What you'll learn about

7.1 Integral As Net Change

- Linear Motion Revisited
- General Strategy
- Consumption Over Time
- Net Change from Data
- Work

We actually did this before, back at 5.2...

...remember these?

Answers:

29. $\int_8^{11} 87\, dt = 261$ miles

30. $\int_0^{60} 25\, dt = 1500$ gallons

31. $\int_6^{7.5} 300\, dt = 450$ calories

In Exercises 29–32, express the desired quantity as a definite integral and evaluate the integral using Theorem 2.

29. Find the distance traveled by a train moving at 87 mph from 8:00 A.M. to 11:00 A.M.

30. Find the output from a pump producing 25 gallons per minute during the first hour of its operation.

31. Find the calories burned by a walker burning 300 calories per hour between 6:00 P.M. and 7:30 P.M.

These were easy, mostly because the integrand was a constant. What if it changed over the course of time?

EXAMPLE 1 Interpreting a Velocity Function

Figure 7.1 shows the velocity

$$\frac{ds}{dt} = v(t) = t^2 - \frac{8}{(t+1)^2} \quad \frac{\text{cm}}{\text{sec}}$$

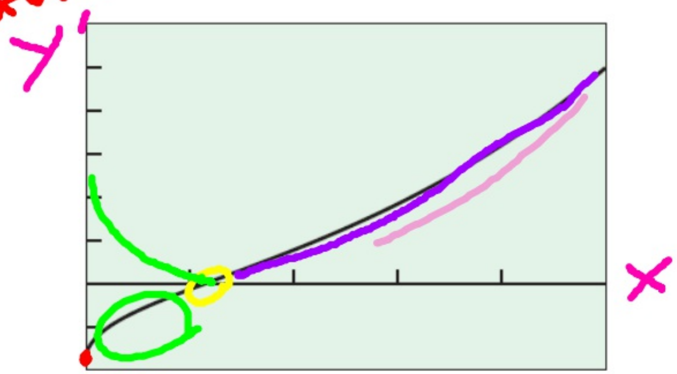
of a particle moving along a horizontal s-axis for $0 \le t \le 5$. Describe the motion.

SOLUTION

Solve Graphically The graph of v (Figure 7.1) starts with $v(0) = -8$, which we interpret as saying that the particle has an initial velocity of 8 cm/sec to the left. It slows to a halt at about $t = 1.25$ sec, after which it moves to the right $(v > 0)$ with increasing speed, reaching a velocity of $v(5) \approx 24.8$ cm/sec at the end. ***Now try Exercise 1(a).***

negative direction

SO. It'll be good for us to be savvy with our graphing calculator....



[0, 5] by [–10, 30]

Figure 7.1 The velocity function in Example 1.

EXAMPLE 2 Finding Position from Displacement

Suppose the initial position of the particle in Example 1 is $s(0) = 9$. What is the particle's position at **(a)** $t = 1$ sec? **(b)** $t = 5$ sec?

$$\begin{aligned}\text{Displacement} &= \int_0^1 v(t)\,dt \\ &= \int_0^1 \left(t^2 - \frac{8}{(t+1)^2}\right) dt \\ &= \left[\frac{t^3}{3} + \frac{8}{t+1}\right]_0^1 \\ &= \frac{1}{3} + \frac{8}{2} - 8 = -\frac{11}{3}.\end{aligned}$$

During the first second of motion, the particle moves $11/3$ cm to the left. It starts at $s(0) = 9$, so its position at $t = 1$ is

$$\text{New position} = \text{initial position} + \text{displacement} = 9 - \frac{11}{3} = \frac{16}{3}.$$

EXAMPLE 2 Finding Position from Displacement

Suppose the initial position of the particle in Example 1 is $s(0) = 9$. What is the particle's position at **(a)** $t = 1$ sec? **(b)** $t = 5$ sec?

(b) If we model the displacement from $t = 0$ to $t = 5$ in the same way, we arrive at

$$\text{Displacement} = \int_0^5 v(t)\,dt = \left[\frac{t^3}{3} + \frac{8}{t+1}\right]_0^5 = 35.$$

The motion has the net effect of displacing the particle 35 cm to the right of its starting point. The particle's final position is

$$\begin{aligned}\text{Final position} &= \text{initial position} + \text{displacement}\\ &= s(0) + 35 = 9 + 35 = 44.\end{aligned}$$

EXAMPLE 3 Calculating Total Distance Traveled

Find the *total distance traveled* by the particle in Example 1.

