

# 8-6 Study Guide and Intervention

## Solving $x^2 + bx + c = 0$

**Factor  $x^2 + bx + c$**  To factor a trinomial of the form  $x^2 + bx + c$ , find two integers,  $m$  and  $p$ , whose sum is equal to  $b$  and whose product is equal to  $c$ .

<b>Factoring <math>x^2 + bx + c</math></b>	$x^2 + bx + c = (x + m)(x + p)$ , where $m + p = b$ and $mp = c$ .
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### Example 1 Factor each polynomial.

a.  $x^2 + 7x + 10$

In this trinomial,  $b = 7$  and  $c = 10$ .

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since  $2 + 5 = 7$  and  $2 \cdot 5 = 10$ , let  $m = 2$  and  $p = 5$ .

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

b.  $x^2 - 8x + 7$

In this trinomial,  $b = -8$  and  $c = 7$ .

Notice that  $m + p$  is negative and  $mp$  is positive, so  $m$  and  $p$  are both negative. Since  $-7 + (-1) = -8$  and  $(-7)(-1) = 7$ ,  $m = -7$  and  $p = -1$ .

$$x^2 - 8x + 7 = (x - 7)(x - 1)$$

### Example 2 Factor $x^2 + 6x - 16$ .

In this trinomial,  $b = 6$  and  $c = -16$ . This means  $m + p$  is positive and  $mp$  is negative. Make a list of the factors of  $-16$ , where one factor of each pair is positive.

Factors of $-16$	Sum of Factors
1, $-16$	$-15$
$-1$ , 16	15
2, $-8$	$-6$
$-2$ , 8	6

Therefore,  $m = -2$  and  $p = 8$ .

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

## Exercises

Factor each polynomial.

- |   |  |   |
|---|--|---|
| 1. $x^2 + 4x + 3$<br>$(x + 3)(x + 1)$     | 2. $m^2 + 12m + 32$<br>$(m + 4)(m + 8)$      | 3. $r^2 - 3r + 2$<br>$(r - 2)(r - 1)$       |
| 4. $x^2 - x - 6$<br>$(x - 3)(x + 2)$      | 5. $x^2 - 4x - 21$<br>$(x - 7)(x + 3)$       | 6. $x^2 - 22x + 121$<br>$(x - 11)(x - 11)$  |
| 7. $t^2 - 4t - 12$<br>$(t + 2)(t - 6)$    | 8. $p^2 - 16p + 64$<br>$(p - 8)(p - 8)$      | 9. $9 - 10x + x^2$<br>$(9 - x)(1 - x)$      |
| 10. $x^2 + 6x + 5$<br>$(x + 5)(x + 1)$    | 11. $a^2 + 8a - 9$<br>$(a - 1)(a + 9)$       | 12. $y^2 - 7y - 8$<br>$(y - 8)(y + 1)$      |
| 13. $x^2 - 2x - 3$<br>$(x - 3)(x + 1)$    | 14. $y^2 + 14y + 13$<br>$(y + 1)(y + 13)$    | 15. $m^2 + 9m + 20$<br>$(m + 4)(m + 5)$     |
| 16. $x^2 + 12x + 20$<br>$(x + 10)(x + 2)$ | 17. $a^2 - 14a + 24$<br>$(a - 2)(a - 12)$    | 18. $18 + 11y + y^2$<br>$(9 + y)(2 + y)$    |
| 19. $x^2 + 2xy + y^2$<br>$(x + y)(x + y)$ | 20. $a^2 - 4ab + 4b^2$<br>$(a - 2b)(a - 2b)$ | 21. $x^2 + 6xy - 7y^2$<br>$(x + 7y)(x - y)$ |

**8-6 Study Guide and Intervention** *(continued)***Solving  $x^2 + bx + c = 0$** 

**Solve Equations by Factoring** Factoring and the Zero Product Property can be used to solve many equations of the form  $x^2 + bx + c = 0$ .

**Example 1** Solve  $x^2 + 6x = 7$ . Check your solutions.

$x^2 + 6x = 7$	Original equation
$x^2 + 6x - 7 = 0$	Rewrite equation so that one side equals 0.
$(x - 1)(x + 7) = 0$	Factor.
$x - 1 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 1$ $x = -7$	Solve each equation.

The solution set is  $\{1, -7\}$ . Since  $1^2 + 6(1) = 7$  and  $(-7)^2 + 6(-7) = 7$ , the solutions check.

**Example 2** **ROCKET LAUNCH** The formula  $h = vt - 16t^2$  gives the height  $h$  of a rocket after  $t$  seconds when the initial velocity  $v$  is given in feet per second. If a rocket is fired with initial velocity 2288 feet per second, how many seconds will it take for the rocket to reach a height of 6720 feet?

$h = vt - 16t^2$	Formula
$6720 = 2288t - 16t^2$	Substitute.
$0 = -16t^2 + 2288t - 6720$	Rewrite equation so that one side equals 0.
$0 = -16(t - 143t + 420)$	Factor out GCF.
$0 = -16(t - 3)(t - 140)$	Factor
$t - 3 = 0$ or $t - 140 = 0$	Zero Product Property
$t = 3$ $t = 140$	Solve each equation.

The rocket reaches 6720 feet in 3 seconds and again in 140 seconds, or 2 minutes 20 seconds after launch.

**Exercises**

Solve each equation. Check the solutions.

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|--------------------------------------|--------------------------------------|--|
| 1. $x^2 - 4x + 3 = 0$ <b>{1, 3}</b>  | 2. $y^2 - 5y + 4 = 0$ <b>{1, 4}</b>  | 3. $m^2 + 10m + 9 = 0$ <b>{-1, -9}</b> |
| 4. $x^2 = x + 2$ <b>{-1, 2}</b>      | 5. $x^2 - 4x = 5$ <b>{-1, 5}</b>     | 6. $x^2 - 12x + 36 = 0$ <b>{6}</b>     |
| 7. $t^2 - 8 = -7t$ <b>{-8, 1}</b>    | 8. $p^2 = 9p - 14$ <b>{2, 7}</b>     | 9. $-9 - 8x + x^2 = 0$ <b>{-1, 9}</b>  |
| 10. $x^2 + 6 = 5x$ <b>{2, 3}</b>     | 11. $a^2 = 11a - 18$ <b>{2, 9}</b>   | 12. $y^2 - 8y + 15 = 0$ <b>{3, 5}</b>  |
| 13. $x^2 = 24 - 10x$ <b>{-12, 2}</b> | 14. $a^2 - 18a = -72$ <b>{6, 12}</b> | 15. $b^2 = 10b - 16$ <b>{2, 8}</b>     |

Use the formula  $h = vt - 16t^2$  to solve each problem.

- 16. FOOTBALL** A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to first reach a height of 32 feet? **1 second**
- 17. ROCKET LAUNCH** If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14,400 feet high? **at 10 seconds and at 90 seconds**