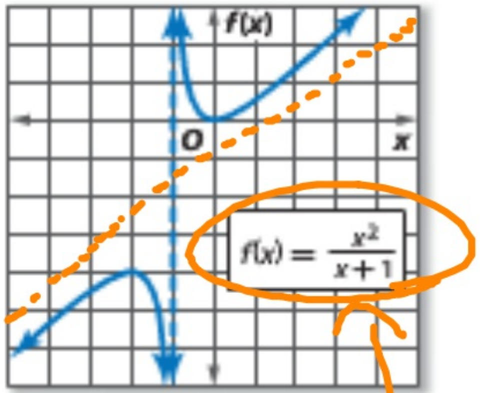
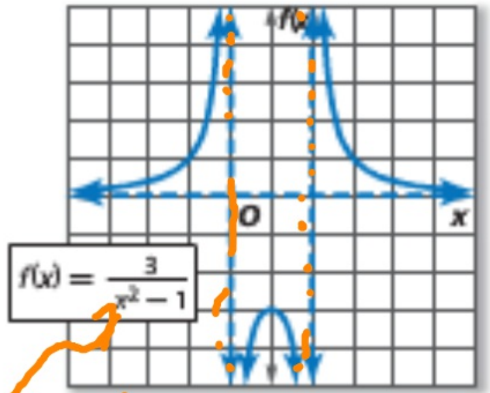
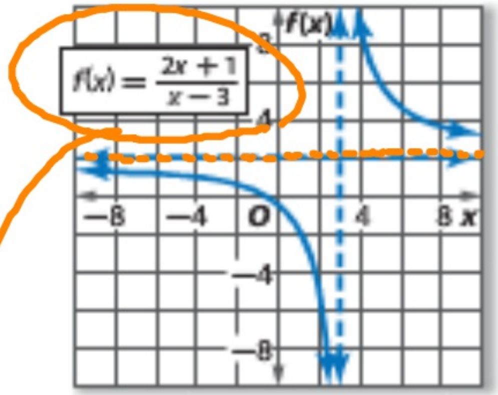


Vertical asymptote:
$x = -1$



Vertical asymptotes:
$x = -1, x = 1$
Horizontal asymptote:
$f(x) = 0$



Vertical asymptote:
$x = 3$
Horizontal asymptote:
$f(x) = 2$

$(x+1)(x-1)$

$f(x) = \frac{x^2}{x+1}$

As $x \to \infty$:
$f(x) \to \frac{x^2}{x} = x$
$f(x) = x$

As $x \to \infty$
$f(x) \to 2x/x$
$= 2$

KeyConcept Vertical and Horizontal Asymptotes

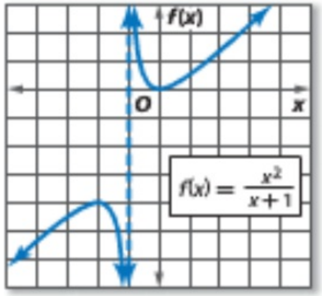
Words

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a **vertical asymptote** whenever $b(x) = 0$.
- $f(x)$ has at most one **horizontal asymptote.**
  - If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote.
  - If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$.
  - If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$.

