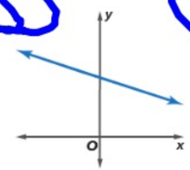
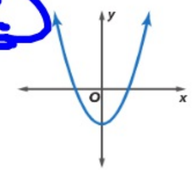
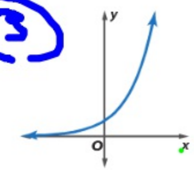
9-6 Analyzing Functions with Successive Differences

Concept Summary Linear and Nonlinear Functions

Linear Function	Quadratic Function	Exponential Function
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$, when $b > 0$
		

① Ex. $y = 3x + 4$

x	0	1	2	3
y	4	7	10	13

+3 +3 +3

adds the same amount

③ $y = 2^x$ multiplies by the same amount

x	0	1	2	3
y	1	2	4	8

×2 ×2 ×2

② $y = x^2$

x	0	1	2	3	4
y	0	1	4	9	16

+1 +3 +5 +7

+2 +2 +2

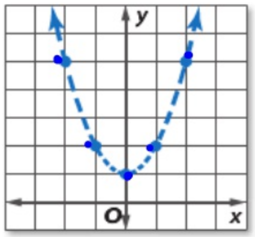
2nd diff. are equal

Example 1 Choose a Model Using Graphs

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

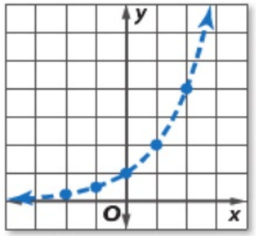
a. {(−2, 5), (−1, 2), (0, 1), (1, 2), (2, 5)}

The ordered pairs appear to represent a quadratic function.



b. $\left\{\left(-2, \frac{1}{4}\right), \left(-1, \frac{1}{2}\right), (0, 1), (1, 2), (2, 4)\right\}$

The ordered pairs appear to represent an exponential function.



Example 1 Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function. **1–4. See margin.**

1. (−2, 8), (−1, 5), (0, 2), (1, −1)

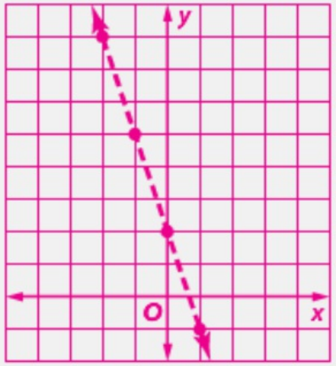
2. (−3, 7), (−2, 3), (−1, 1), (0, 1), (1, 3)

3. (−3, 8), (−2, 4), (−1, 2), (0, 1), (1, 0.5)

4. (0, 2), (1, 2.5), (2, 3), (3, 3.5)

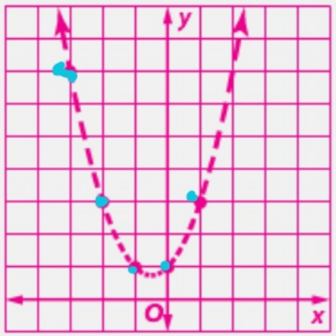
Additional Answers

1.

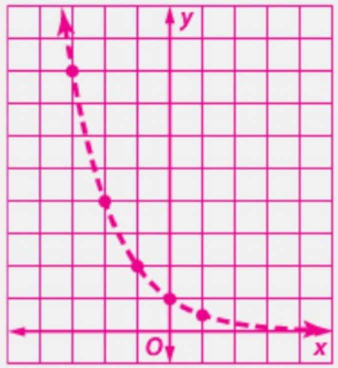


linear

2.

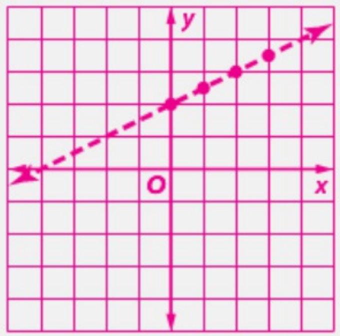


3.



exponential

4.



linear

Example 2 Choose a Model Using Differences or Ratios

Look for a pattern in each table of values to determine which kind of model best describes the data.

a.

x	−2	−1	0	1	2
y	−8	−3	2	7	12

First differences: $-8 \quad -3 \quad 2 \quad 7 \quad 12$

$5 \quad 5 \quad 5 \quad 5$

Since the first differences are all equal, the table of values represents a linear function.

b.

x	−1	0	1	2	3
y	8	4	2	1	0.5

First differences: $8 \quad 4 \quad 2 \quad 1 \quad 0.5$

$-4 \quad -2 \quad -1 \quad -0.5$

The first differences are not all equal. So, the table of values does not represent a linear function. Find the second differences and compare.

