

Calculus Practice Midterm

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Write an equation for the line described.

1) Passes through (5, 5) with slope $-\frac{2}{5}$ 1) _____

2) Passes through (-5, -3) and is parallel to the line $8x + 7y = -19$ 2) _____

Solve the problem.

3) Assume that a watermelon dropped from a tall building falls $y = 16t^2$ ft in t sec. Find the watermelon's average speed during the first 4 sec of fall. 3) _____

4) Assume that a watermelon dropped from a tall building falls $y = 16t^2$ ft in t sec. Find the watermelon's speed at the instant $t = 5$ sec. 4) _____

Determine the limit algebraically, if it exists.

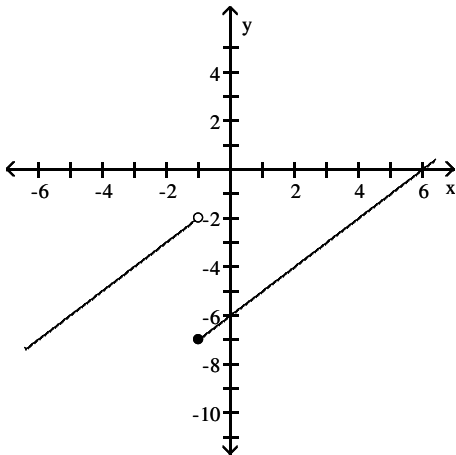
5) $\lim_{x \rightarrow 6} \frac{x + 6}{(x - 6)^2}$ 5) _____

6) $\lim_{x \rightarrow -6} \frac{x^2 - 36}{x + 6}$ 6) _____

7) $\lim_{x \rightarrow 0} \frac{7 \sin x}{4x}$ 7) _____

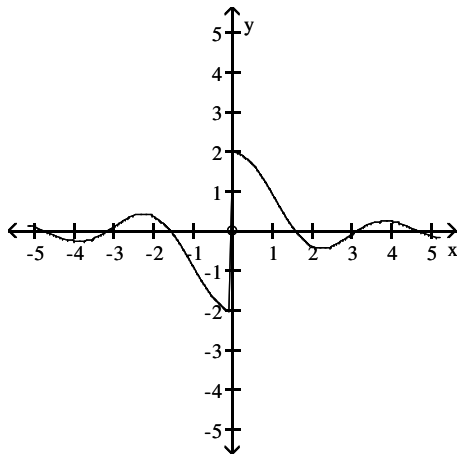
Determine the limit graphically, if it exists.

8) Find $\lim_{x \rightarrow -1^-} f(x)$ and $\lim_{x \rightarrow -1^+} f(x)$. 8) _____



9) $\lim_{x \rightarrow 0} f(x)$

9) _____



Find the indicated limit.

10) $\lim_{x \rightarrow 0^-} \frac{7x}{|x|}$

10) _____

Evaluate or determine that the limit does not exist for each of the limits (a) $\lim_{x \rightarrow d^-} f(x)$, (b) $\lim_{x \rightarrow d^+} f(x)$, and (c) $\lim_{x \rightarrow d} f(x)$ for the given function f and number d .

11)

$$f(x) = \begin{cases} \frac{1}{x+4}, & \text{for } x > -4, \\ x^2 - 4x, & \text{for } x \leq -4 \end{cases}$$

$d = -4$

11) _____

Find the limit, if it exists.

12) $\lim_{x \rightarrow \infty} \frac{x^2 + 3x + 19}{x^3 + 2x^2 + 10}$

12) _____

13) $\lim_{x \rightarrow \infty} \frac{4x + 1}{13x - 7}$

13) _____

14) $\lim_{x \rightarrow -\infty} \frac{3x^3 + 3x^2}{x - 6x^2}$

14) _____

Find the vertical asymptotes of the graph of $f(x)$.

15) $f(x) = \frac{1}{x^2 - 81}$

15) _____

16) $f(x) = \csc x$ (fair warning- There are three other trig functions that has asymptotes)

16) _____

Find the limit of $f(x)$ as (a) $x \rightarrow -\infty$, (b) $x \rightarrow \infty$, (c) $x \rightarrow 0^-$, and (d) $x \rightarrow 0^+$.

$$17) f(x) = \begin{cases} \frac{x-6}{x-3}, & x \leq 0 \\ \frac{1}{x^2}, & x > 0 \end{cases} \quad 17) \underline{\hspace{2cm}}$$

Find a value for a so that the function $f(x)$ is continuous.

$$18) f(x) = \begin{cases} x^2 - 2, & x < 3 \\ 5ax, & x \geq 3 \end{cases} \quad 18) \underline{\hspace{2cm}}$$

Solve the problem.

$$19) \text{ If } y = x^3 - 9x - 3, \text{ find an equation of the tangent line to the graph of } y \text{ at } x = 3. \quad 19) \underline{\hspace{2cm}}$$

Find dy/dx .