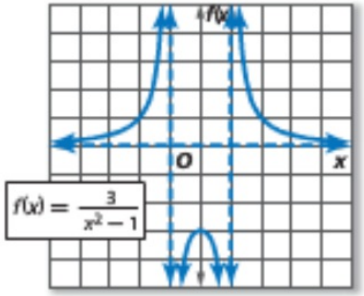
Examples

No horizontal asymptote

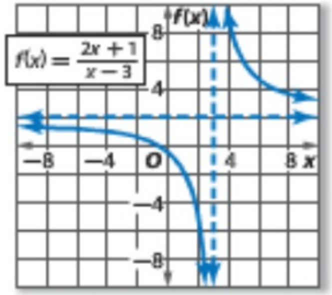


Vertical asymptote:
$x = -1$

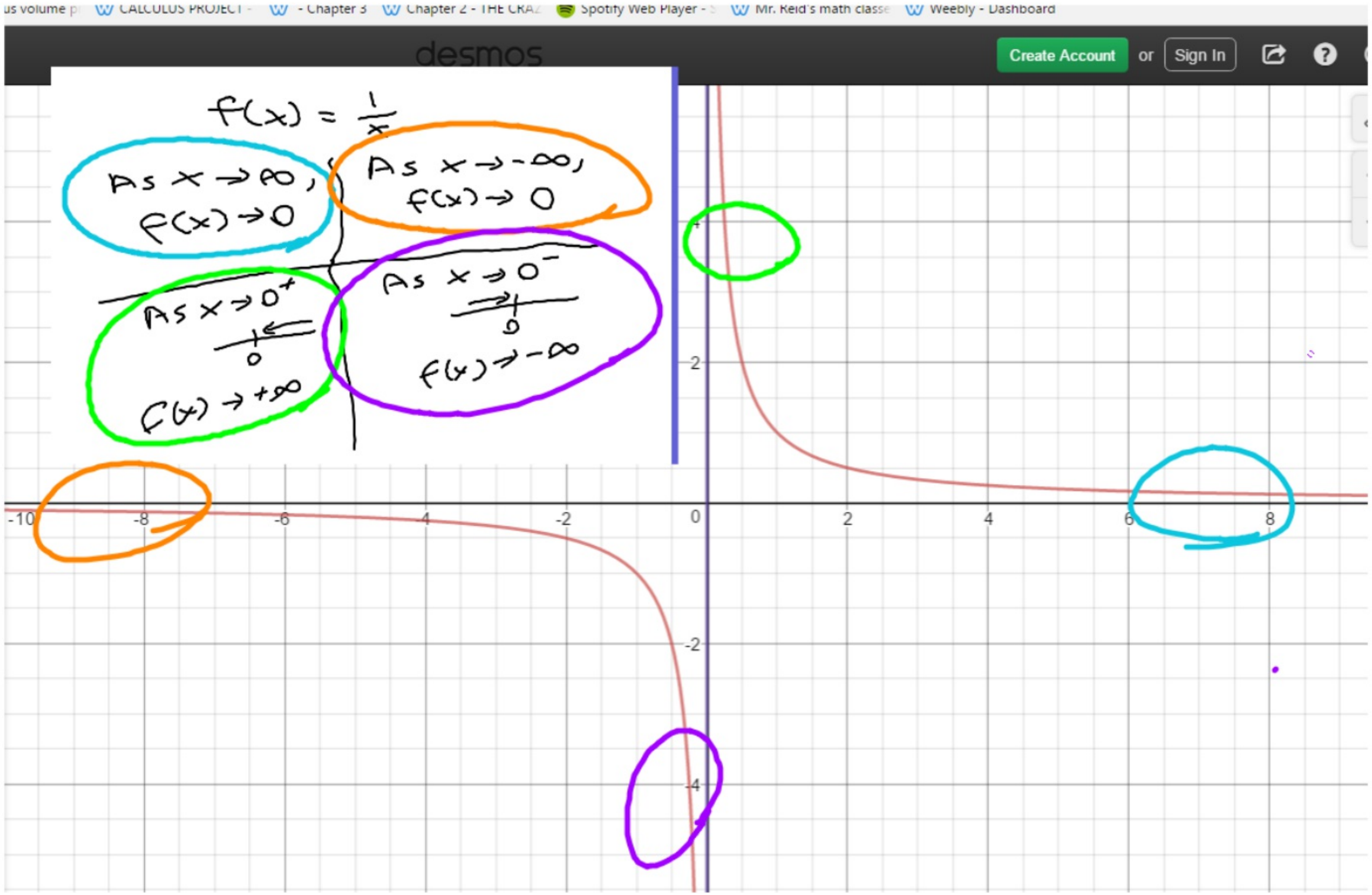
One horizontal asymptote



Vertical asymptotes:
$x = -1, x = 1$
Horizontal asymptote:
$f(x) = 0$



Vertical asymptote:
$x = 3$
Horizontal asymptote:
$f(x) = 2$



CALCULUS PROJECT
Chapter 3
Chapter 2 - THE CRAZ
Spotify Web Player
Mr. Reid's math classe
Weebly - Dashboard
desmos
Create Account
or
Sign In
f(x) = 1/x
As x → ∞, f(x) → 0
As x → -∞, f(x) → 0
As x → 0⁺
0
f(x) → +∞
As x → 0⁻
0
f(x) → -∞
-10
-8
-6
-4
-2
0
2
4
6
8
2
-2
-4

