5-Minute Check

Over Lesson 7–6

1 The number of people who carry cell phones increases by 29% each year. In 2002, there were 180 million cell phone users. Which equation models the number of people with cell phones y if it is t years after 2002?

A. $y = 180(1 + 0.29)^t$

B. $y = 180(1 + 2.9)^t$

C. $y = 180(0.29)$

D. $y = 180(1 + 0.29)t$

5-Minute Check Over Lesson 7–6

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5-Minute Check Over Lesson 7–6

3 In 2004, there were 243 million vehicles in the U.S. This number is increasing by 1.6% each year. If *y* represents cars and *t* represents the number of years after 2004, which equation models the number of cars in the U.S.?

A. $y = 243(0.016)^t$

B. $y = 243(1 + 0.016)^t$

C. $y = 243(0.16)t$

D. $y = 243(1 + 0.016)t$

5-Minute Check Over Lesson 7–6

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Then

You related arithmetic sequences to linear functions.

Now

- Identify and generate geometric sequences.
- Relate geometric sequences to exponential functions.

EXAMPLE 1 Identify Geometric Sequences

A. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain.

0, 8, 16, 24, 32, ...

0 8 16 24 32

8 – 0 = 8 16 – 8 = 8 24 – 16 = 8 32 – 24 = 8

Answer: The common difference is 8. So, the sequence is arithmetic.

EXAMPLE 1 Identify Geometric Sequences

B. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain.

64, 48, 36, 27, ...

64 48 36 27

$\frac{48}{64} = \frac{3}{4}$ $\frac{36}{48} = \frac{3}{4}$ $\frac{27}{36} = \frac{3}{4}$

Answer: The common ratio is $\frac{3}{4}$, so the sequence is geometric.

EXAMPLE 1 Check Your Progress

B. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*.
1, 2, 4, 14, 54, ...

A. arithmetic

B. geometric

C. neither

EXAMPLE 1 Check Your Progress

B. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*.
1, 2, 4, 14, 54, ...

A. arithmetic

B. geometric

C. neither

EXAMPLE 2 Find Terms of Geometric Sequences

A. Find the next three terms in the geometric sequence.

1, –8, 64, –512, ...

Step 1 Find the common ratio.

1 –8 64 –512

$\frac{-8}{1} = -8$ $\frac{64}{-8} = -8$ $\frac{-512}{64} = -8$

The common ratio is –8.

EXAMPLE 2 Find Terms of Geometric Sequences

Step 2 Multiply each term by the common ratio to find the next three terms.

–512 4096 –32,768 262,144

× (–8) × (–8) × (–8)

Answer: The next 3 terms in the sequence are 4096; –32,768; and 262,144.

EXAMPLE 2 Find Terms of Geometric Sequences

B. Find the next three terms in the geometric sequence.

40, 20, 10, 5, ...

Step 1 Find the common ratio.

40 20 10 5

$$\frac{40}{20} = \frac{1}{2} \quad \frac{10}{20} = \frac{1}{2} \quad \frac{5}{10} = \frac{1}{2}$$

The common ratio is $\frac{1}{2}$.

EXAMPLE 2 Find Terms of Geometric Sequences

Step 2 Multiply each term by the common ratio to find the next three terms.

$$5 \quad \frac{5}{2} \quad \frac{5}{4} \quad \frac{5}{8}$$

$$\times \frac{1}{2} \qquad \times \frac{1}{2} \qquad \times \frac{1}{2}$$

Answer: The next 3 terms in the sequence are $\frac{5}{2}$, $\frac{5}{4}$, and $\frac{5}{8}$.

...ncept *n*th term of a Geometric Sequence

...term a_n of a geometric sequence with first term a_1 and common ratio r is given by the ... formula, where n is any positive integer and $a_1, r \neq 0$.

$$a_n = a_1 r^{n-1}$$

x	y	
1	2	$2(2)^0$ $r = 2$
2	4	$2(2)^1$ $a_1 = 2$
3	8	$2(2)^2$
4	16	$2(2)^3$

EXAMPLE 3 Find the *n*th Term of a Geometric Sequence

A. Write an equation for the *n*th term of the geometric sequence 1, –2, 4, –8, ...

The first term of the sequence is 1. So, $a_1 = 1$. Now find the common ratio.