First differences: $-4 \quad -2 \quad -1 \quad -0.5$

Second differences: $2 \quad 1 \quad 0.5$

The second differences are not all equal. So, the table of values does not represent a quadratic function. Find the ratios of the *y*-values and compare.

$8 \quad 4 \quad 2 \quad 1 \quad 0.5$

Ratios: $\frac{4}{8} = \frac{1}{2} \quad \frac{2}{4} = \frac{1}{2} \quad \frac{1}{2} \quad \frac{0.5}{1} = \frac{1}{2}$

The ratios of successive *y*-values are equal. Therefore, the table of values can be modeled by an exponential function.

Example 2 Look for a pattern in each table of values to determine which kind of model best describes the data.

5. quadratic

x	0	1	2	3	4
y	5	8	17	32	53

+3 +9 +15 +21

6. linear

x	−3	−2	−1	0
y	−6.75	−7.5	−8.25	−9

7. exponential

x	−1	0	1	2	3
y	3	6	12	24	48

8. quadratic

x	3	4	5	6	7
y	−1.5	0	2.5	6	10.5

(5) +3 +9 +15 +21
+6 +6 +6
2nd diff.

(6) −.75 −.75 −.75...

(7) ×2 ×2 ×2 ×2

(8) +1.5 +2.5 +3.5 +4.5
+1 +1 +1
2nd diff.

Example 3 Write an Equation

Determine which kind of model best describes the data. Then write an equation for the function that models the data.

x	−4	−3	−2	−1	0
y	32	18	8	2	0

Step 1 Determine which model fits the data.

32 18 8 2 0

First differences: −14 −10 −6 −2

Second differences: 4 4 4

Since the second differences are equal, a quadratic function models the data.

Step 2 Write an equation for the function that models the data.

The equation has the form $y = ax^2$. Find the value of a by choosing one of the ordered pairs from the table of values. Let's use $(-1, 2)$.

$y = ax^2$ Equation for quadratic function

$2 = a(-1)^2$ $x = -1$ and $y = 2$

$2 = a$ An equation that models the data is $y = 2x^2$.

Guided Practice

3A. linear; $y = -4x + 3$

x	−2	−1	0	1	2
y	11	7	3	−1	−5

3B. exponential; $y = 3(2)^x$

x	−3	−2	−1	0	1
y	0.375	0.75	1.5	3	6

Example 3 Determine which kind of model best describes the data. Then write an equation for the function that models the data.

9.

x	−1	0	1	2	3
y	1	3	9	27	81

exponential; $y = 3 \cdot 3^x$

10.

x	−5	−4	−3	−2	−1
y	125	80	45	20	5

quadratic; $y = 5x^2$

11.

x	−3	−2	−1	0	1
y	1	1.5	2	2.5	3

linear; $y = \frac{1}{2}x + \frac{5}{2}$

12.

x	−1	0	1	2
y	−1.25	−1	−0.75	−0.5

linear; $y = \frac{1}{4}x - 1$

+25 +20 +15

+.5 +.5

+.25 +.25 +.25

(11)

$m = \frac{1}{2}$

$b = 2.5$

$y = \frac{1}{2}x + 2.5$

linear

$y = mx + b$

slope

y-int

$(x = 0)$

(10)

choose $(-1, 5)$ (x, y)

$y = ax^2$

$5 = a(-1)^2$

$5 = a$

$y = 5x^2$

quadratic

$y = ax^2$

Real-World Example 4 Write an Equation for a Real-World Situation

BOOK CLUB The table shows the number of book club members for four consecutive years. Determine which model best represents the data. Then write a function that models the data.

Understand We need to find a model for the data, and then write a function.

Time (years)	0	1	2	3	4
Members	5	10	20	40	80

Plan Find a pattern using successive differences or ratios. Then use the general form of the equation to write a function.

Solve The constant ratio is 2. This is the value of the base. An exponential function of the form $y = ab^x$ models the data.

$y = ab^x$ Equation for exponential function

$5 = a(2)^0$ $x = 0$, $y = 5$, and $b = 2$

$5 = a$ The equation that models the data is $y = 5 \cdot 2^x$.

Check You used (0, 5) to write the function. Verify that every other ordered pair satisfies the equation.