$$f(x) = \frac{1}{x}$$

As $x \to \infty$, $f(x) \to 0$

As $x \to -\infty$, $f(x) \to 0$

As $x \to 0^+$

$f(x) \to +\infty$

As $x \to 0^-$

$f(x) \to -\infty$

Example 1 Graph with no Horizontal Asymptote

As $x \to \infty$,

$f(x) \to \frac{x^3}{x} = x^2$

Graph $f(x) = \frac{x^3}{x-1}$.

Step 1 Find the zeros.

$x^3 = 0$ Set $a(x) = 0$.

$x = 0$ Take the cube root of each side.

There is a zero at $x = 0$.

Step 2 Draw the asymptotes.

Find the vertical asymptote.

$x - 1 = 0$ Set $b(x) = 0$.

$x = 1$ Add 1 to each side.

There is a vertical asymptote at $x = 1$.

The degree of the numerator is greater than the degree of the denoı
So, there is no horizontal asymptote.

Check Your Understanding

= Step-b

Example 1 Graph each function. **1, 2. See Chapter 8 Answer Appendix.**

1. $f(x) = \dfrac{x^4 - 2}{x^2 - 1}$

2. $f(x) = \dfrac{x^3}{x + 2}$

$\rightarrow \dfrac{x^4}{x^2} = x^2$

$x = -2$

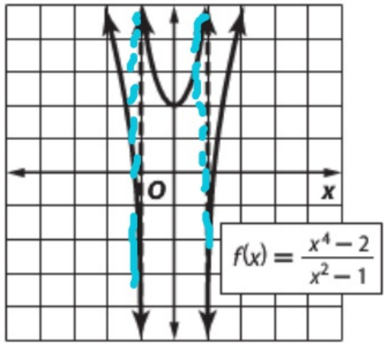
As $x \rightarrow \infty$

$f(x) \rightarrow \dfrac{x^3}{x} = x^2$

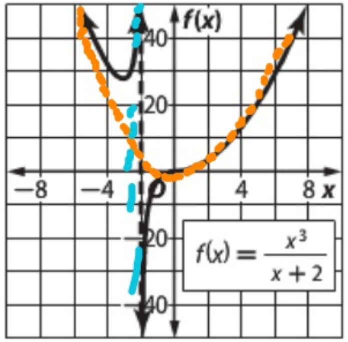
$f(x) = \dfrac{\;}{(x-1)(x+1)}$

Lesson 8-4

1.



2.



Real-World Example 2 Use Graphs of Rational Functions

AVERAGE SPEED A boat traveled upstream at r_1 miles per hour. During the return trip to its original starting point, the boat traveled at r_2 miles per hour. The average speed for the entire trip R is given by the formula $R = \frac{2r_1r_2}{r_1 + r_2}$.

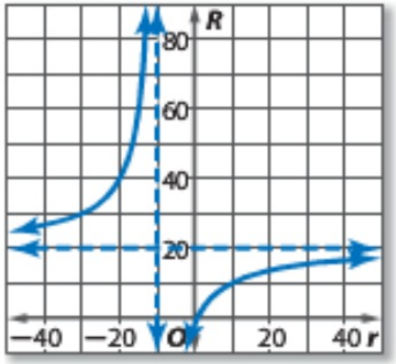
a. Let r_1 be the independent variable, and let R be the dependent variable. Draw the graph if $r_2 = 10$ miles per hour.

The function is $R = \frac{2r_1(10)}{r_1 + (10)}$ or $R = \frac{20r_1}{r_1 + 10}$.

The vertical asymptote is $r_1 = -10$.

Graph the vertical asymptote and the function.

Notice that the horizontal asymptote is $R = 20$.



b. What is the R-intercept of the graph?

c. What domain and range values are meaningful in the context of the problem?