$1 \quad -2 \quad 4 \quad -8$

$\frac{-2}{1} = -2 \quad \frac{4}{-2} = -2 \quad \frac{-8}{4} = -2$

The common ratio is –2.

$a_n = a_1 r^{n-1}$ Formula for the *n*th term

$a_n = 1(-2)^{n-1}$ $a_1 = 1$ and $r = -2$

Answer: $a_n = 1(-2)^{n-1}$

EXAMPLE 3 Find the *n*th Term of a Geometric Sequence

B. Find the 12th term of the sequence.
1, –2, 4, –8, ...

$a_n = a_1 r^{n-1}$	Formula for the *n*th term
$a_{12} = 1(-2)^{12-1}$	For the *n*th term, $n = 12$.
$= 1(-2)^{11}$	Simplify.
$= 1(-2048)$	$(-2)^{11} = -2048$
$= -2048$	Multiply.

Answer: The 12th term of the sequence is –2048.

EXAMPLE 3 Check Your Progress

A. Write an equation for the *n*th term of the geometric sequence 3, –12, 48, –192, ...

A. $a_n = 3(-4)^{n-1}$

B. $a_n = 3\left(\frac{1}{4}\right)^{n-1}$

C. $a_n = 3\left(\frac{1}{3}\right)^{n-1}$

D. $a_n = 4(-3)^{n-1}$

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EXAMPLE 3 Check Your Progress

B. Find the 7th term of this sequence using the equation $a_n = 3(-4)^{n-1}$.

A. 768

B. –3072

C. 12,288

D. –49,152

Example 1 **Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.**

1. 200, 40, 8, … **2.** 2, 4, 16, … **3.** −6, −3, 0, 3, … **4.** 1, −1, 1, −1, …

Example 2 **Find the next three terms in each geometric sequence.** **5–8. See margin.**

5. 10, 20, 40, 80, … **6.** 100, 50, 25, … **7.** 4, −1, $\frac{1}{4}$, … **8.** −7, 21, −63, …

Example 3 **Write an equation for the *n*th term of each geometric sequence, and find the indicated term.**

9. the fifth term of −6, −24, −96, … $a_n = -6 \cdot (4)^{n-1}$; −1536

10. the seventh term of −1, 5, −25, … $a_n = -1 \cdot (-5)^{n-1}$; −15,625

11. the tenth term of 72, 48, 32, … $a_n = 72 \cdot \left(\frac{2}{3}\right)^{n-1}$; $\frac{4096}{2187}$

12. the ninth term of 112, 84, 63, … $a_n = 112 \cdot \left(\frac{3}{4}\right)^{n-1}$; $\frac{45,927}{4096}$

Example 4 **13. EXPERIMENT** In a physics class experiment, Diana drops a ball from a height of 16 feet. Each bounce has 70% the height of the previous bounce. Draw a graph to represent the height of the ball after each bounce. **See margin.**

1. Geometric; the common ratio is $\frac{1}{5}$.

2. Neither; there is no common ratio or difference.

3. Arithmetic; the common difference is 3.

4. Geometric; the common ratio is −1.

Example 1 **Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.**

1. 200, 40, 8, … **2.** 2, 4, 16, … **3.** −6, −3, 0, 3, … **4.** 1, −1, 1, −1, …

1. Geometric; the common ratio is $\frac{1}{5}$.

Example 2 **Find the next three terms in each geometric sequence.** **5–8. See margin.**

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12. the ninth term of 112, 84, 63, … a_n

9) $y = a_1(r)^{n-1}$

$y = (-6)(4)^{5-1}$

$y = (-6)(4^4)$

$= (-6)(256) = -1536$

Example 4 **13.** **EXPERIMENT** In a physics class experiment, Diana drops a ball from a height of 16 feet. Each bounce has 70% the height of the previous bounce. Draw a graph to represent the height of the ball after each bounce. **See margin.**

