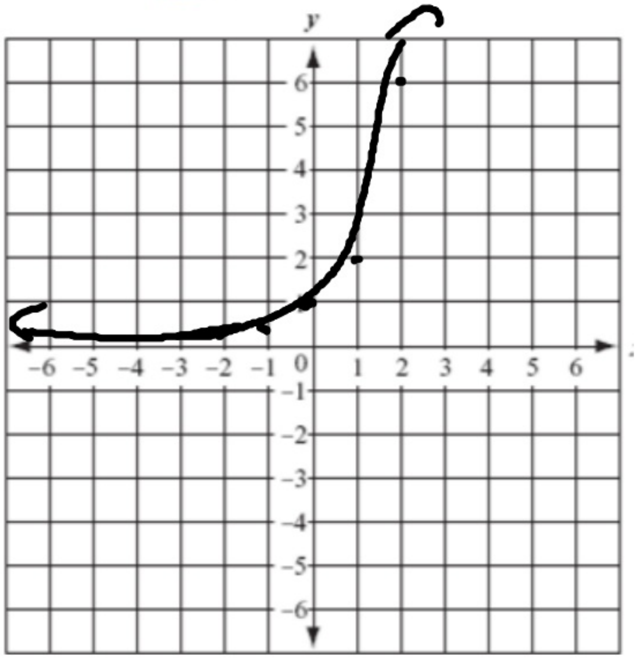
$$f(x) = ab^{x-h} + k$$

h – Horizontal Translation	*k* – Vertical Translation
h units right if *h* is positive $|h|$ units left if *h* is negative	*k* units up if *k* is positive $|k|$ units down if *k* is negative

a – Orientation and Shape	
If $a < 0$, the graph is reflected in the *x*-axis.	If $|a| > 1$, the graph is stretched vertically. If $0 < |a| < 1$, the graph is compressed vertically.

Examples 1–2 Graph each function. State the domain and range. **1–6. See Chapter 7 Answer Appendix.**

1. $f(x) = 2^x$
2. $f(x) = 5^x$
3. $f(x) = 3^{x-2} + 4$
4. $f(x) = 2^{x+1} + 3$
5. $f(x) = 0.25(4)^x - 6$
6. $f(x) = 3(2)^x + 8$



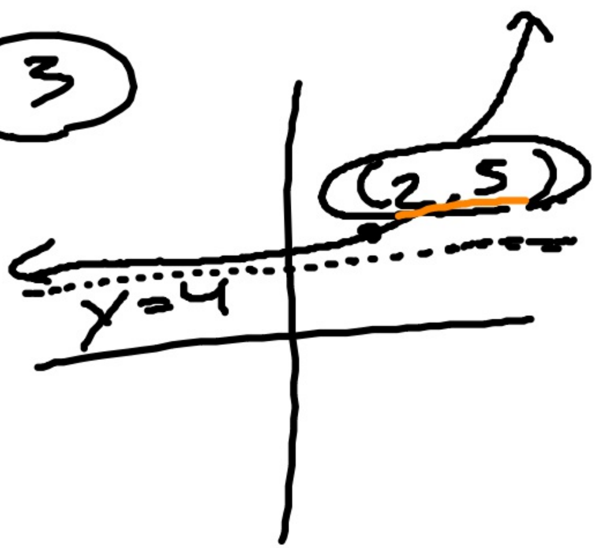
① $f(x) = 2^x$

x	y
-2	1/4
-1	1/2
0	$2^0 = 1$
1	2
2	4

Examples 1–2 **Graph each function. State the domain and range.** **1–6. See Chapter 7 Answer Appendix.**

1. $f(x) = 2^x$
2. $f(x) = 5^x$
3. $f(x) = 3^{x-2} + 4$
4. $f(x) = 2^{x+1} + 3$
5. $f(x) = 0.25(4)^x - 6$
6. $f(x) = 3(2)^x + 8$