Example 2

3. CCSS **REASONING** Eduardo is a kicker for his high school football team. So far this season, he has made 7 out of 11 field goals. He would like to improve his field goal percentage. If he can make x consecutive field goals, his field goal percentage can be determined using the function $P(x) = \frac{7 + x}{11 + x}$.

   a. Graph the function. **See Chapter 8 Answer Appendix.** [redacted]

   b. What part of the graph is meaningful in the context of this problem? [redacted]

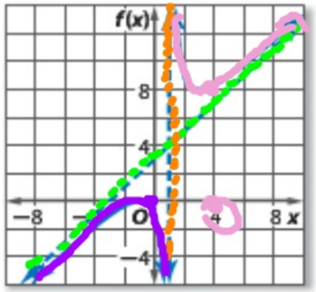
   c. Describe the meaning of the intercept of the vertical axis. [redacted]

   d. What is the equation of the horizontal asymptote? Explain its meaning with respect to Eduardo's field goal percentage. [redacted] the degree of $b(x)$ equals 1. The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.

Example $f(x) = \frac{x^4 + 3x^3}{x^3 - 1}$

Vertical asymptote: $x = 1$

Oblique asymptote: $f(x) = x + 3$



let $x = 0$, $y = 0$

$(x-1)(x^2 + \ldots\ldots)$

$x = 4 \ldots$

$\frac{256+192}{63} \approx 7 \ldots$

$$\frac{-x}{x^3} = \frac{-1}{x^2}$$

as $x \to \infty$, fraction $\to 0$

$$x + 3 + \frac{-x-4}{x^3-1}$$

$$x^3 - 1 \overline{)\, x^4 + 3x^3 + 0x^2 + 0x - 1}$$

$-(x^4 \qquad -x)$

$3x^3 \qquad -x \quad -1$

$-(3x^3 \qquad -3)$

$-x \quad -4$

Example 3 Determine Oblique Asymptotes

Graph $f(x) = \frac{x^2 + 4x + 4}{2x - 1}$.

Step 1 Find the zeros.

$x^2 + 4x + 4 = 0$ Set $a(x) = 0$.

$(x + 2)^2 = 0$ Factor.

$x + 2 = 0$ Take the square root of each side.

$x = -2$ Subtract 2 from each side.

There is a zero at $x = -2$.

Step 2 Find the asymptotes.

$2x - 1 = 0$ Set $b(x) = 0$.

$2x = 1$ Add 1 to each side.

$x = \frac{1}{2}$ Divide each side by 2.

There is a vertical asymptote at $x = \frac{1}{2}$.

The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote.

The difference between the degree of the numerator and the degree of the denominator is 1, so there is an oblique asymptote.

Divide the numerator by the denominator to determine the equation of the oblique asymptote.

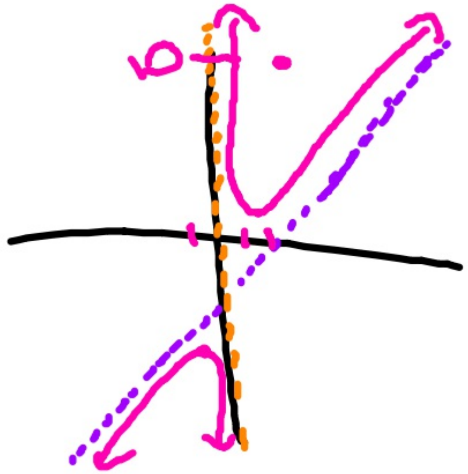
The equation of the asymptote is the quotient excluding any remainder.

$$\begin{array}{r} \frac{1}{2}x + \frac{9}{4} \\ 2x - 1\overline{)x^2 + 4x + 4} \\ \underline{(-)x^2 - \frac{1}{2}x} \\ \frac{9}{2}x + 4 \\ \underline{(-)\frac{9}{2}x - \frac{9}{4}} \\ \frac{25}{4} \end{array}$$

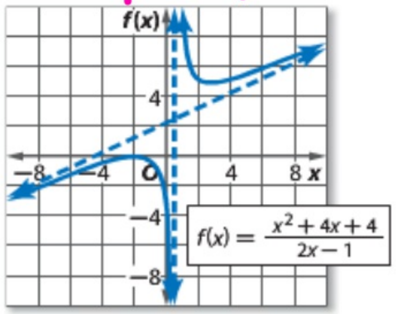
Thus, the oblique asymptote is the line $f(x) = \frac{1}{2}x + \frac{9}{4}$.

