# Lesson-by-Lesson Review

## 4-1 Graphing Quadratic Functions

Complete parts **a**–**c** for each quadratic function.

**a.** Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

**b.** Make a table of values that includes the vertex.

**c.** Use this information to graph the function.

**9.** $f(x) = x^2 + 5x + 12$ **10.** $f(x) = x^2 - 7x + 15$

**11.** $f(x) = -2x^2 + 9x - 5$ **12.** $f(x) = -3x^2 + 12x - 1$

Determine whether each function has a maximum or minimum value and find the maximum or minimum value. Then state the domain and range of the function.

**13.** $f(x) = -x^2 + 3x - 1$ **14.** $f(x) = -3x^2 - 4x + 5$

**15.** **BUSINESS** Sal's Shirt Store sells 100 T-shirts per week at a rate of $10 per shirt. Sal estimates that he will sell 5 fewer shirts for each $1 increase in price. What price will maximize Sal's T-shirt income? **75 T-shirts at $15 each**

### Example 1

Consider the quadratic function $f(x) = x^2 - 4x + 11$. Find the *y*-intercept, the equation for the axis of symmetry, and the *x*-coordinate of the vertex.

In the function, $a = 1$, $b = -4$, and $c = 11$. The *y*-intercept is $c = 11$.

Use *a* and *b* to find the equation of the axis of symmetry.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

$= -\frac{-4}{2(1)}$ $a = 1$ and $b = -4$

$= 2$ Simplify.

The equation of the axis of symmetry is $x = 2$. Therefore, the *x*-coordinate of the vertex is 2.

**13. max; 1.25; D = all real numbers; $R = \{f(x) \mid f(x) \leq 1.25\}$**

**9–12. See margin.**

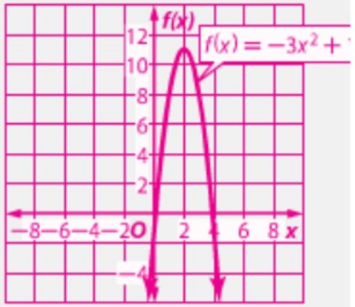
**14. max; $\frac{19}{3}$; D = all real numbers; $R = \left\{f(x) \middle| f(x) \leq \frac{19}{3}\right\}$**

**12a.** *y*-int: −1; $x = 2$; 2

**12b.**

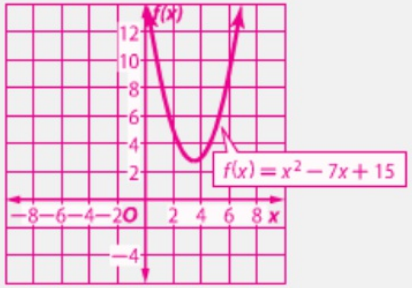
| x | f(x) |
|---|---|
| 0 | −1 |
| 1 | 8 |
| 2 | 11 |
| 3 | 8 |
| 4 | −1 |

**12c.**




## Additional Answers

**10c.**

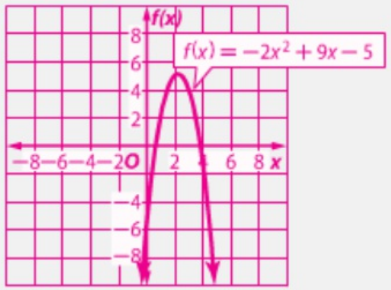



**11a.** *y*-int: −5; $x = \frac{9}{4}$; $\frac{9}{4}$

**11b.**

| x | f(x) |
|---|---|
| 1 | 2 |
| 2 | 5 |
| $\frac{9}{4}$ | $\frac{41}{8}$ |
| 3 | 4 |
| 4 | −1 |

