9-3 Study Guide and Intervention

Transformations of Quadratic Functions

Translations A **translation** is a change in the position of a figure either up, down, left, right, or diagonal. Adding or subtracting constants in the equations of functions translates the graphs of the functions.

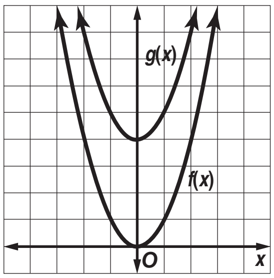
The graph of $g(x) = x^2 + k$ translates the graph of $f(x) = x^2$ vertically.
If $k > 0$, the graph of $f(x) = x^2$ is translated k units up.
If $k < 0$, the graph of $f(x) = x^2$ is translated $\lvert k \rvert$ units down.

The graph of $g(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.
If $h > 0$, the graph of $f(x) = x^2$ is translated h units to the right.
If $h < 0$, the graph of $f(x) = x^2$ is translated $\lvert h \rvert$ units to the left.

Example **Describe how the graph of each function is related to the graph of $f(x) = x^2$.**

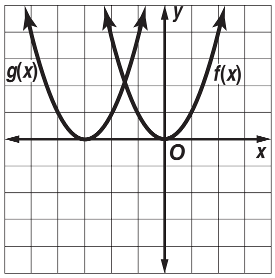
a. $g(x) = x^2 + 4$

The value of k is 4, and $4 > 0$. Therefore, the graph of $g(x) = x^2 + 4$ is a translation of the graph of $f(x) = x^2$ up 4 units.



b. $g(x) = (x + 3)^2$

The value of h is –3, and $-3 < 0$. Thus, the graph of $g(x) = (x + 3)^2$ is a translation of the graph of $f(x) = x^2$ to the left 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 1$

Translation of $f(x) = x^2$ up 1 unit.

2. $g(x) = (x - 6)^2$

Translation of $f(x) = x^2$ to the right 6 units.

3. $g(x) = (x + 1)^2$

Translation of $f(x) = x^2$ to the left 1 unit.

4. $g(x) = 20 + x^2$

Translation of $f(x) = x^2$ up 20 units.

5. $g(x) = (-2 + x)^2$

Translation of $f(x) = x^2$ to the right 2 units.

6. $g(x) = -\frac{1}{2} + x^2$

Translation of $f(x) = x^2$ down $\frac{1}{2}$ unit.

7. $g(x) = x^2 + \frac{8}{9}$

Translation of $f(x) = x^2$ up $\frac{8}{9}$ unit.

8. $g(x) = x^2 - 0.3$

Translation of $f(x) = x^2$ down 0.3 unit.

9. $g(x) = (x + 4)^2$

Translation of $f(x) = x^2$ to the left 4 units.

Lesson 9-3

9-3 Study Guide and Intervention *(continued)*

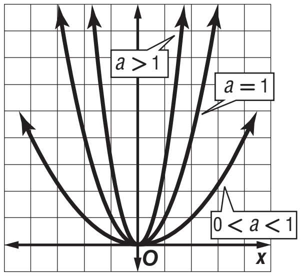
Transformations of Quadratic Functions

Dilations and Reflections A **dilation** is a transformation that makes the graph narrower or wider than the parent graph. A **reflection** flips a figure over the x- or y-axis.

The graph of $f(x) = ax^2$ stretches or compresses the graph of $f(x) = x^2$.

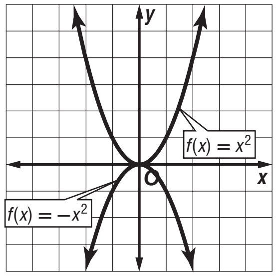
If $|a| > 1$, the graph of $f(x) = x^2$ is stretched vertically.

If $0 < |a| < 1$, the graph of $f(x) = x^2$ is compressed vertically.



The graph of the function $-f(x)$ flips the graph of $f(x) = x^2$ across the x-axis.

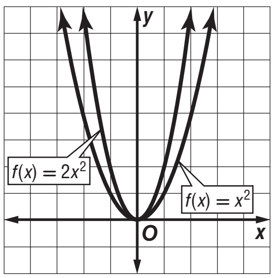
The graph of the function $f(-x)$ flips the graph of $f(x) = x^2$ across the y-axis.



Example **Describe how the graph of each function is related to the graph of $f(x) = x^2$.**

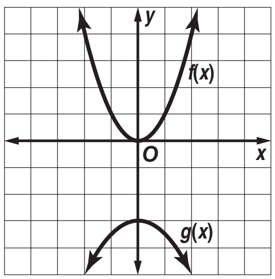
a. $g(x) = 2x^2$

The function can be written as $f(x) = ax^2$ where $a = 2$. Because $|a| > 1$, the graph of $y = 2x^2$ is the graph of $y = x^2$ that is stretched vertically.



b. $g(x) = -\frac{1}{2}x^2 - 3$

The negative sign causes a reflection across the x-axis. Then a dilation occurs in which $a = \frac{1}{2}$ and a translation in which $k = -3$. So the graph of $g(x) = -\frac{1}{2}x^2 - 3$ is reflected across the x-axis, dilated wider than the graph of $f(x) = x^2$, and translated down 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = -5x^2$

Compression of $f(x) = x^2$ narrower than the graph of $f(x) = x^2$ reflected over the x-axis.

2. $g(x) = -(x + 1)^2$

Translation of $f(x) = x^2$ to the left 1 unit and reflected over the x-axis.

3. $g(x) = -\frac{1}{4}x^2 - 1$

Dilation of $f(x) = x^2$ wider than the graph of $f(x) = x^2$ reflected over the x-axis translated down 1 unit.