Example 3 Determine Oblique Asymptotes

Graph $f(x) = \frac{x^2 + 4x + 4}{2x - 1}$.



Step 3 Draw the asymptotes, and then use a table of values to graph the function.

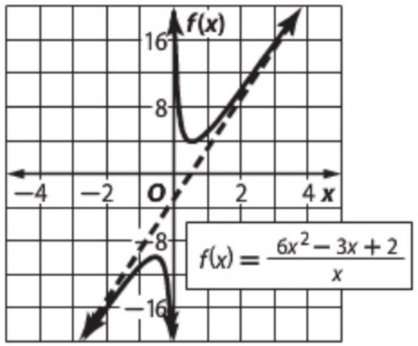


Examples 3–4 Graph each function. **4–7. See Chapter 8 Answer Appendix.**

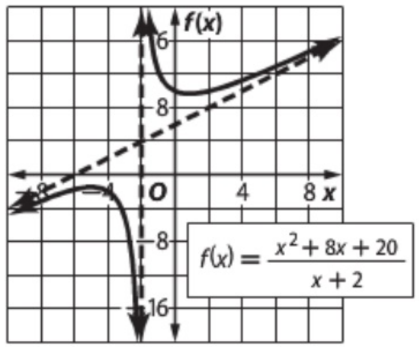
4. $f(x) = \frac{6x^2 - 3x + 2}{x}$

5. $f(x) = \frac{x^2 + 8x + 20}{x + 2}$

4.



5.

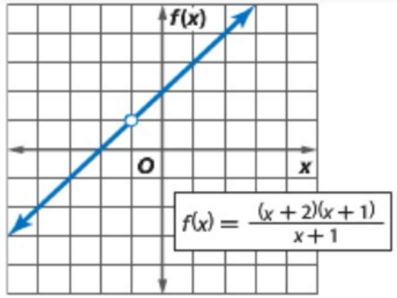


In some cases, graphs of rational functions may have **point discontinuity**, which looks like a hole in the graph. This is because the function is undefined at that point.

KeyConcept Point Discontinuity

Words If $f(x) = \frac{a(x)}{b(x)}$, $b(x) \neq 0$, and $x - c$ is a factor of both $a(x)$ and $b(x)$, then there is a point discontinuity at $x = c$.

Example
$$f(x) = \frac{(x+2)(x+1)}{x+1}$$
$$= x + 2;\ x \neq -1$$



Example 4 Graph with Point Discontinuity

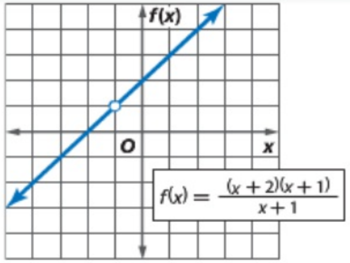
Graph $f(x) = \frac{x^2 - 16}{x - 4}$.

KeyConcept Point Discontinuity

Words If $f(x) = \frac{a(x)}{b(x)}$, $b(x) \neq 0$, and $x - c$ is a factor of both $a(x)$ and $b(x)$, then there is a point discontinuity at $x = c$.

Example $f(x) = \frac{(x+2)(x+1)}{x+1}$

$= x + 2;\ x \neq -1$



6. $f(x) = \dfrac{x^2 - 4x - 5}{x + 1}$

7. $f(x) = \dfrac{x^2 + x - 12}{x + 4}$

Practice and Problem Solving

Extra Practice is on page R8.

Example 1

Graph each function. **8–11. See margin.**

8. $f(x) = \dfrac{x^4}{6x + 12}$

9. $f(x) = \dfrac{x^3}{8x - 4}$

10. $f(x) = \dfrac{x^4 - 16}{x^2 - 1}$

11. $f(x) = \dfrac{x^3 + 64}{16x - 24}$

12a. $c(t) = \dfrac{9.5t - 75}{t - 15}$; **See margin for graph.**

Example 2

12. SCHOOL SPIRIT As president of Student Council, Brandy is getting T-shirts made for a pep rally. Each T-shirt costs \$9.50, and there is a set-up fee of \$75. The student council plans to sell the shirts, but each of the 15 council members will get one for free.