SOLUTION

Solve Analytically We partition the time interval as in Example 2 but record every position shift as *positive* by taking absolute values. The Riemann sum approximating total distance traveled is

$$\sum |v(t_k)|\,\Delta t,$$

and we are led to the integral

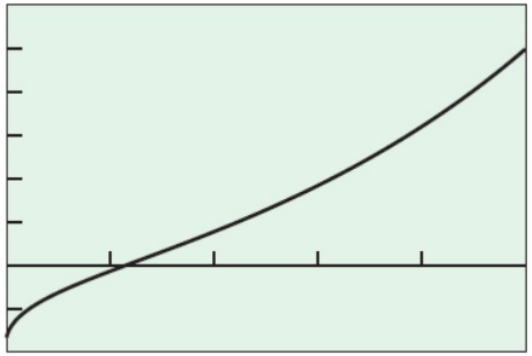
$$\text{Total distance traveled} = \int_0^5 |v(t)|\,dt = \int_0^5 \left|t^2 - \frac{8}{(t+1)^2}\right| dt.$$

Evaluate Numerically We have

$$\text{NINT}\left(\left|t^2 - \frac{8}{(t+1)^2}\right|, t, 0, 5\right) \approx 42.59.$$

Now try Exercise 1(c).

Why do we take the absolute value?



[0, 5] by [–10, 30]

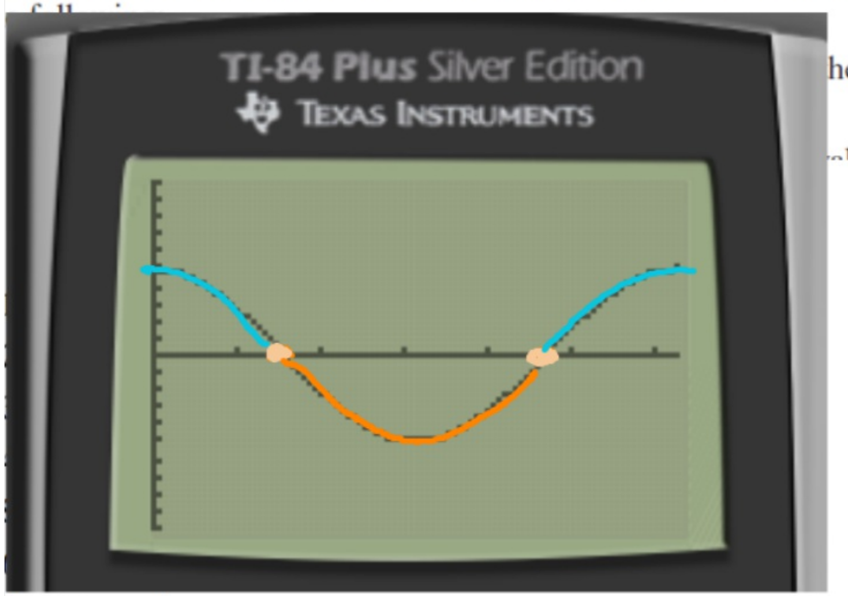
in

Compare the motion of the particle to the graph of v(t)...

$$\textbf{1. } v(t) = 5\cos t, \quad 0 \le t \le 2\pi$$

$s(0) = 3$

Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a
rticle moving along the x-axis. Use analytic methods to do each of



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al If

7.

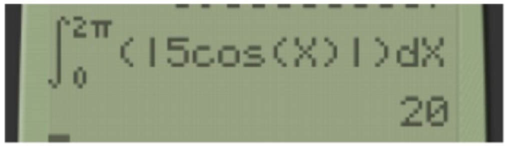
b) $\int_0^{2\pi} 5\cos t \, dt + 3$

c) $\int_0^{2\pi} |5\cos t| \, dt$

1. (a) Right: $0 \le t < \pi/2$, $3\pi/2 < t \le 2\pi$
Left: $\pi/2 < t < 3\pi/2$
Stopped: $t = \pi/2, 3\pi/2$

(b) 0; 3 **(c)** 20

2. (a) Right: $0 < t < \pi/3$
Left: $\pi/3 < t \le \pi/2$
Stopped: $t = 0, \pi/3$
(b) 2; 5 **(c)** 6



In Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to do each of the following:

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval. If $s(0) = 3$, what is the particle's final position?

(c) Find the total distance traveled by the particle.

1. $v(t) = 5\cos t, \quad 0 \le t \le 2\pi$ See page 389.

2. $v(t) = 6\sin 3t, \quad 0 \le t \le \pi/2$ See page 389.

3. $v(t) = 49 - 9.8t, \quad 0 \le t \le 10$ See page 389.

4. $v(t) = 6t^2 - 18t + 12, \quad 0 \le t \le 2$ See page 389.