(3)



x	y
0	$3^{-2}+4 = 4\frac{1}{9}$
1	$3^{-1}+4 = 4\frac{1}{3}$
2	$3^{0}+4 = 5$
3	$3^{1}+4 = 7$
4	$3^{2}+4 = 13$

"center"

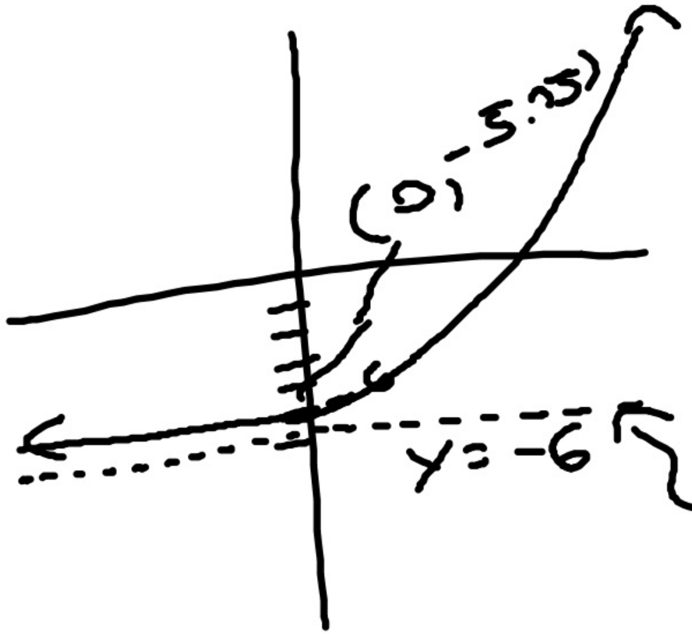
Examples 1–2 **Graph each function. State the domain and range.** **1–6. See Chapter 7 Answer Appendix.**

1. $f(x) = 2^x$	**2.** $f(x) = 5^x$	**3** $f(x) = 3^{x-2} + 4$
4. $f(x) = 2^{x+1} + 3$	**5.** $f(x) = 0.25(4)^x - 6$	**6.** $f(x) = 3(2)^x + 8$

5

$.25 \cdot \frac{1}{4}$

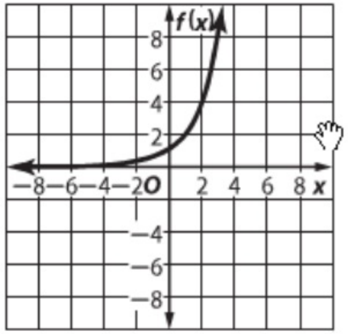
x	y
-2	$.25(4^{-2}) - 6 = -5.98$
-1	$.25(4^{-1}) - 6 = -5.93$
0	$.25(4^{0}) - 6 = -5.75$
1	$.25(4^{1}) - 6 = -5$
2	$.25(4^{2}) - 6 = -2$



"Asymptote"

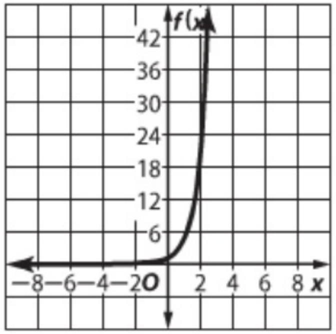
Lesson 7-1

1.



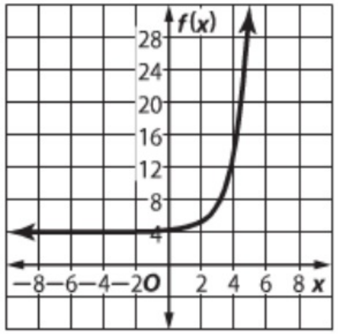
D = {all real numbers};
R = {$f(x)$ | $f(x) > 0$}

2.