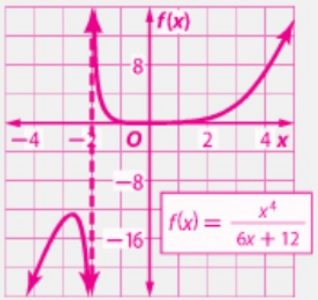
a. Write a function for the average cost of a T-shirt to be sold. Graph the function.

b. What is the average cost if 200 shirts are ordered? if 500 shirts are ordered? **\$10.68; \$9.95**

c. How many T-shirts must be ordered to bring the average cost under \$9.75? **more than 885**

Additional Answers

8.



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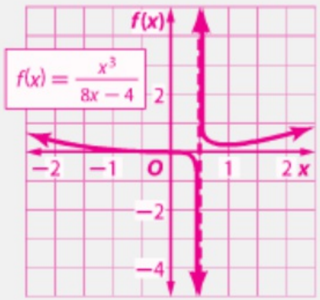

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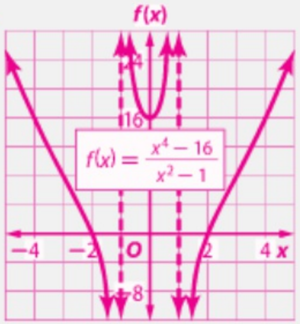

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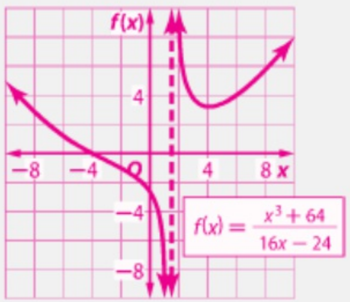


12a.

Examples 2–3 Graph each function. **13–26. See Chapter 8 Answer Appendix.**

13. $f(x) = \frac{x}{x + 2}$

14. $f(x) = \frac{5}{(x - 1)(x + 4)}$

15. $f(x) = \frac{4}{(x - 2)^2}$

16. $f(x) = \frac{x - 3}{x + 1}$

17. $f(x) = \frac{1}{(x + 4)^2}$

18. $f(x) = \frac{2x}{(x + 2)(x - 5)}$

19. $f(x) = \frac{(x - 4)^2}{x + 2}$

20. $f(x) = \frac{(x + 3)^2}{x - 5}$

21. $f(x) = \frac{x^3 + 1}{x^2 - 4}$

22. $f(x) = \frac{4x^3}{2x^2 + x - 1}$

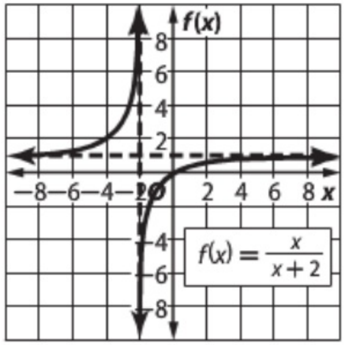
23. $f(x) = \frac{3x^2 + 8}{2x - 1}$

24. $f(x) = \frac{2x^2 + 5}{3x + 4}$

25. $f(x) = \frac{x^4 - 2x^2 + 1}{x^3 + 2}$

26. $f(x) = \frac{x^4 - x^2 - 12}{x^3 - 6}$

13.



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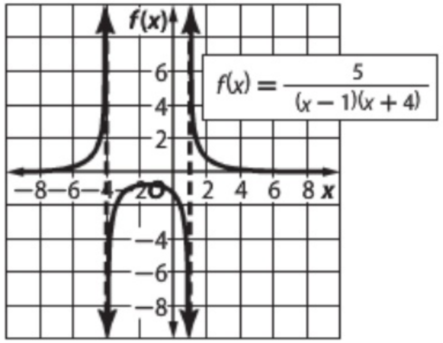

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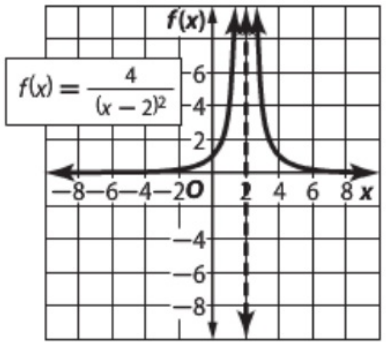