5. $v(t) = 5\sin^2 t\cos t, \quad 0 \le t \le 2\pi$ See page 389.

6. $v(t) = \sqrt{4 - t}, \quad 0 \le t \le 4$ See page 389.

7. $v(t) = e^{\sin t}\cos t, \quad 0 \le t \le 2\pi$ See page 389.

8. $v(t) = \dfrac{t}{1 + t^2}, \quad 0 \le t \le 3$ See page 389.

1. (a) Right: $0 \le t < \pi/2$, $3\pi/2 < t \le 2\pi$
Left: $\pi/2 < t < 3\pi/2$
Stopped: $t = \pi/2, 3\pi/2$
(b) 0; 3 **(c)** 20

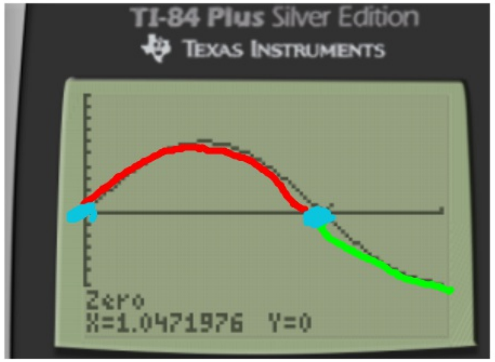
2. (a) Right: $0 < t < \pi/3$
Left: $\pi/3 < t \le \pi/2$
Stopped: $t = 0, \pi/3$
(b) 2; 5 **(c)** 6

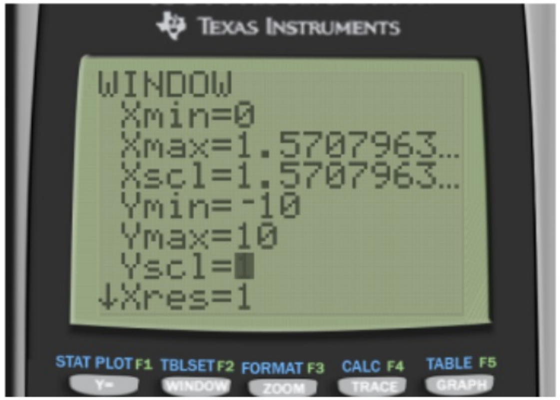
3. (a) Right: $0 \le t < 5$
Left: $5 < t \le 10$
Stopped: $t = 5$
(b) 0; 3 **(c)** 245

4. (a) Right: $0 \le t < 1$
Left: $1 < t < 2$
Stopped: $t = 1, 2$
(b) 4; 7 **(c)** 6

5. (a) Right: $0 < t < \pi/2$, $3\pi/2 < t < 2\pi$
Left: $\pi/2 < t < \pi$, $\pi < t < 3\pi/2$
Stopped: $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$
(b) 0 ; 3 **(c)** 20/3

2. $v(t) = 6 \sin 3t, \quad 0 \le t \le \pi/2$ See page 389.



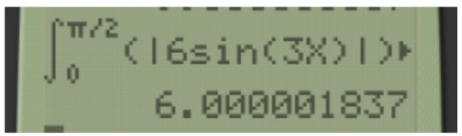


2. (a) Right: $0 < t < \pi/3$
Left: $\pi/3 < t \le \pi/2$
Stopped: $t = 0, \pi/3$
(b) 2; 5 (c) 6

② b) $\int_0^{\pi/2} 6\sin 3t \, dt + 3$

$\Rightarrow 2 + 3 = 5$

c)



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6. **(a)** Right: $0 \le t < 4$
Left: never
Stopped: $t = 4$
(b) 16/3; 25/3
(c) 16/3

7. **(a)** Right: $0 \le t < \pi/2$, $3\pi/2 < t \le 2\pi$
Left: $\pi/2 < t < 3\pi/2$
Stopped: $t = \pi/2, 3\pi/2$
(b) 0; 3
(c) $2e - (2/e) \approx 4.7$

8. **(a)** Right: $0 < t \le 3$
Left: never
Stopped: $t = 0$
(b) $(\ln 10)/2 \approx 1.15$; 4.15
(c) $(\ln 10)/2 \approx 1.15$

(11) $a(t) = 32 \text{ ft}/\text{sec}^2$ $\quad v(0) = 90$

a) $v(t) = -32t \text{ ft}/\text{sec} + 90$ initial velocity

$v(3) = -32(3) + 90$

$= -96 + 90 = -6$

11. ***Projectile*** Recall that the acceleration due to Earth's gravity is 32 ft/sec^2. From ground level, a projectile is fired straight upward with velocity 90 feet per second.