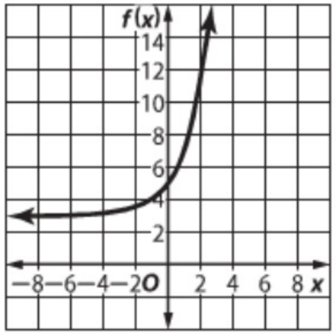
D = {all real numbers};
R = {$f(x)$ | $f(x) > 0$}

3.



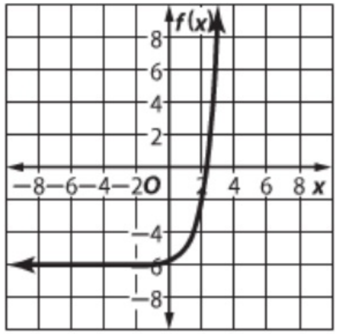
D = {all real numbers};
R = {$f(x)$ | $f(x) > 4$}

4.



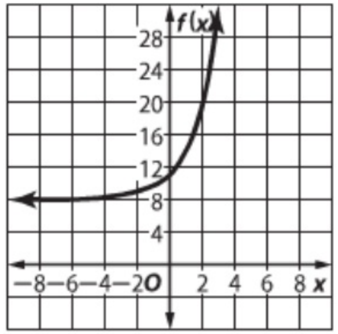
D = {all real numbers};
R = {$f(x)$ | $f(x) > 3$}

5.



D = {all real numbers};
R = {$f(x)$ | $f(x) > -6$}

6.



D = {all real numbers};
R = {$f(x)$ | $f(x) > 8$}

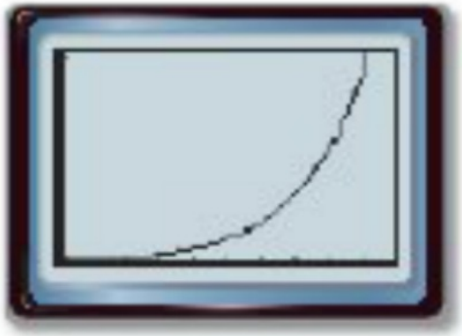
Real-World Example 3 Graph Exponential Growth Functions

CENSUS The first U.S. census was conducted in 1790. At that time, the population was 3,929,214. Since then, the U.S. population has grown by approximately 2.03% annually. Draw a graph showing the population growth of the U.S. since 1790.

First, write an equation using $a = 3{,}929{,}214$, and $r = 0.0203$.

$y = 3{,}929{,}214(1.0203)^t$

Then graph the equation.



[0, 250] scl: 25 by [0, 400,000,000] scl: 40,000,000

StudyTip

Interest The formula for simple interest, $i = prt$, illustrates linear growth over time. However, the formula for compound interest, $A(t) = a(1 + r)^t$, illustrates exponential growth over time. This is why investments with compound interest make more money.

Example 3

7. CCSS **SENSE-MAKING** A virus spreads through a network of computers such that each minute, 25% more computers are infected. If the virus began at only one computer, graph the function for the first hour of the spread of the virus. **See margin.**

KeyConcept Parent Function of Exponential Decay Functions

Parent Functions: $f(x) = b^x, 0 < b < 1$

Type of graph: continuous, one-to-one, and decreasing

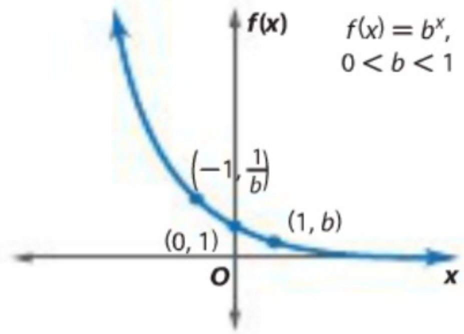
Domain: all real numbers

Range: positive real numbers

Asymptote: x-axis

Intercept: (0, 1)