$$20) y = (x^2 - 5x + 2)(4x^3 - x^2 + 5) \quad 20) \underline{\hspace{2cm}}$$

$$21) y = \frac{8x^2 + x - 1}{x^3 - 9x^2} \quad 21) \underline{\hspace{2cm}}$$

$$22) y = 3x^2 + 8x + 3x^{-3} \quad 22) \underline{\hspace{2cm}}$$

$$23) y = 3 \sec^2 x \quad 23) \underline{\hspace{2cm}}$$

$$24) y = \frac{1}{\sqrt{5-4x}} \quad 24) \underline{\hspace{2cm}}$$

$$25) y = \sqrt{6 + \sin 2x} \quad 25) \underline{\hspace{2cm}}$$

$$26) f(x) = 9e^{-8x} \quad 26) \underline{\hspace{2cm}}$$

$$27) y = 11^{-x} \quad 27) \underline{\hspace{2cm}}$$

$$28) y = \ln 7x^2 \quad 28) \underline{\hspace{2cm}}$$

$$29) y = \log(5x - 8) \quad 29) \underline{\hspace{2cm}}$$

Find dy/dx by implicit differentiation. If applicable, express the result in terms of x and y .

$$30) \cos xy + x^3 = y^3 \quad 30) \underline{\hspace{2cm}}$$

Find the derivative of the given function.

$$31) y = \tan^{-1} \sqrt{3x} \quad 31) \underline{\hspace{2cm}}$$

Find the extreme values of the function on the interval and where they occur.

32) $g(x) = -x^2 + 10x - 21$ on $3 \leq x \leq 7$ 32) _____

Find the extreme values of the function and where they occur.

33) $y = \frac{4x}{x^2 + 1}$ 33) _____

Give an appropriate answer.

34) Find the value or values of c that satisfy $\frac{f(b) - f(a)}{b - a} = f'(c)$ for the function $f(x) = x + \frac{27}{x}$ on the interval $[3, 9]$. 34) _____

Find the derivative at each critical point and determine the local extreme values.

35) $y = x^{2/3}(x^2 - 16)$; $x \geq 0$ 35) _____

Use analytic methods to find the local extrema.

36) $h(x) = \frac{x - 1}{x^2 + 3x + 5}$ 36) _____

Use analytic methods to find those values of x for which the given function is increasing and those values of x for which it is decreasing.

37) $f(x) = x^4 - 2$ 37) _____

Find all possible functions with the given derivative.

38) $f'(x) = 4 \cos 4x$ 38) _____

Find the function with the given derivative whose graph passes through the point P .

39) $f'(x) = x^2 + 9$, $P(3, 55)$ 39) _____

Find the points of inflection.

40) $y = \frac{4}{3}x^3 - 12x^2 + 10x + 50$ 40) _____

Solve the problem.

41) Suppose $c(x) = x^3 - 18x^2 + 10,000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items. 41) _____

42) A piece of land is shaped like a right triangle. Two people start at the right angle at the same time, and walk at the same speed along different legs of the triangle while spraying the land. If the area covered is changing at $2 \text{ m}^2/\text{s}$, how fast are the people moving when they are 4 m from the right angle? (Round approximations to two decimal places.) 42) _____

Answer Key

Testname: CALCULUS PRACTICE MIDTERM

1) $y = -\frac{2}{5}x + 7$

2) $y = -\frac{8}{7}x - \frac{61}{7}$

3) 64 ft/sec

4) 160 ft/sec

5) Does not exist

6) -12

7) $\frac{7}{4}$

8) -2; -7

9) Does not exist

10) -7

11) (a) 32

(b) Does not exist

(c) Does not exist

12) 0

13) $\frac{4}{13}$

14) ∞

15) $x = -9$ and $x = 9$

16) $x = n\pi$, n is any integer

17) (a) 1

(b) 0

(c) 2

(d) ∞

18) $a = \frac{7}{15}$

19) $y = 18x - 57$

20) $20x^4 - 84x^3 + 39x^2 + 6x - 25$

21) $\frac{-8x^4 - 2x^3 + 12x^2 - 18x}{(x^3 - 9x^2)^2}$

22) $6x + 8 - 9x^{-4}$

23) $6 \tan x \sec^2 x$

24) $\frac{2}{(5 - 4x)^{3/2}}$

25) $\frac{1 \cos 2x}{\sqrt{6 + \sin 2x}}$

26) $-72e^{-8x}$

27) $-\ln 11 (11^{-x})$

28) $\frac{2}{x}$

29) $\frac{5}{(5x - 8) \ln 10}$

Answer Key

Testname: CALCULUS PRACTICE MIDTERM

30) $\frac{3x^2 - y \sin xy}{3y^2 + x \sin xy}$

31) $\frac{3}{2(1 + 3x)\sqrt{3x}}$

32) Maximum value is 4 at $x = 5$; minimum value is 0 at $x = 7$ and 0 at $x = 3$

33) The minimum is -2 at $x = -1$. The maximum is 2 at $x = 1$.

34) $3\sqrt{3}$

35)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	Undefined	local max	0
$x = 2$	0	minimum	-19.048813

36) Local minimum at $x = -2$; local maximum at $x = 4$

37) Increasing on $(-1, 0)$ and $(1, \infty)$, decreasing on $(-\infty, -1)$ and $(0, 1)$

38) $\sin 4x + C$

39) $f(x) = \frac{x^3}{3} + 9x + 19$

40) $(3, 8)$

41) 9 items

42) 0.50 m/s