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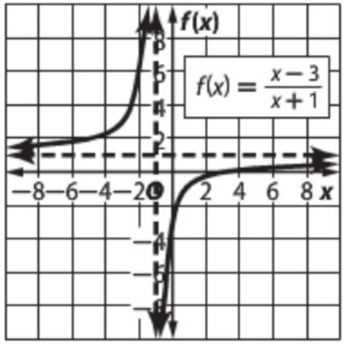

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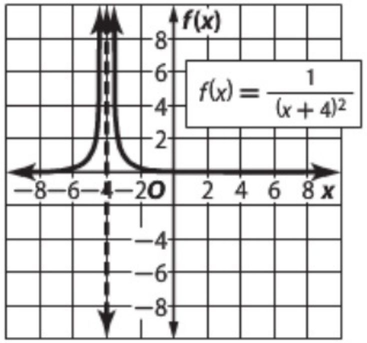

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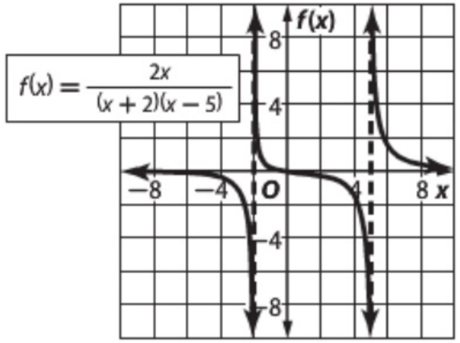

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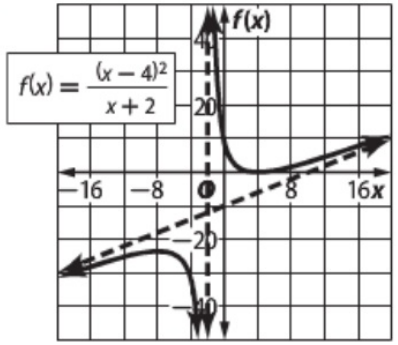

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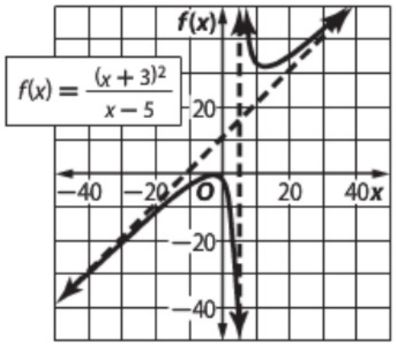

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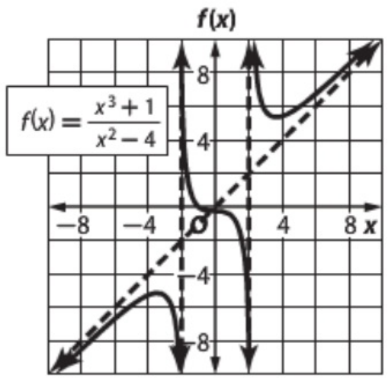


27. CCSS **PERSEVERANCE** The current in amperes in an electrical circuit with three resistors in a series is given by the equation $I = \frac{V}{R_1 + R_2 + R_3}$, where V is the voltage in volts in the circuit and R_1, R_2, and R_3 are the resistances in ohms of the three resistors.

a. Let R_1 be the independent variable, and let I be the dependent variable. Graph the function if $V = 120$ volts, $R_2 = 25$ ohms, and $R_3 = 75$ ohms. **See Chapter 8 Answer Appendix.**

b. Give the equation of the vertical asymptote and the R_1- and I-intercepts of the graph. **$R_1 = -100$; no R_1-intercept; 1.2**

c. Find the value of I when the value of R_1 is 140 ohms. **0.5 amperes**

d. What domain and range values are meaningful in the context of the problem?
$R_1 \geq 0$ and $0 < I \leq 1.2$