**11c.**



## 4-2 Solving Quadratic Equations by Graphing

**Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.**

**16.** $x^2 - x - 20 = 0$ **{−4, 5}**

**17.** $2x^2 - x - 3 = 0$ $\left\{-1, \frac{3}{2}\right\}$

**18.** $4x^2 - 6x - 15 = 0$ **18. between −1 and −2; between 2 and 3**

**19.** **BASEBALL** A baseball is hit upward at 120 feet per second. Use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, $v_0$ is the object's initial velocity in feet per second, and $t$ is the time in seconds. Ignoring the height of the ball when it was hit, how long does it take for the ball to hit the ground? **7.5 seconds**

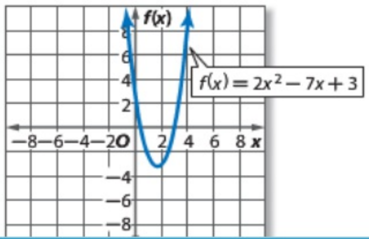
### Example 2

**Solve $2x^2 - 7x + 3 = 0$ by graphing.**

The equation of the axis of symmetry is $-\frac{-7}{2(2)}$ or $x = \frac{7}{4}$.

| $x$ | 0 | 1 | $\frac{7}{4}$ | 2 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | −2 | $-2\frac{5}{8}$ | −3 | 0 |

The zeros of the related function are $\frac{1}{2}$ and 3. Therefore, the solutions of the equation are $\frac{1}{2}$ and 3.




## 4-3 Solving Quadratic Equations by Factoring

**Write a quadratic equation in standard form with the given roots.** **20–25. See margin.**

**20.** 5, 6

**21.** −3, −7

**22.** −4, 2

**23.** $-\frac{2}{3}, 1$

**24.** $\frac{1}{6}, 5$

**25.** $-\frac{1}{4}, -1$

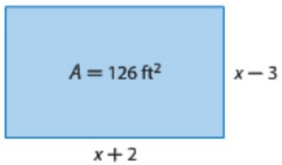
**Solve each equation by factoring.**

**26.** $2x^2 - 2x - 24 = 0$ **{−3, 4}**

**27.** $2x^2 - 5x - 3 = 0$ $\left\{-\frac{1}{2}, 3\right\}$

**28.** $3x^2 - 16x + 5 = 0$ $\left\{\frac{1}{3}, 5\right\}$

**29.** Find $x$ and the dimensions of the rectangle below.




**$x = 12$; 9 feet by 14 feet**

### Example 3

**Write a quadratic equation in standard form with $-\frac{1}{2}$ and 4 as its roots.**

$(x - p)(x - q) = 0$ Write the pattern.

$\left[x - \left(-\frac{1}{2}\right)\right](x - 4) = 0$ Replace $p$ with $-\frac{1}{2}$ and $q$ with 4.

$\left(x + \frac{1}{2}\right)(x - 4) = 0$ Simplify.

$x^2 - \frac{7}{2}x - 2 = 0$ Multiply.

$2x^2 - 7x - 4 = 0$ Multiply each side by 2 so that $b$ and $c$ are integers.

### Example 4

**Solve $2x^2 - 3x - 5 = 0$ by factoring.**

$2x^2 - 3x - 5 = 0$ Original equation

$(2x - 5)(x + 1) = 0$ Factor the trinomial.

$2x - 5 = 0$ or $x + 1 = 0$ Zero Product Property

$x = \frac{5}{2}$ $\quad x = -1$

The solution set is $\left\{-1, \frac{5}{2}\right\}$ or $\left\{x \mid x = -1, \frac{5}{2}\right\}$.

### Additional Answers

**20.** $x^2 - 11x + 30 = 0$

**21.** $x^2 + 10x + 21 = 0$

**22.** $x^2 + 2x - 8 = 0$

**23.** $3x^2 - x - 2 = 0$

**24.** $6x^2 - 31x + 5 = 0$

**25.** $4x^2 + 5x + 1 = 0$

## 4-4 Complex Numbers

Simplify. **32. $2 + 5i$**

30. $\sqrt{-8}$ $2i\sqrt{2}$

31. $(2 - i) + (13 + 4i)$ $15 + 3i$

32. $(6 + 2i) - (4 - 3i)$

33. $(6 + 5i)(3 - 2i)$ $28 + 3i$

34. **ELECTRICITY** The impedance in one part of a series circuit is $3 + 2j$ ohms, and the impedance in the other part of the circuit is $4 - 3j$ ohms. Add these complex numbers to find the total impedance in the circuit. **$7 - j$ ohms**

Solve each equation.

