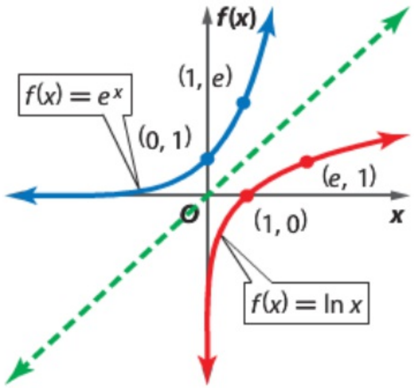
7-7 Base e and Natural Logarithms

1 Base *e* and Natural Logarithms Like π and $\sqrt{2}$, the number e is an irrational number. The value of e is 2.71828… . It is referred to as the **natural base, *e***. An exponential function with base e is called a **natural base exponential function**.

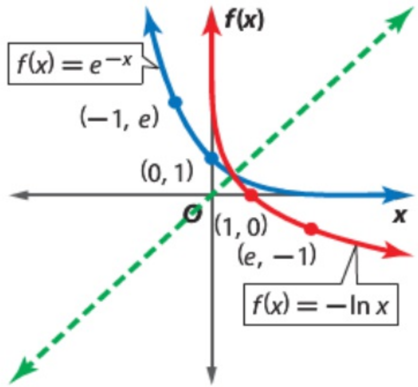
KeyConcept Natural Base Functions

The function $f(x) = e^x$ is used to model continuous exponential growth.
The function $f(x) = e^{-x}$ is used to model continuous exponential decay.

The inverse of a natural base exponential function is called the **natural logarithm**. This logarithm can be written as $\log_e x$, but is more often abbreviated as $\ln x$.



Exponential Growth



Exponential Decay

Write each exponential equation in logarithmic form.

a. $e^x = 8$

$e^x = 8 \rightarrow \log_e 8 = x$

$\ln 8 = x$

b. $e^5 = x$

$e^5 = x \rightarrow \log_e x = 5$

$\ln x = 5$

Example 2 Write Equivalent Expressions

Write each logarithmic equation in exponential form.

a. $\ln x \approx 0.7741$

$\ln x \approx 0.7741 \rightarrow \log_e x = 0.7741$

$x \approx e^{0.7741}$

b. $\ln 10 = x$

$\ln 10 = x \rightarrow \log_e 10 = x$

$10 = e^x$

Examples 1–2 **Write an equivalent exponential or logarithmic function.**

1. $e^x = 30$ $\ln 30 = x$
2. $\ln x = 42$ $e^{42} = x$
3. $e^3 = x$ $\ln x = 3$
4. $\ln 18 = x$ $e^x = 18$

5 $3\ln 2 + 2\ln 4$

$\ln 2^3 + \ln 4^2$

$= \ln(2^3 \cdot 4^2)$

6 $5\ln 3 - 2\ln 9$

$\ln 3^5 - \ln 9^2$

$= \ln \frac{3^5}{9^2}$

Example 3 **Write each as a single logarithm.**

5. $3 \ln 2 + 2 \ln 4$ **7 ln 2**

6. $5 \ln 3 - 2 \ln 9$ **ln 3**

7. $3 \ln 6 + 2 \ln 9$ **ln 17496**

8. $3 \ln 5 + 4 \ln x$ **ln $125x^4$**

$\ln(8 \cdot 16)$

$\ln(128)$

$\ln 2^7$

Example 4 Solve Base e Equations

Solve $4e^{-2x} - 5 = 3$. Round to the nearest ten-thousandth.

9
$5e^{x} - 24 = 16$
$+24 \quad +24$
$5e^{x} = 40$
$\frac{5e^{x}}{5} = \frac{40}{5}$
$e^{x} = 8 \rightarrow \ln 8 = x$

Example 4 Solve each equation. Round to the nearest ten-thousandth.

9. $5e^{x} - 24 = 16$ **2.0794**

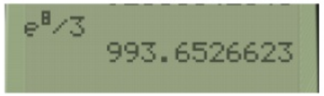
10. $-3e^{x} + 9 = 4$ **0.5108**

11. $3e^{-3x} + 4 = 6$ **0.1352**

12. $2e^{-x} - 3 = 8$ **−1.7047**

13 $\ln 3x = 8$

$$\frac{e^8}{3} = \frac{3x}{3}$$



Example 5 **Solve each equation or inequality. Round to the nearest ten-thousandth.**

13. $\ln 3x = 8$ **993.6527**

14. $-4 \ln 2x = -26$ **332.5708**

15. $\ln (x + 5)^2 < 6$

16. $\ln (x - 2)^3 > 15$ **$\{x \mid x > 150.4132\}$**

17. $e^x > 29$ **$\{x \mid x > 3.3673\}$**

18. $5 + e^{-x} > 14$ **$\{x \mid x < -2.1972\}$**

15. $\{x \mid -25.0855 < x < 15.0855,\ x \neq -5\}$

Example 5 Solve Natural Log Equations and Inequalities

Solve each equation or inequality. Round to the nearest ten-thousandth.

a. $3 \ln 4x = 24$

$3 \ln 4x = 24$	Original equation
$\ln 4x = 8$	Divide each side by 3.
$e^{\ln 4x} = e^8$	Property of Equality for Exponential Functions
$4x = e^8$	$e^{\ln x} = x$
$x = \frac{e^8}{4}$	Divide each side by 4.
$x \approx 745.2395$	Use a calculator.

b. $\ln (x - 8)^4 < 4$

$\ln (x - 8)^4 < 4$	Original equation
$e^{\ln (x-8)^4} < e^4$	Write each side using exponents and base e.
$(x - 8)^4 < e^4$	$e^{\ln x} = x$
$x - 8 < e$	Property of Equality for Exponential Functions
$x < e + 8$	Add 8 to each side.
$x < 10.7183$	Use a calculator.

