9-5 Study Guide and Intervention

Solving Quadratic Equations by Using the Quadratic Formula

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation

$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.

$x^2 + 2x - 3 = 0$ Simplify.

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{16}}{2}$$

$x = \frac{-2 + 4}{2}$ or $x = \frac{-2 - 4}{2}$

$= 1$ $\quad$ $= -3$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{6 + \sqrt{44}}{2}$$

$x = \frac{6 + \sqrt{44}}{2}$ or $x = \frac{6 - \sqrt{44}}{2}$

≈ 6.3 $\quad$ ≈ -0.3

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$ **1, 2**

2. $x^2 - 8x = -16$ **4**

3. $16x^2 - 8x = -1$ $\mathbf{\frac{1}{4}}$

4. $x^2 + 5x = 6$ **−6, 1**

5. $3x^2 + 2x = 8$ $\mathbf{-2, \frac{4}{3}}$

6. $8x^2 - 8x - 5 = 0$ **−0.4, 1.4**

7. $-4x^2 + 19x = 21$ $\mathbf{\frac{7}{4}, 3}$

8. $2x^2 + 6x = 5$ **−3.7, 0.7**

9. $48x^2 + 22x - 15 = 0$ $\mathbf{-\frac{5}{6}, \frac{3}{8}}$

10. $8x^2 - 4x = 24$ $\mathbf{-\frac{3}{2}, 2}$

11. $2x^2 + 5x = 8$ **−3.6, 1.1**

12. $8x^2 + 9x - 4 = 0$ **−1.5, 0.3**

13. $2x^2 + 9x + 4 = 0$ $\mathbf{-4, -\frac{1}{2}}$

14. $8x^2 + 17x + 2 = 0$ $\mathbf{-2, -\frac{1}{8}}$

9-5 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Using the Quadratic Formula

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real solutions for a quadratic equation.

Case 1: $b^2 - 4ac < 0$	no real solutions
Case 2: $b^2 - 4ac = 0$	one real solution
Case 3: $b^2 - 4ac > 0$	two real solutions

Example **State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.**

a. $\mathbf{12x^2 + 5x = 4}$

Write the equation in standard form.

$12x^2 + 5x = 4$ Original equation

$12x^2 + 5x - 4 = 4 - 4$ Subtract 4 from each side.

$12x^2 + 5x - 4 = 0$ Simplify.

Now find the discriminant.

$b^2 - 4ac = (5)^2 - 4(12)(-4)$

$= 217$

Since the discriminant is positive, the equation has two real solutions.

b. $\mathbf{2x^2 + 3x = -4}$

$2x^2 + 3x = -4$ Original equation

$2x^2 + 3x + 4 = -4 + 4$ Add 4 to each side.

$2x^2 + 3x + 4 = 0$ Simplify.

Find the discriminant.

$b^2 - 4ac = (3)^2 - 4(2)(4)$

$= -23$

Since the discriminant is negative, the equation has no real solutions.

Exercises

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

1. $3x^2 + 2x - 3 = 0$

40, 2 real solutions

2. $3x^2 - 7x - 8 = 0$

145, 2 real solutions

3. $2x^2 - 10x - 9 = 0$

172, 2 real solutions

4. $4x^2 = x + 4$

65, 2 real solutions

5. $3x^2 - 13x = 10$

289, 2 real solutions

6. $6x^2 - 10x + 10 = 0$

–140, no real solutions

7. $2x^2 - 20 = -x$

161, 2 real solutions

8. $6x^2 = -11x - 40$

–839, no real solutions

9. $9 - 18x + 9x^2 = 0$

0, 1 real solution

10. $12x^2 + 9 = -6x$

–396, no real solutions

11. $9x^2 = 81$

2916, 2 real solutions

12. $16x^2 + 16x + 4 = 0$

0, 1 real solution

13. $8x^2 + 9x = 2$

145, 2 real solutions

14. $4x^2 - 4x + 4 = 3$

0, 1 real solution

15. $3x^2 - 18x = -14$

156, 2 real solutions