35. $2x^2 + 50 = 0$ $\pm 5i$

36. $4x^2 + 16 = 0$ $\pm 2i$

37. $3x^2 + 15 = 0$ $\pm i\sqrt{5}$

38. $8x^2 + 16 = 0$ $\pm i\sqrt{2}$

39. $4x^2 + 1 = 0$ $\pm \frac{1}{2}i$

**Example 5**

Simplify $(12 + 3i) - (-5 + 2i)$.

$(12 + 3i) - (-5 + 2i)$

$= [12 - (-5)] + (3 - 2)i$ — Group the real and imaginary parts.

$= 17 + i$ — Simplify.

**Example 6**

Solve $3x^2 + 12 = 0$.

$3x^2 + 12 = 0$ — Original equation

$3x^2 = -12$ — Subtract 12 from each side.

$x^2 = -4$ — Divide each side by 3.

$x = \pm\sqrt{-4}$ — Square Root Property

$x = \pm 2i$ — $\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1}$

## 4-5 Completing the Square

Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

**40–45. See margin.**

40. $x^2 + 18x + c$

41. $x^2 - 4x + c$

42. $x^2 - 7x + c$

43. $x^2 + 2.4x + c$

44. $x^2 - \frac{1}{2}x + c$

45. $x^2 + \frac{6}{5}x + c$

Solve each equation by completing the square.

46. $x^2 - 6x - 7 = 0$ $\{-1, 7\}$

47. $x^2 - 2x + 8 = 0$ $\{1 \pm i\sqrt{7}\}$

48. $2x^2 + 4x - 3 = 0$ $\left\{\frac{-2 \pm \sqrt{10}}{2}\right\}$

49. $2x^2 + 3x - 5 = 0$ $\left\{1, -\frac{5}{2}\right\}$

50. **FLOOR PLAN** Mario's living room has a length 6 feet wider than the width. The area of the living room is 280 square feet. What are the dimensions of his living room? **20 feet by 14 feet**

**Example 7**

Find the value of $c$ that makes $x^2 + 14x + c$ a perfect square. Then write the trinomial as a perfect square.

**Step 1** Find one half of 14.

**Step 2** Square the result of Step 1.

**Step 3** Add the result of Step 2 to $x^2 + 14x$.

The trinomial $x^2 + 14x + 49$ can be written as $(x + 7)^2$.

**Example 8**

Solve $x^2 + 12x - 13 = 0$ by completing the square.

$x^2 + 12x - 13 = 0$

$x^2 + 12x = 13$

$x^2 + 12x + 36 = 13 + 36$

$(x + 6)^2 = 49$

$x + 6 = \pm 7$

$x + 6 = 7$ or $x + 6 = -7$

$x = 1$ $\quad x = -13$

The solution set is $\{-13, 1\}$ or $\{x \mid x = -13, 1\}$.

## Additional Answers

40. $81; (x + 9)^2$

41. $4; (x - 2)^2$

42. $\frac{49}{4}; \left(x - \frac{7}{2}\right)^2$

43. $1.44; (x + 1.2)^2$

44. $\frac{1}{16}; \left(x - \frac{1}{4}\right)^2$

45. $\frac{9}{25}; \left(x + \frac{3}{5}\right)^2$

## 4-6 The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

**a.** Find the value of the discriminant.

**b.** Describe the number and type of roots.

**c.** Find the exact solutions by using the Quadratic Formula.

**51.** $x^2 - 10x + 25 = 0$ **51–57. See margin.**

**52.** $x^2 + 4x - 32 = 0$

**53.** $2x^2 + 3x - 18 = 0$

**54.** $2x^2 + 19x - 33 = 0$

**55.** $x^2 - 2x + 9 = 0$

**56.** $4x^2 - 4x + 1 = 0$

**57.** $2x^2 + 5x + 9 = 0$

**58.** **PHYSICAL SCIENCE** Lauren throws a ball with an initial velocity of 40 feet per second. The equation for the height of the ball is $h = -16t^2 + 40t + 5$, where $h$ represents the height in feet and $t$ represents the time in seconds. When will the ball hit the ground? **about 2.62 seconds**

