

## Calculus Practice Final 2017

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

**Express the limit as a definite integral.**

$$1) \lim_{n \rightarrow \infty} \sum_{k=1}^n (3c \frac{2}{k} - 6c_k + 16) \Delta x_k \quad [-9, 2] \quad 1) \underline{\hspace{2cm}}$$

**Use areas to evaluate the integral.**

$$2) \int_a^b 8x \, dx, \quad 0 < a < b \quad 2) \underline{\hspace{2cm}}$$

**Find the average value over the given interval.**

$$3) y = 6x + 1; [1, 8] \quad 3) \underline{\hspace{2cm}}$$

**Find  $dy/dx$ .**

$$4) \int_{\pi/4}^{\cot x} \csc^2 t \, dt \quad 4) \underline{\hspace{2cm}}$$

$$5) \int_0^x \sqrt{4t+7} \, dt \quad 5) \underline{\hspace{2cm}}$$

**Solve the initial value problem explicitly.**

$$6) \frac{du}{dx} = 10x^9 - 4x^3 + 5 \text{ and } u = 2 \text{ when } x = 1 \quad 6) \underline{\hspace{2cm}}$$

**Solve the initial value problem using the Fundamental Theorem. Your answer will contain a definite integral.**

$$7) \frac{dy}{dx} = \cos(x^2) \text{ and } y = 8 \text{ when } x = 3 \quad 7) \underline{\hspace{2cm}}$$

**Solve the initial value problem.**

$$8) \frac{dy}{dx} = x \sin 3x \text{ and } y = 6 \text{ when } x = 0 \quad 8) \underline{\hspace{2cm}}$$

**Solve the problem.**

$$9) \text{ The decay equation for a radioactive substance is known to be } y = y_0 e^{-0.031t}, \text{ with } t \text{ in days. About how long will it take for the amount of substance to decay to 54\% of its original value?} \quad 9) \underline{\hspace{2cm}}$$

**Evaluate the integral.**

$$10) \int \frac{2x+23}{x^2+11x+28} \, dx \quad 10) \underline{\hspace{2cm}}$$

11)  $\int \frac{x^2 - 9}{x^2 - 36} dx$

11) \_\_\_\_\_

12)  $\int \frac{x^4}{x^2 - 9} dx$

12) \_\_\_\_\_

**Solve the problem.**

13) A car moving with an initial velocity of 3 mph accelerates at the rate of  $a(t) = 2.1t$  mph per second for 9 seconds. How fast is the car going when the 9 seconds are up?

13) \_\_\_\_\_

14) A car moving with an initial velocity of 9 mph accelerates at the rate of  $a(t) = 2.6t$  mph per second for 8 seconds. How far did the car travel during those 8 seconds?

14) \_\_\_\_\_

**Find the area enclosed by the given curves.**

15) Find the area of the region in the first quadrant bounded on the left by the y-axis, below by the line  $y = \frac{1}{3}x$ , above left by  $y = x + 4$ , and above right by  $y = -x^2 + 10$ .

15) \_\_\_\_\_

**Find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.**

16)  $y = \sqrt{2x}, y = 2, x = 0$

16) \_\_\_\_\_

**Find the volume of the solid generated by revolving the region about the given line.**

17) The region bounded above by the line  $y = 4$ , below by the curve  $y = 4 - x^2$ , and on the right by the line  $x = 2$ , about the line  $y = 4$

17) \_\_\_\_\_

**Find the volume of the solid generated by revolving the region about the y-axis.**

18) The region enclosed by  $x = \frac{y^2}{5}, x = 0, y = -5, y = 5$

18) \_\_\_\_\_

**Find the exact length of the curve analytically by antidifferentiation.**

19)  $x = \frac{2}{3}(y - 1)^{3/2}$  from  $y = 1$  to  $y = 4$

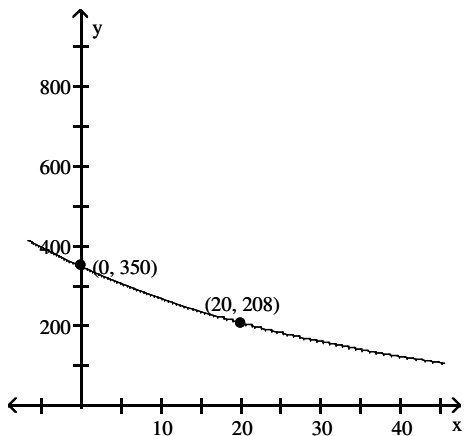
19) \_\_\_\_\_

20)  $y = \int_0^x \sqrt{4 \sin^2 t - 1} dt, 0 \leq x \leq \frac{\pi}{2}$

20) \_\_\_\_\_

Find the exponential function  $y = y_0 e^{kt}$  whose graph passes through the two given points.

21)



21) \_\_\_\_\_

**Solve the problem.**

22) Suppose that the accompanying table shows the velocity of a car every second for 8 seconds. Use the Trapezoidal Rule to approximate the distance traveled by the car in the 8 seconds.

22) \_\_\_\_\_

Time (sec)	Velocity (ft/sec)
0	19
1	20
2	21
3	23
4	22
5	24
6	21
7	19
8	20

23) A particle moves with velocity  $v(t) = 2t + 3$  find the distance traveled between  $t = 0$  and  $t = 2$ .

23) \_\_\_\_\_

**Evaluate the integral.**

24)  $\int (\sqrt{t} - \sqrt[6]{t}) dt$

24) \_\_\_\_\_

**Verify that  $\int f(u) du \neq \int f(u) dx$ .**

25)  $f(u) = \sqrt{u}$  and  $u = x^4$  ( $x > 0$ ).

25) \_\_\_\_\_

## Answer Key

### Testname: CALCULUS PRACTICE FINAL

$$1) \int_{-9}^2 (3x^2 - 6x + 16) dx$$

$$2) 4(b^2 - a^2)$$

$$3) 28$$

$$4) -\csc^2 x \csc^2 (\cot x)$$

$$5) \sqrt{4x+7}$$

$$6) u = x^{10} - x^4 + 5x - 3$$

$$7) y = \int_3^x \cos(t^2) dt + 8$$

$$8) y = -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} + 6$$

$$9) 19.9 \text{ days}$$

$$10) \ln \left| \frac{(x+4)^5}{(x+7)^3} \right| + C$$

$$11) x + \frac{9}{4} \ln \left| \frac{x-6}{x+6} \right| + C$$

$$12) \frac{x^3}{3} + 9x + \frac{27}{2} \ln \left| \frac{x-3}{x+3} \right| + C$$

$$13) 88.05 \text{ mph}$$

$$14) 0.082 \text{ mi}$$

$$15) \frac{73}{6}$$

$$16) 4\pi$$

$$17) \frac{32}{5}\pi$$

$$18) 50\pi$$

$$19) \frac{14}{3}$$

$$20) 2$$

$$21) y = 350e^{-0.026t}$$

$$22) 169.5 \text{ feet}$$

$$23) 10$$

$$24) \frac{2}{3}t^{3/2} - \frac{6}{7}t^{7/6} + C$$

$$25) \int f(u) du = \int \sqrt{u} du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(x^4)^{3/2} + C = \frac{2}{3}x^6 + C$$

$$f(u) = \sqrt{u} = \sqrt{x^4} = x^2.$$

$$\int f(u) dx = \int x^2 dx = \frac{1}{3}x^3 + C$$

$$\text{So } \int f(u) du \neq \int f(u) dx$$