Example 4

13. PLANTS The table shows the height of a plant for four consecutive weeks. Determine which kind of function best models the height. Then write a function that models the data. **linear; $y = 0.5x + 3$**

Week	0	1	2	3	4
Height (in.)	3	3.5	4	4.5	5

Practice and Problem Solving

Extra Practice is on page R9.

Example 1

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

14–19. See Ch. 9 Answer Appendix.

14. (−1, 1), (0, −2), (1, −3), (2, −2), (3, 1)

15. (1, 2.75), (2, 2.5), (3, 2.25), (4, 2)

16. (−3, 0.25), (−2, 0.5), (−1, 1), (0, 2)

17. (−3, −11), (−2, −5), (−1, −3), (0, −5)

18. (−2, 6), (−1, 1), (0, −4), (1, −9)

19. (−1, 8), (0, 2), (1, 0.5), (2, 0.125)

Examples 2–3

Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

20.

x	−3	−2	−1	0
y	−8.8	−8.6	−8.4	−8.2

linear; $y = 0.2x - 8.2$

21.

x	−2	−1	0	1	2
y	10	2.5	0	2.5	10

quadratic; $y = 2.5x^2$

22.

x	−1	0	1	2	3
y	0.75	3	12	48	192

exponential; $y = 3 \cdot 4^x$

23.

x	−2	−1	0	1	2
y	0.008	0.04	0.2	1	5

exponential; $y = 0.2 \cdot 5^x$

24.

x	0	1	2	3	4
y	0	4.2	16.8	37.8	67.2

quadratic; $y = 4.2x^2$

25.

x	−3	−2	−1	0	1
y	14.75	9.75	4.75	−0.25	−5.25

linear; $y = -5x - 0.25$

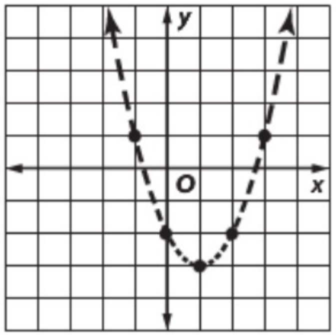
Example 4

26. WEB SITES A company tracked the number of visitors to its Web site over 4 days. Determine which kind of model best represents the number of visitors to the Web site with respect to time. Then write a function that models the data. quadratic; $y = 0.9x^2$

Day	0	1	2	3	4
Visitors (in thousands)	0	0.9	3.6	8.1	14.4

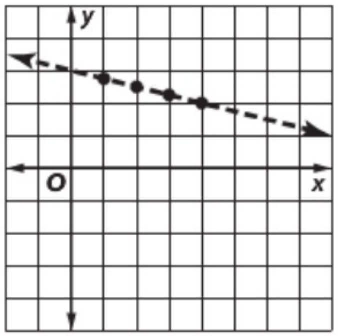
Lesson 9-6

14.



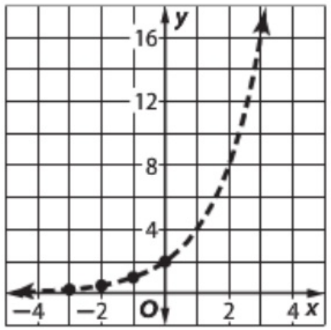
quadratic

15.



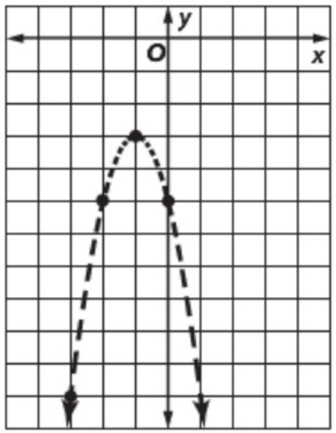
linear

16.



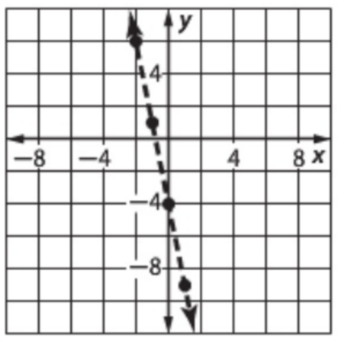
exponential

17.



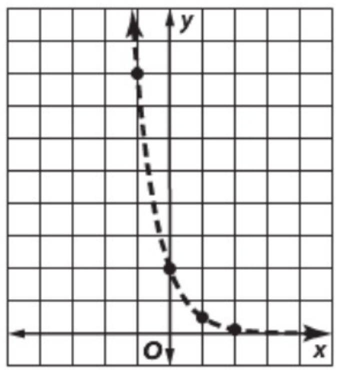
quadratic

18.



linear

19.



exponential