(a) What is its velocity after 3 seconds? -6 ft/sec

(b) When does it hit the ground? 5.625 sec

(c) When it hits the ground, what is the net distance it has traveled? 0

(d) When it hits the ground, what is the total distance it has traveled? 253.125 feet

$s(t) = -16t^2 + 90t + C$ ← initial displacement

$C = 0$

$s(t) = -16t^2 + 90t = 0$

$t(-16t + 90) = 0$

$-16t + 90 = 0$

$t = 90/16$

$$h(t) = y(t) = y_0 + v_0\, t + \frac{1}{2}\, a\, t^2 .$$

from college...

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90/16
                5.625
```

d) $s(t) = \int_0^{5.625} |v(t)|\, dt$

```
∫0^5.625 (|-32X+90|)
             253.125
```

9. An automobile accelerates from rest at $1 + 3\sqrt{t}$ mph/sec for 9 seconds.

$v(0) = 0$

(a) What is its velocity after 9 seconds? 63 mph

(b) How far does it travel in those 9 seconds? 344.52 feet

a

velocity acceleration

$$v(t) = \int a(t)\,dt$$

$$3t^{1/2}$$

$$= \int (1 + 3\sqrt{t})\,dt$$

$$= t + 2t^{3/2}$$

$$v(t) = t + 2t^{3/2}$$

$$v(9) = 9 + 2(9)^{3/2}$$

$$9 + 2(27)$$

$$9 + 54 = 63$$

b) $v(t) = t + 2t^{3/2}$

total distance $= \int_0^9 t + 2t^{3/2}\,dt$

$$= \frac{1}{2}t^2 + \frac{4}{5}t^{5/2} \Big|_0^9$$

$$= \frac{1}{2}(81) + \frac{4}{5}(3)^5$$

1 mile = 5280 ft

1 hr = 3600 sec

$$\left(\frac{5280}{3600}\right)(234.9)$$

