7-1 Graphing Exponential Functions

Graph each function. State the domain and range. **11–16. See margin.**

11. $f(x) = 3^x$

12. $f(x) = -5(2)^x$

13. $f(x) = 3(4)^x - 6$

14. $f(x) = 3^{2x} + 5$

15. $f(x) = 3\left(\frac{1}{4}\right)^{x+3} - 1$

16. $f(x) = \frac{3}{5}\left(\frac{2}{3}\right)^{x-2} + 3$

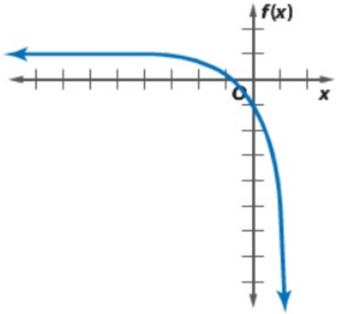
17. POPULATION A city with a population of 120,000 decreases at a rate of 3% annually.

a. Write the function that represents this situation. $f(x) = 120{,}000(0.97)^x$

b. What will the population be in 10 years? **about 88,491**

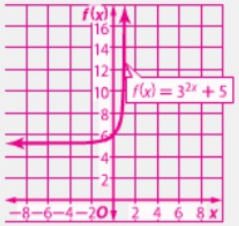
Example 1

Graph $f(x) = -2(3)^x + 1$. State the domain and range.



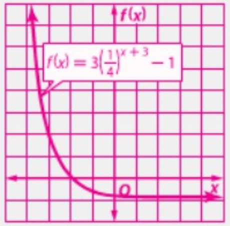
The domain is all real numbers, and the range is all real numbers less than 1.

14.



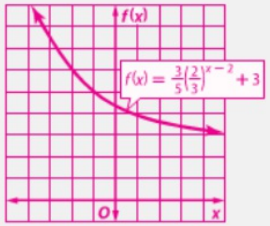
D = {all real numbers}
R = {f(x) | f(x) > 5}

15.



D = {all real numbers}
R = {f(x) | f(x) > −1}

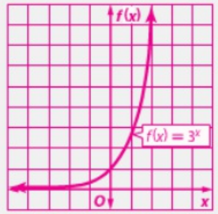
16.



D = {all real numbers}
R = {f(x) | f(x) > 3}

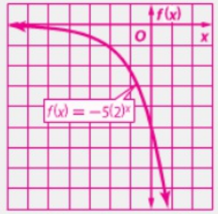
Additional Answers

11.



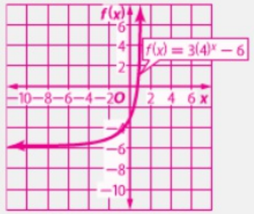
D = {all real numbers}
R = {f(x) | f(x) > 0}

12.



D = {all real numbers}
R = {f(x) | f(x) < 0}

13.



D = {all real numbers}
R = {f(x) | f(x) > −6}

7-2 Solving Exponential Equations and Inequalities

Solve each equation or inequality. **22.** $x > -\frac{2}{3}$

18. $16^x = \frac{1}{64}$ $-\frac{3}{2}$

19. $3^{4x} = 9^{3x+7}$ -7

20. $64^{3n} = 8^{2n-3}$ $-\frac{3}{4}$

21. $8^{3-3y} = 256^{4y}$ $\frac{9}{41}$

22. $9^{x-2} > \left(\frac{1}{81}\right)^{x+2}$

23. $27^{3x} \leq 9^{2x-1}$ $x \leq -\frac{2}{5}$

24. **BACTERIA** A bacteria population started with 5000 bacteria. After 8 hours there were 28,000 in the sample.

a. Write an exponential function that could be used to model the number of bacteria after x hours if the number of bacteria changes at the same rate. $y = 5000(1.240)^x$

b. How many bacteria can be expected in the sample after 32 hours? **about 4,880,496**

Example 2

Solve $4^{3x} = 32^{x-1}$ for x.

$4^{3x} = 32^{x-1}$	Original equation
$(2^2)^{3x} = (2^5)^{x-1}$	Rewrite so each side has the same base.
$2^{6x} = 2^{5x-5}$	Power of a Power
$6x = 5x - 5$	Property of Equality for Exponential Functions
$x = -5$	Subtract $5x$ from each side.

The solution is -5.

7-3 Logarithms and Logarithmic Functions

25. Write $\log_2 \frac{1}{16} = -4$ in exponential form. $2^{-4} = \frac{1}{16}$

26. Write $10^2 = 100$ in logarithmic form. $\log_{10} 100 = 2$

Evaluate each expression.