### Example 9

**Solve $x^2 - 4x - 45 = 0$ by using the Quadratic Formula.**

In $x^2 - 4x - 45 = 0$, $a = 1$, $b = -4$, and $c = -45$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-45)}}{2(1)}$$

$$= \frac{4 \pm 14}{2}$$

Write as two equations.

$$x = \frac{4 + 14}{2} \quad \text{or} \quad x = \frac{4 - 14}{2}$$

$$= 9 \qquad\qquad = -5$$

The solution set is $\{-5, 9\}$ or $\{x \mid x = -5, 9\}$.

**51a.** 0
**51b.** 1 real rational root
**51c.** $\{5\}$
**52a.** 144
**52b.** 2 rational real roots
**52c.** $\{-8, 4\}$
**53a.** 153
**53b.** 2 irrational real roots
**53c.** $\left\{\frac{-3 \pm 3\sqrt{17}}{4}\right\}$
**54a.** 625
**54b.** 2 real rational roots
**54c.** $\left\{-11, \frac{3}{2}\right\}$
**55a.** $-32$
**55b.** 2 complex roots
**55c.** $\{1 \pm 2i\sqrt{2}\}$
**56a.** 0
**56b.** 1 real rational root
**56c.** $\left\{\frac{1}{2}\right\}$
**57a.** $-47$
**57b.** 2 complex roots
**57c.** $\left\{\frac{-5 \pm i\sqrt{47}}{4}\right\}$

## 4-7 Transformations of Quadratic Graphs

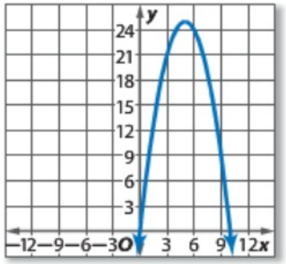
**Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. Then graph the function.**

**59–62. See margin.**

**59.** $y = -3(x - 1)^2 + 5$ **60.** $y = 2x^2 + 12x - 8$

**61.** $y = -\frac{1}{2}x^2 - 2x + 12$ **62.** $y = 3x^2 + 36x + 25$

**63.** The graph at the right shows a product of 2 numbers with a sum of 10. Find a function that models this product and use it to determine the two numbers that would give a maximum product.




***f*(*x*) = $-x^2 + 10x$; 5 and 5**

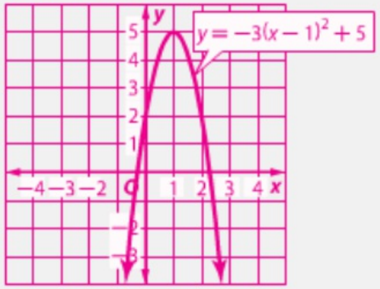
### Example 10

**Write the quadratic function $y = 3x^2 + 24x + 15$ in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.**

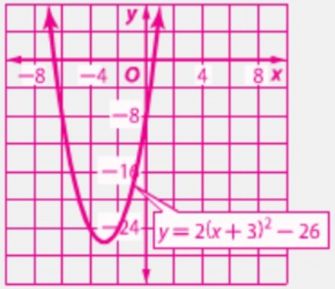
| | |
|---|---|
| $y = 3x^2 + 24x + 15$ | Original equation |
| $y = 3(x^2 + 8x) + 15$ | Group and factor. |
| $y = 3(x^2 + 8x + 16) + 15 - 3(16)$ | Complete the square. |
| $y = 3(x + 4)^2 - 33$ | Rewrite $x^2 + 8x + 16$ as a perfect square. |

So, $a = 3$, $h = -4$, and $k = -33$. The vertex is at $(-4, -33)$ and the axis of symmetry is $x = -4$. Since $a$ is positive, the graph opens up.