5. 160, 320, 640

6. 12.5, 6.25, 3.125

7. $-\frac{1}{16}, \frac{1}{64}, -\frac{1}{256}$

8. 189, −567, 1701

9) multiply by 4…

$\frac{-24}{-6} = 4 = r$

$a_1 = -6$

$n = 5$ $a_5 = -1536$

10. the seventh term of −1, 5, −25, … $a_n = -1 \cdot (-5)^{n-1}$; −15,625

11. the tenth term of 72, 48, 32, … $a_n = 72 \cdot \left(\frac{2}{3}\right)^{n-1}$; $\frac{4096}{2187}$

12. the ninth term of 112, 84, 63, … $a_n = 112 \cdot \left(\frac{3}{4}\right)^{n-1}$; $\frac{45,927}{4096}$

difference.

3. Arithmetic; the common difference is 3.

4. Geometric; the common ratio is −1.

Example 4 13. **EXPERIMENT** In a physics class experiment, Diana drops a ball from a height of 16 feet. Each bounce has 70% the height of the previous bounce. Draw a graph to represent the height of the ball after each bounce. See margin.

PEMDAS

$a_n = a_1 \cdot (r)^{n-1}$

(10) $n = 7$, $r = -5$, $a_1 = -1$

$\frac{32}{48} = \frac{48}{72} = \frac{2}{3}$

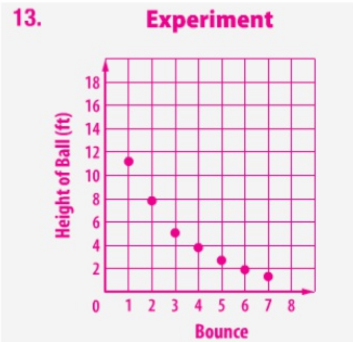
(11) $a_1 = 72$, $n = 10$, $r = 2/3$

$a_{10} = 72\left(\frac{2}{3}\right)^{10-1}$

$= 72\left(\frac{2}{3}\right)^9$

$= 72\left(\frac{2^9}{3^9}\right)$

$= 72\left(\frac{512}{19683}\right)$



Practice and Problem Solving

Extra Practice is on R7.

Example 1 **Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.** **14–19. See margin.**

14. 4, 1, 2, …

15. 10, 20, 30, 40, …

16. 4, 20, 100, …

17. 212, 106, 53, …

18. −10, −8, −6, −4, …

19. 5, −10, 20, 40, …

Example 2 **Find the next three terms in each geometric sequence.** **20–25. See margin.**

20. 2, −10, 50, …

21 36, 12, 4, …

22. 4, 12, 36, …

23. 400, 100, 25, …

24. −6, −42, −294, …

25. 1024, −128, 16, …

Example 3

26. The first term of a geometric series is 1 and the common ratio is 9. What is the 8th term of the sequence? **4,782,969**

27. The first term of a geometric series is 2 and the common ratio is 4. What is the 14th term of the sequence? **134,217,728**

28. What is the 15th term of the geometric sequence −9, 27, −81, …? **−43,046,721**

29. What is the 10th term of the geometric sequence 6, −24, 96, …? **−1,572,864**

14. Neither; there is no common ratio or difference.

15. Arithmetic; the common difference is 10.

16. Geometric; the common ratio is 5.

17. Geometric; the common ratio is $\frac{1}{2}$.

18. Arithmetic; the common difference is 2.

19. Neither; there is no common ratio or difference.

20. −250, 1250, −6250

21. $\frac{4}{3}, \frac{4}{9}, \frac{4}{27}$

22. 108, 324, 972

23. $\frac{25}{4}, \frac{25}{16}, \frac{25}{64}$

24. −2058; −14,406; −100,842

25. $-2, \frac{1}{4}, -\frac{1}{32}$