27. $\log_4 256$ **4**

28. $\log_2 \frac{1}{8}$ **−3**

Graph each function. **29, 30. See margin.**

29. $f(x) = 2 \log_{10} x + 4$

30. $f(x) = \frac{1}{6} \log_{\frac{1}{3}} (x - 2)$

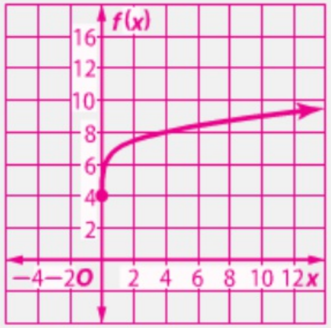
Example 3

Evaluate $\log_2 64$.

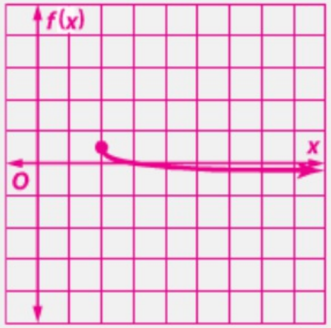
$\log_2 64 = y$	Let the logarithm equal y.
$64 = 2^y$	Definition of logarithm
$2^6 = 2^y$	$64 = 2^6$
$6 = y$	Property of Equality for Exponential Functions

Additional Answers

29.



30.



7-4 Solving Logarithmic Equations and Inequalities

Solve each equation or inequality.

31. $\log_4 x = \frac{3}{2}$ **64**

32. $\log_2 \frac{1}{64} = x$ **−6**

33. $\log_4 x < 3$ $\{x \mid 0 < x < 64\}$

34. $\log_5 x < -3$ $\left\{x \mid 0 < x < \frac{1}{125}\right\}$

35. $\log_9 (3x - 1) = \log_9 (4x)$ **no solution**

36. $\log_2 (x^2 - 18) = \log_2 (-3x)$ **−6**

37. $\log_3 (3x + 4) \leq \log_3 (x - 2)$ **no solution**

38. **EARTHQUAKE** The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude M is given by $M = \log_{10} x$, where x represents the amplitude of the seismic wave causing ground motion. How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 10 as an aftershock with a Richter scale rating of 7? **1000**

Example 4

Solve $\log_{27} x < \frac{2}{3}$.

$\log_{27} x < \frac{2}{3}$	Original inequality
$x < 27^{\frac{2}{3}}$	Logarithmic to Exponential Inequality
$x < 9$	Simplify.

Example 5

Solve $\log_5 (p^2 - 2) = \log_5 p$.

$\log_5 (p^2 - 2) = \log_5 p$	Original equation
$p^2 - 2 = p$	Property of Equality
$p^2 - p - 2 = 0$	Subtract p from each side.
$(p - 2)(p + 1) = 0$	Factor.
$p - 2 = 0$ or $p + 1 = 0$	Zero Product Property
$p = 2$ $\quad$ $p = -1$	Solve each equation.

The solution is $p = 2$, since $\log_5 p$ is undefined for $p = -1$.

7-5 Properties of Logarithms

Use $\log_5 16 \approx 1.7227$ and $\log_5 2 \approx 0.4307$ to approximate the value of each expression.

39. $\log_5 8$ **1.2920**

40. $\log_5 64$ **2.5841**

41. $\log_5 4$ **0.8614**

42. $\log_5 \frac{1}{8}$ **−1.2921**

43. $\log_5 \frac{1}{2}$ **−0.4307**

Solve each equation. Check your solution.

44. $\log_5 x - \log_5 2 = \log_5 15$ **30**

45. $3 \log_4 a = \log_4 27$ **3**

46. $2 \log_3 x + \log_3 3 = \log_3 36$ $\mathbf{2\sqrt{3}}$

47. $\log_4 n + \log_4 (n - 4) = \log_4 5$ **5**

48. **SOUND** Use the formula $L = 10 \log_{10} R$, where L is the loudness of a sound and R is the sound's relative intensity, to find out how much louder 20 people talking would be than one person talking. Suppose the sound of one person talking has a relative intensity of 80 decibels. **361.6 times**

Example 6

Use $\log_5 16 \approx 1.7227$ and $\log_5 2 \approx 0.4307$ to approximate $\log_5 32$.

$\log_5 32 = \log_5 (16 \cdot 2)$	Replace 32 with 16.
$= \log_5 16 + \log_5 2$	Product Property
$\approx 1.7227 + 0.4307$	Use a calculator.
≈ 2.1534	

Example 7

Solve $\log_3 3x + \log_3 4 = \log_3 36$.