Model



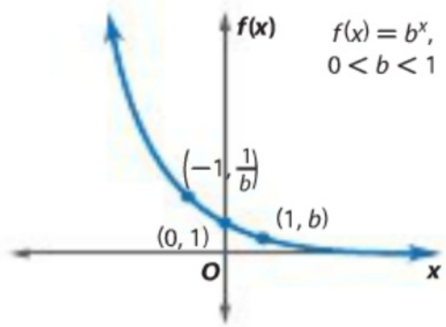
Graph each function. State the domain and range.

a. $y = \left(\frac{1}{3}\right)^x$

KeyConcept Parent Function of Exponential Decay Functions

Parent Functions:	$f(x) = b^x, 0 < b < 1$
Type of graph:	continuous, one-to-one, and decreasing
Domain:	all real numbers
Range:	positive real numbers
Asymptote:	x-axis
Intercept:	(0, 1)

Model



b. $y = 2\left(\frac{1}{4}\right)^{x+2} - 3$

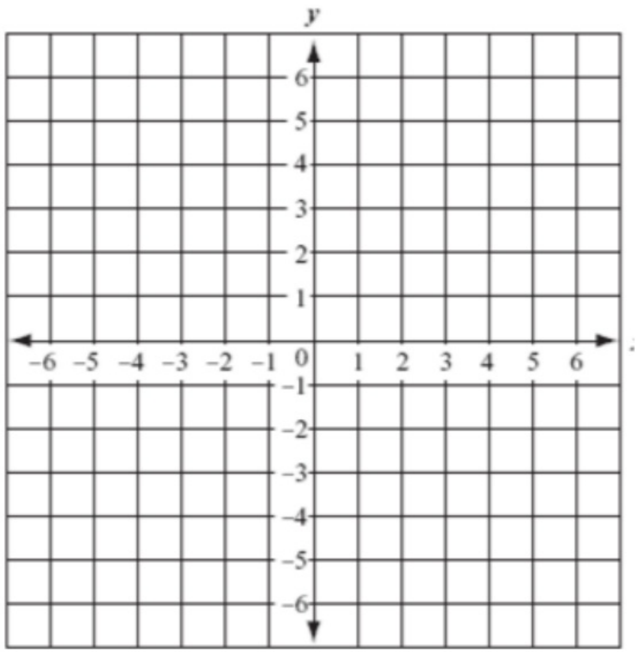
Example 4 **Graph each function. State the domain and range.** **8–11. See Chapter 7 Answer Appendix.**

8. $f(x) = 2\left(\frac{2}{3}\right)^{x-3} - 4$

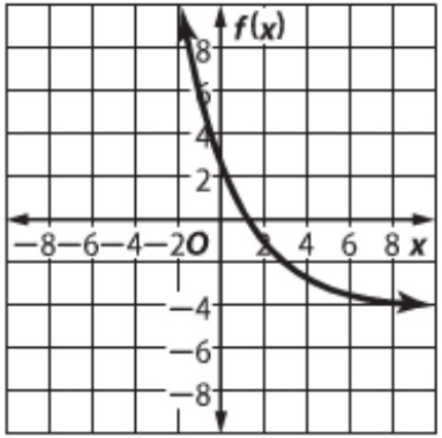
9. $f(x) = -\frac{1}{2}\left(\frac{3}{4}\right)^{x+1} + 5$

10. $f(x) = -\frac{1}{3}\left(\frac{4}{5}\right)^{x-4} + 3$

11. $f(x) = \frac{1}{8}\left(\frac{1}{4}\right)^{x+6} + 7$

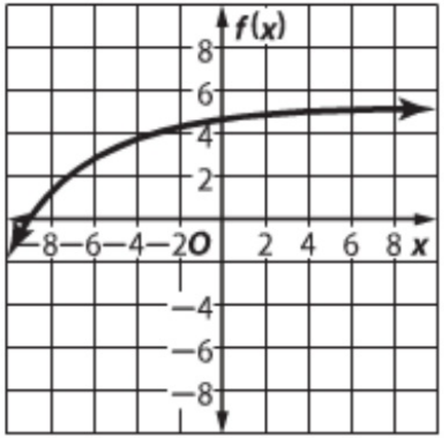


8.