KeyConcept Continuously Compounded Interest

Calculate continuously compounded interest using the following formula:

$$A = Pe^{rt},$$

where A is the amount in the account after t years, P is the principal amount invested, and r is the annual interest rate.

Real-World Example 6 Solve Base e Inequalities

FINANCIAL LITERACY When Angelina was born, her grandparents deposited $3000 into a college savings account paying 4% interest compounded continuously.

a. Assuming there are no deposits or withdrawals from the account, what will the balance be after 10 years?

$A = Pe^{rt}$	Continuous Compounding Formula
$= 3000e^{(0.04)(10)}$	$P = 3000$, $r = 0.04$, and $t = 10$
$= 3000e^{0.4}$	Simplify.

KeyConcept Continuously Compounded Interest

Calculate continuously compounded interest using the following formula:

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Real-World Example 6 Solve Base e Inequalities

FINANCIAL LITERACY **When Angelina was born, her grandparents deposited $3000 into a college savings account paying 4% interest compounded continuously.**

b. How long will it take the balance to reach at least $10,000?

KeyConcept Continuously Compounded Interest

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Real-World Example 6 Solve Base e Inequalities

FINANCIAL LITERACY When Angelina was born, her grandparents deposited \$3000 into a college savings account paying 4% interest compounded continuously.

c. If her grandparents want Angelina to have \$10,000 after 18 years, how much would they need to invest?

KeyConcept Continuously Compounded Interest

Calculate continuously compounded interest using the following formula:

$$A = Pe^{rt},$$

where A is the amount in the account after t years, P is the principal amount invested, and r is the the annual interest rate.

Example 6

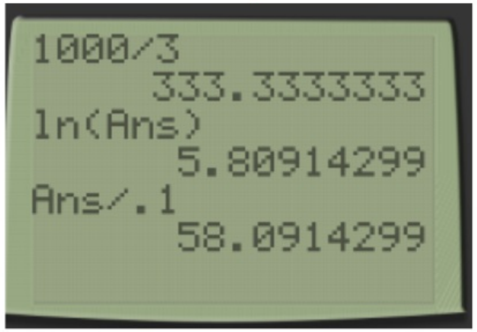
19. SCIENCE A virus is spreading through a computer network according to the formula $v(t) = 30e^{0.1t}$, where v is the number of computers infected and t is the time in minutes. How long will it take the virus to infect 10,000 computers? **about 58 min**

$$v(t) = 30e^{0.1t}$$

$$\frac{10{,}00\cancel{0}}{3\cancel{0}} = \frac{30e^{0.1t}}{30}$$

$$333.\bar{3} = e^{0.1t}$$

$$\frac{\ln(333.\bar{3})}{0.1} = \frac{\cancel{0.1}t}{\cancel{0.1}}$$



Examples 5–6 **40.** CCSS **SENSE-MAKING** The value of a certain car depreciates according to $v(t) = 18500e^{-0.186t}$, where t is the number of years after the car is purchased new.

a. What will the car be worth in 18 months? **\$13,996**

b. When will the car be worth half of its original value? **about 3.73 yr**

c. When will the car be worth less than \$1000? **about 15.69 yr**

c. $18500e^{-.186t} < 1000$

45. $\{x \mid x < -239.8802 \text{ or } x > 239.8802\}$

Solve each inequality. Round to the nearest ten-thousandth.

46. $\{x \mid 6 < x \leq 26.0855\}$

41. $e^x \leq 8.7$ $\{x \mid x \leq 2.1633\}$ **42.** $e^x \geq 42.1$ $\{x \mid x \geq 3.7400\}$ **43.** $\ln(3x+4)^3 > 10$ $\{x \mid x > 8.0105\}$

44. $4 \ln x^2 < 72$ $\{x \mid -8103.0839 < x < 8103.0839,\ x \neq 0\}$ **45.** $\ln(8x^4) > 24$ **46.** $-2[\ln(x-6)^{-1}] \leq 6$

c(i)

a) $v(t) = 18500e^{-0.186t}$

$t = \frac{18}{12}$

$t = 1.5$ yrs.

$v(1.5) = 18500e^{(-0.186)(1.5)}$

$v(1.5) = 18500e^{(-.279)}$

b) $9250 = 18500e^{-0.18t}$

$\frac{1}{2} = e^{-.18t}$

$\ln\left(\frac{1}{2}\right) = -.18t$

$t = \frac{\ln\left(\frac{1}{2}\right)}{-.18}$