Example 4 Graph each function. **28–35. See Chapter 8 Answer Appendix.**

28. $f(x) = \frac{x^2 - 2x - 8}{x - 4}$

29. $f(x) = \frac{x^2 + 4x - 12}{x - 2}$

30. $f(x) = \frac{x^2 - 25}{x + 5}$

31. $f(x) = \frac{x^2 - 64}{x - 8}$

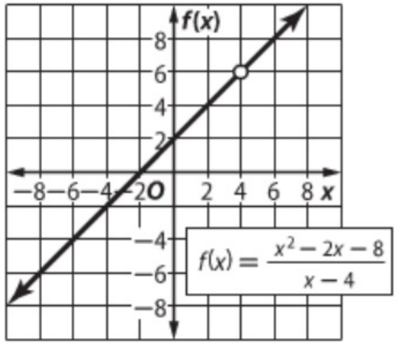
32. $f(x) = \frac{(x - 4)(x^2 - 4)}{x^2 - 6x + 8}$

33. $f(x) = \frac{(x + 5)(x^2 + 2x - 3)}{x^2 + 8x + 15}$

34. $f(x) = \frac{3x^4 + 6x^3 + 3x^2}{x^2 + 2x + 1}$

35. $f(x) = \frac{2x^4 + 10x^3 + 12x^2}{x^2 + 5x + 6}$

28.



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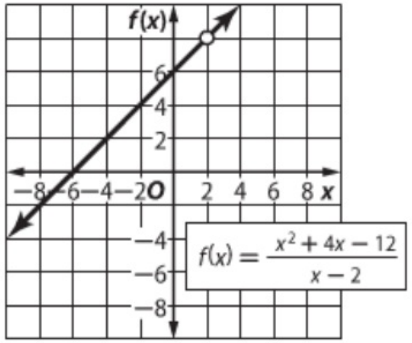

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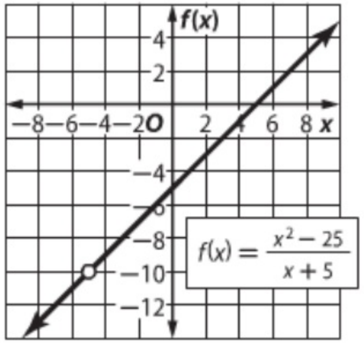

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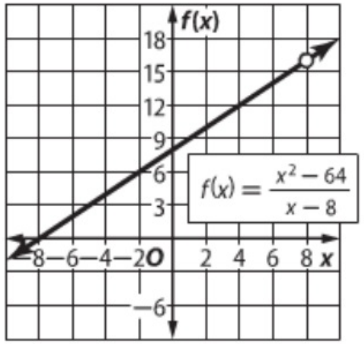

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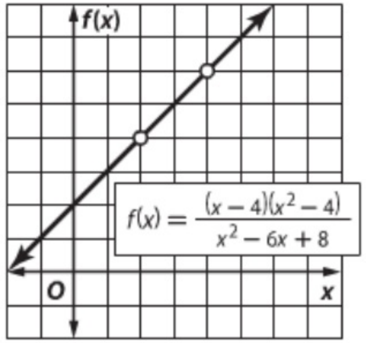

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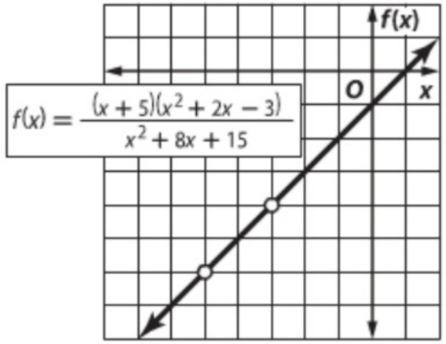

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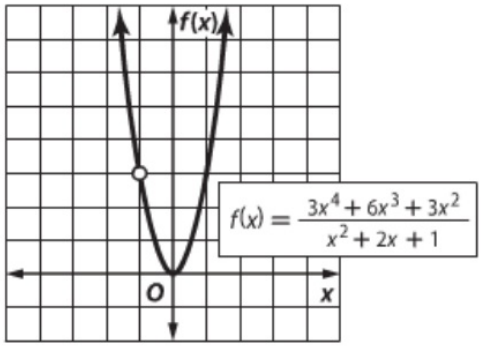

35.