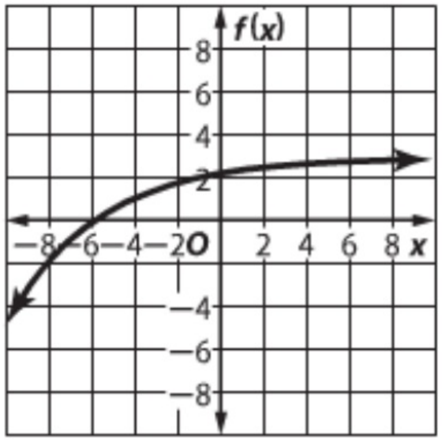
D = {all real numbers};
R = $\{f(x) \mid f(x) > -4\}$

9.



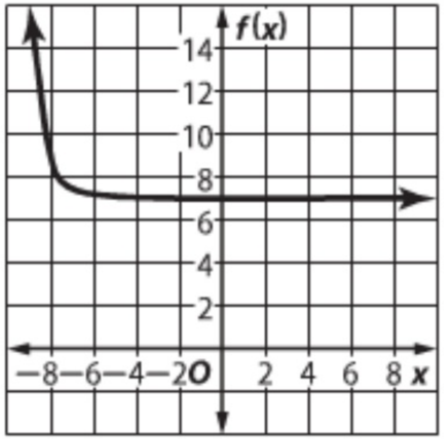
D = {all real numbers};
R = $\{f(x) \mid f(x) < 5\}$

10.



D = {all real numbers};
R = $\{f(x) \mid f(x) < 3\}$

11.



D = {all real numbers};
R = $\{f(x) \mid f(x) > 7\}$

Real-World Example 5 Graph Exponential Decay Functions

TEA A cup of green tea contains 35 milligrams of caffeine. The average teen can eliminate approximately 12.5% of the caffeine from their system per hour.

a. Draw a graph to represent the amount of caffeine remaining after drinking a cup of green tea.

b. Estimate the amount of caffeine in a teenager's body 3 hours after drinking a cup of green tea.

Example 5

12. **FINANCIAL LITERACY** A new SUV depreciates in value each year by a factor of 15%. Draw a graph of the SUV's value for the first 20 years after the initial purchase. **See margin.**

Examples 1–2 Graph each function. State the domain and range. **13–18. See Chapter 7 Answer Appendix.**

13. $f(x) = 2(3)^x$

14. $f(x) = -2(4)^x$

15. $f(x) = 4^{x+1} - 5$

16. $f(x) = 3^{2x} + 1$

17. $f(x) = -0.4(3)^{x+2} + 4$

18. $f(x) = 1.5(2)^x + 6$

Example 3

19 **SCIENCE** The population of a colony of beetles grows 30% each week for 10 weeks. If the initial population is 65 beetles, graph the function that represents the situation. **See Chapter 7 Answer Appendix.**

Example 4 Graph each function. State the domain and range. **20–25. See Chapter 7 Answer Appendix.**

20. $f(x) = -4\left(\frac{3}{5}\right)^{x+4} + 3$

21. $f(x) = 3\left(\frac{2}{5}\right)^{x-3} - 6$

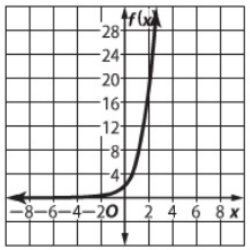
22. $f(x) = \frac{1}{2}\left(\frac{1}{5}\right)^{x+5} + 8$

23. $f(x) = \frac{3}{4}\left(\frac{2}{3}\right)^{x+4} - 2$

24. $f(x) = -\frac{1}{2}\left(\frac{3}{8}\right)^{x+2} + 9$

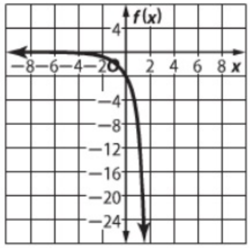
25. $f(x) = -\frac{5}{4}\left(\frac{4}{5}\right)^{x+4} + 2$

13.