$\log_3 3x + \log_3 4 = \log_3 36$	Original equation
$\log_3 3x(4) = \log_3 36$	Product Property
$3x(4) = 36$	Definition of logarithm
$12x = 36$	Multiply.
$x = 3$	Divide each side by 12.

7-6 Common Logarithms

Solve each equation or inequality. Round to the nearest ten-thousandth.

49. $3^x = 15$ **$x \approx 2.4650$**

50. $6^{x^2} = 28$ **$x \approx \pm 1.3637$**

51. $8^{m+1} = 30$ **$m \approx 0.6356$**

52. $12^{r-1} = 7r$ **$r \approx 4.6102$**

53. $3^{5n} > 24$ **$\{n \mid n > 0.5786\}$**

54. $5^{x+2} \leq 3^x$ **$\{x \mid x \leq -6.3013\}$**

55. **SAVINGS** You deposited $1000 into an account that pays an annual interest rate r of 5% compounded quarterly. Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

a. How long will it take until you have $1500 in your account? **about 8.2 years**

b. How long it will take for your money to double? **about 13.9 years**

Example 8

Solve $5^{3x} > 7^{x+1}$.

$5^{3x} > 7^{x+1}$	Original inequality
$\log 5^{3x} > \log 7^{x+1}$	Property of Inequality
$3x \log 5 > (x + 1) \log 7$	Power Property
$3x \log 5 > x \log 7 + \log 7$	Distributive Property
$3x \log 5 - x \log 7 > \log 7$	Subtract $x \log 7$.
$x(3 \log 5 - \log 7) > \log 7$	Distributive Property
$x > \frac{\log 7}{3 \log 5 - \log 7}$	Divide by $3 \log 5 - \log 7$.
$x > 0.6751$	Use a calculator.

The solution set is $\{x \mid x > 0.6751\}$.

7-7 Base e and Natural Logarithms

Solve each equation or inequality. Round to the nearest ten-thousandth. **56–61. See margin.**

56. $4e^x - 11 = 17$

57. $2e^{-x} + 1 = 15$

58. $\ln 2x = 6$

59. $2 + e^x > 9$

60. $\ln (x + 3)^5 < 5$

61. $e^{-x} > 18$

62. **SAVINGS** If you deposit $2000 in an account paying 6.4% interest compounded continuously, how long will it take for your money to triple? Use $A = Pe^{rt}$. **about 17.2 years**

Example 9

Solve $3e^{5x} + 1 = 10$. Round to the nearest ten-thousandth.

$3e^{5x} + 1 = 10$	Original equation
$3e^{5x} = 9$	Subtract 1 from each side.
$e^{5x} = 3$	Divide each side by 3.
$\ln e^{5x} = \ln 3$	Property of Equality
$5x = \ln 3$	$\ln e^x = x$
$x = \frac{\ln 3}{5}$	Divide each side by 5.
$x \approx 0.2197$	Use a calculator.

Additional Answers

56. 1.9459

57. −1.9459

58. 201.7144

59. $\{x \mid x > 1.9459\}$

60. $\{x \mid -3 < x < -0.2817\}$

61. $\{x \mid x < -2.8904\}$

7-8 Using Exponential and Logarithmic Functions

63. **CARS** Abe bought a used car for $2500. It is expected to depreciate at a rate of 25% per year. What will be the value of the car in 3 years? **$1054.69**

64. **BIOLOGY** For a certain strain of bacteria, k is 0.728 when t is measured in days. Using the formula $y = ae^{kt}$, how long will it take 10 bacteria to increase to 675 bacteria? **≈5.8 days**

65. **POPULATION** The population of a city 20 years ago was 24,330. Since then, the population has increased at a steady rate each year. If the population is currently 55,250, find the annual rate of growth for this city. **about 4.1%**

Example 10

A certain culture of bacteria will grow from 250 to 2000 bacteria in 1.5 hours. Find the constant k for the growth formula. Use $y = ae^{kt}$.

$y = ae^{kt}$	Exponential Growth Formula
$2000 = 250e^{k(1.5)}$	Replace y with 2000, a with 250, and t with 1.5.
$8 = e^{1.5k}$	Divide each side by 250.
$\ln 8 = \ln e^{1.5k}$	Property of Equality
$\ln 8 = 1.5k$	Inverse Property
$\frac{\ln 8}{1.5} = k$	Divide each side by 1.5.
$1.3863 \approx k$	Use a calculator.