**8-7 Study Guide and Intervention**

***Solving ax2* + *bx* + *c* = 0**

**Factor** $ax^{2}$ **+ *bx* + *c*** To factor a trinomial of the form $ax^{2}$ + *bx* + *c*, find two integers, *m* and *p* whose product is equal to *ac* and whose sum is equal to *b*. If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

**Example 1: Factor 2**$x^{2}$ **+ 15*x* + 18.**

In this example, *a* = 2, *b* = 15, and *c* = 18. You need to find two numbers that have a sum of 15 and a product of 2 ⋅ 18 or 36. Make a list of the factors of 36 and look for

the pair of factors with a sum of 15.

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| **Factors of 36** | **Sum of Factors** |
| 1, 36 | 37 |
| 2, 18 | 20 |
| 3, 12 | 15 |

Use the pattern *a*$x^{2}$ + *mx* + *px* + *c*, with *a* = 2, *m* = 3,

*p* = 12, and *c* = 18.

2$x^{2}$ + 15*x* + 18 = 2$x^{2}$ + 3*x* + 12*x* + 18

 = (2$x^{2}$ + 3*x*) + (12*x* + 18)

 = *x*(2*x* + 3) + 6(2*x* + 3)

 = (*x* + 6)(2*x* + 3)

Therefore, 2$x^{2}$ + 15*x* + 18 = (*x* + 6)(2*x* + 3).

**Example 2: Factor 3**$x^{2}$ **– 3*x* – 18.**

Note that the GCF of the terms 3$x^{2}$, 3*x*, and 18 is 3.
First factor out this GCF. 3$x^{2}$ – 3*x* – 18 = 3($x^{2}$ – *x* – 6).

Now factor $x^{2}$– *x* – 6. Since *a* = 1, find the two factors
of –6 with a sum of –1.

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| **Factors of –6** | **Sum of Factors** |
| 1, –6 | –5 |
| –1, 6 | 5 |
| –2, 3 | 1 |
| 2, –3 | –1 |

Now use the pattern (*x* + *m*)(*x* + *p*) with *m* = 2 and *p* = –3.

$x^{2}$ – *x* – 6 = (*x* + 2)(*x* – 3)

The complete factorization is

3$x^{2}$ – 3*x* – 18 = 3(*x* + 2)(*x* – 3).

**Exercises**

**Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.**

 **1.** 2$x^{2}$ – 3*x* – 2 **2.** 3$m^{2}$ – 8*m* – 3 **3.** 16$r^{2}$ – 8*r* + 1

 **4.** 6$x^{2}$ + 5*x* – 6 **5.** 3$x^{2}$ + 2*x* – 8 **6.** 18$x^{2}$ – 27*x* – 5

 **7.** 2$a^{2}$ + 5*a* + 3 **8.** 18$y^{2}$ + 9*y* – 5 **9.** –4$t^{2}$ + 19*t* – 21

**10.** 8$x^{2}$ – 4*x* – 24 **11.** 28$p^{2}$ + 60*p* – 25 **12.** 48$x^{2}$ + 22*x* – 15

**13.** 3$y^{2}$ – 6*y* – 24 **14.** 4$x^{2}$ + 26*x* – 48 **15.** 8$m^{2}$ – 44*m* + 48

**16.** 6$x^{2}$ – 7*x* + 18 **17.** 2$a^{2}$ – 14*a* + 18 **18.** 18 + 11*y* + 2$y^{2}$

**8-7 Study Guide and Intervention** *(continued)*

***Solving ax*2 + *bx* + *c* = 0**

**Solve Equations by Factoring** Factoring and the Zero Product Property can be used to solve some equations of the form *a*$x^{2}$ + *bx* + *c* = 0.

**Example: Solve 12**$x^{2}$ **+ 3*x* = 2 – 2*x*. Check your solutions.**

 12$x^{2}$ + 3*x* = 2 – 2*x* Original equation

 12$x^{2}$ + 5*x* – 2 = 0 Rewrite equation so that one side equals 0.

 (3*x* + 2)(4*x* – 1) = 0 Factor the left side.

 3*x* + 2 = 0 or 4*x* – 1 = 0 Zero Product Property

 *x* = – $\frac{2}{3}$ *x* = $\frac{1}{4}$ Solve each equation.

The solution set is $\left\{- \frac{2}{3} , \frac{1}{4}\right\}$ .

Since 12$\left(- \frac{2}{3}\right)^{2}$ + 3$\left(- \frac{2}{3}\right)$ = 2 – 2$\left(- \frac{2}{3}\right)$ and 12$\left(\frac{1}{4}\right)^{2}$+ 3$\left(\frac{1}{4}\right)$ = 2 – 2$\left(\frac{1}{4}\right)$ , the solutions check.

**Exercises**

**Solve each equation. Check the solutions.**

 **1.** 8$x^{2}$ + 2*x* – 3 = 0 **2.** 3$n^{2}$ – 2*n* – 5 = 0 **3.** 2$d^{2}$ – 13*d* – 7 = 0

 **4.** 4$x^{2}$ = *x* + 3 **5.** 3$x^{2}$ – 13*x* = 10 **6.** 6$x^{2}$ – 11*x* – 10 = 0

 **7.** 2$k^{2}$ – 40 = –11*k* **8.** 2$p^{2}$ = –21*p* – 40 **9.** –7 – 18*x* + 9$x^{2}$ = 0

**10.** 12$x^{2}$ – 15 = – 8*x* **11.** 7$a^{2}$ = –65*a* – 18 **12.** 16$y^{2}$ – 2*y* – 3 = 0

**13.** 8$x^{2}$ + 5*x* = 3 + 7*x* **14.** 4$a^{2}$ – 18*a* + 5 = 15 **15.** 3$b^{2}$ – 18*b* = 10*b* – 49

**16.** The difference of the squares of two consecutive odd integers is 24. Find the integers.

**17. GEOMETRY** The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width.
The area is 300 square yards. What are the dimensions?



**18. GEOMETRY** A rectangle with an area of 24 square inches is formed by cutting strips of equal width from a rectangular piece of paper. Find the dimensions of the new rectangle if the original rectangle measures 8 inches by 6 inches.