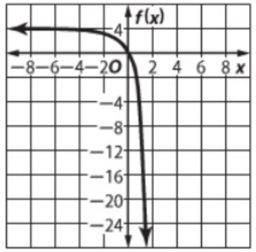
D = {all real numbers};
R = $\{f(x) \mid f(x) > 0\}$

14.



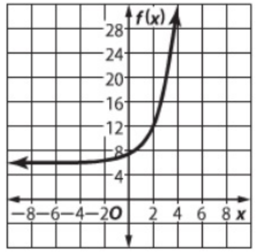
D = {all real numbers};
R = $\{f(x) \mid f(x) < 0\}$

17.



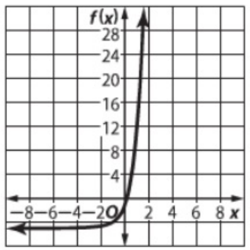
D = {all real numbers};
R = $\{f(x) \mid f(x) < 4\}$

18.



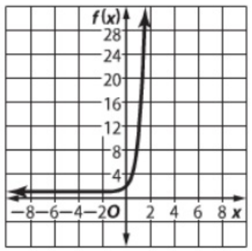
D = {all real numbers};
R = $\{f(x) \mid f(x) > 6\}$

15.



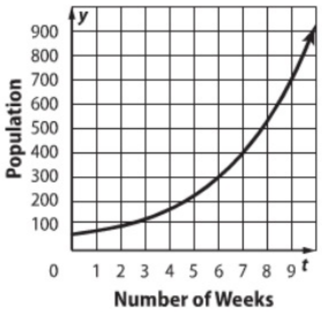
D = {all real numbers};
R = $\{f(x) \mid f(x) > -5\}$

16.



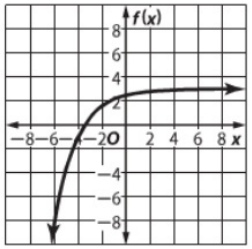
D = {all real numbers};
R = $\{f(x) \mid f(x) > 1\}$

19.



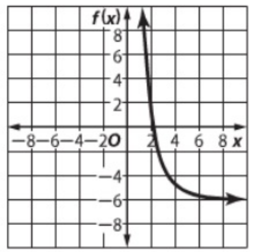
Number of Weeks

20.



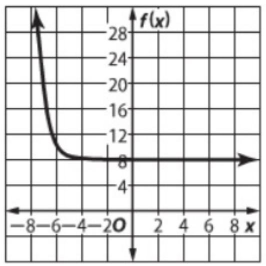
D = {all real numbers};
R = $\{f(x) \mid f(x) < 3\}$

21.



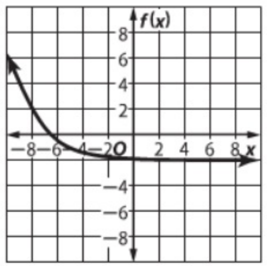
D = {all real numbers};
R = $\{f(x) \mid f(x) > -6\}$

22.



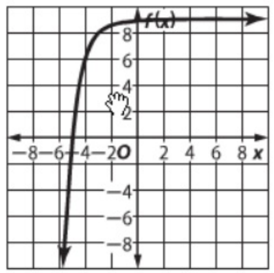
D = {all real numbers};
R = $\{f(x) \mid f(x) > 8\}$

23.



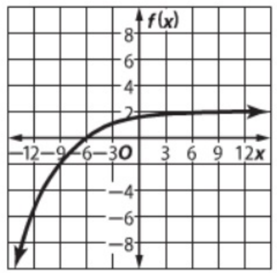
D = {all real numbers};
R = $\{f(x) \mid f(x) > -2\}$

24.



D = {all real numbers};
R = $\{f(x) \mid f(x) < 9\}$

25.



D = {all real numbers};
R = $\{f(x) \mid f(x) < 2\}$