```
5280/3600
          1.466666667
Ans*234.9
               344.52
```

$= 344.52$

10. A particle travels with velocity

$$v(t) = (t - 2)\sin t \ \ \text{m/sec}$$

for $0 \le t \le 4$ sec.

(a) What is the particle's displacement? ≈ -1.44952 meters

(b) What is the total distance traveled? ≈ 1.91411 meters

absolute value!

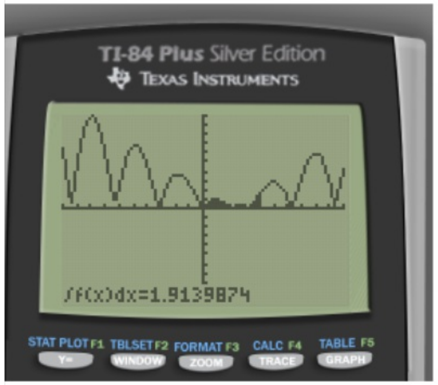
$$s(t) = \int v(t)$$

$$= \int_0^4 (t-2)\sin t\, dt$$

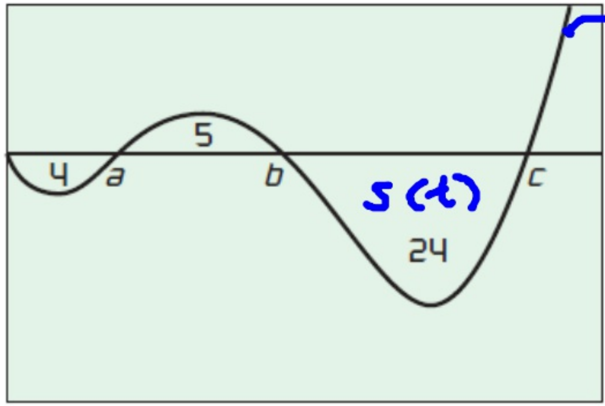
$$= -1.44952$$

$$\int |v(t)|$$

$$\int_0^4 |(t-2)\sin t|\, dt$$



In Exercises 12–16, a particle moves along the x-axis (units in cm). Its initial position at $t = 0$ sec is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the *areas* of the enclosed regions.



v(t)

12 $-4+5-24$

13 $4+5+24$

12. What is the particle's displacement between $t = 0$ and $t = c$? -23 cm

13. What is the total distance traveled by the particle in the same time period? 33 cm

a: 11 b: 16 c: -8

14. Give the positions of the particle at times a, b, and c.

15. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$? $t = a$

(positive)
a(t) = slope of v(t)

16. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$? $t = c$

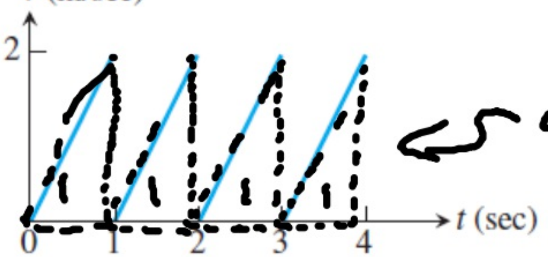
In Exercises 17–20, the graph of the velocity of a particle moving on the x-axis is given. The particle starts at $x = 2$ when $t = 0$.

(a) Find where the particle is at the end of the trip.

(b) Find the total distance traveled by the particle.

$s(0) = 2$

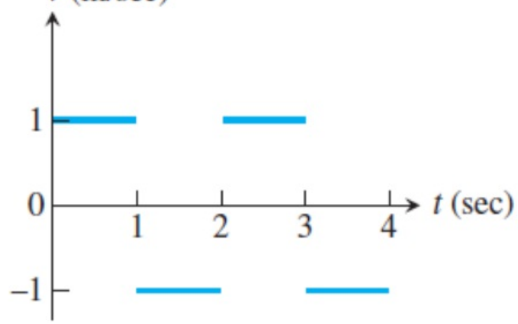
17. (a) 6 (b) 4 meters



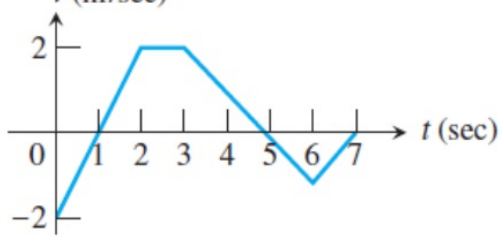
area: 4

starts @ 2, ends @ $2+4=6$

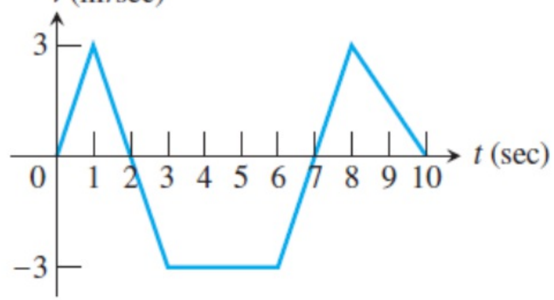
18. (a) 2 (b) 4 meters



19. (a) 5 (b) 7 meters



20. (a) −2.5 (b) 19.5 meters



21. ***U.S. Oil Consumption*** The rate of consumption of oil in the United States during the 1980s (in billions of barrels per year) is modeled by the function $C = 27.08 \cdot e^{t/25}$, where t is the number of years after January 1, 1980. Find the total consumption of oil in the United States from January 1, 1980 to January 1, 1990. $\approx$332.965 billion barrels

22. ***Home Electricity Use*** The rate at which your home consumes electricity is measured in kilowatts. If your home consumes electricity at the rate of 1 kilowatt for 1 hour, you will be charged for 1 "kilowatt-hour" of electricity. Suppose that the average consumption rate for a certain home is modeled by the function $C(t) = 3.9 - 2.4 \sin(\pi t/12)$, where $C(t)$ is measured in kilowatts and t is the number of hours past midnight. Find the average daily consumption for this home, measured in kilowatt-hours.

23. *Population Density* Population density measures the number of people per square mile inhabiting a given living area. Washerton's population density, which decreases as you move away from the city center, can be approximated by the function $10{,}000(2 - r)$ at a distance r miles from the city center.

(a) If the population density approaches zero at the edge of the city, what is the city's radius?

(b) A thin ring around the center of the city has thickness Δr and radius r. If you straighten it out, it suggests a rectangular strip. Approximately what is its area?

(c) Writing to Learn Explain why the population of the ring in part (b) is approximately

$$10{,}000(2 - r)(2\pi r)\,\Delta r.$$

(d) Estimate the total population of Washerton by setting up and evaluating a definite integral.

24. ***Oil Flow*** Oil flows through a cylindrical pipe of radius 3 inches, but friction from the pipe slows the flow toward the outer edge. The speed at which the oil flows at a distance r inches from the center is $8(10 - r^2)$ inches per second.

(a) In a plane cross section of the pipe, a thin ring with thickness Δr at a distance r inches from the center approximates a rectangular strip when you straighten it out. What is the area of the strip (and hence the approximate area of the ring)?

(b) Explain why we know that oil passes through this ring at approximately $8(10 - r^2)(2\pi r)\,\Delta r$ cubic inches per second.

(c) Set up and evaluate a definite integral that will give the rate (in cubic inches per second) at which oil is flowing through the pipe.