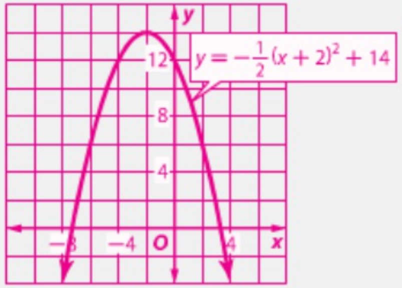
**59.** $y = -3(x - 1)^2 + 5$; $(1, 5)$; $x = 1$; opens down



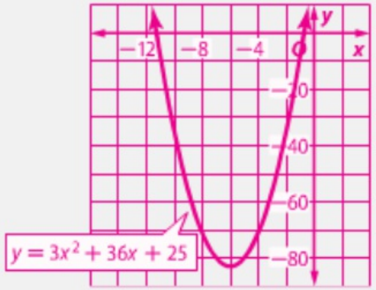

**60.** $y = 2(x + 3)^2 - 26$; $(-3, -26)$; $x = -3$; opens up




**61.** $y = -\frac{1}{2}(x + 2)^2 + 14$; $(-2, 14)$; $x = -2$; opens down




**62.** $y = 3(x + 6)^2 - 83$; $(-6, -83)$; $x = 6$; opens up



4–67. See Chapter 4 Answer Appendix.

## 4-8 Quadratic Inequalities

Graph each quadratic inequality.

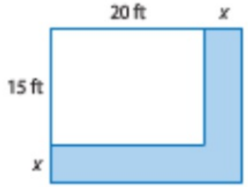
**64.** $y \geq x^2 + 5x + 4$

**65.** $y < -x^2 + 5x - 6$

**66.** $y > x^2 - 6x + 8$

**67.** $y \leq x^2 + 10x - 4$

**68.** Solomon wants to put a deck along two sides of his garden. The deck width will be the same on both sides and the total area of the garden and deck cannot exceed 500 square feet. How wide can the deck be? **between 0 and 5 ft**




Solve each inequality using a graph or algebraically.

**69.** $x^2 + 8x + 12 > 0$ $\{x \mid x < -6 \text{ or } x > -2\}$

**70.** $6x + x^2 \geq -9$ {all real numbers}

**71.** $2x^2 + 3x - 20 > 0$ $\left\{x \,\middle|\, x < -4 \text{ or } x > \frac{5}{2}\right\}$

**72.** $4x^2 - 3 < -5x$ $\{x \mid -1.69 < x < 0.44\}$

**73.** $3x^2 + 4 > 8x$ $\left\{x \,\middle|\, x < \frac{2}{3} \text{ or } x > 2\right\}$

### Example 11

Graph $y > x^2 + 3x + 2$.

**Step 1** Graph the related function, $y > x^2 + 3x + 2$. Because the inequality symbol $>$ is used, the parabola should be dashed.

**Step 2** Test a point not on the graph of the parabola such as (0, 0).

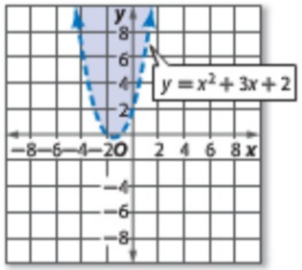
$$y > x^2 + 3x + 2$$

$$(0) \overset{?}{>} (0)^2 + 3(0) + 2$$

$$0 \not> 2$$

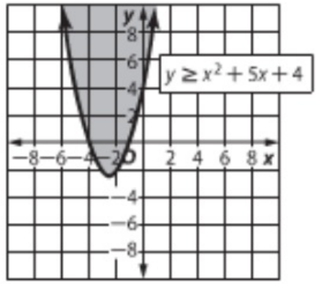
So, (0, 0) is not a solution of the inequality.

**Step 3** Shade the region that does not contain the point (0, 0).

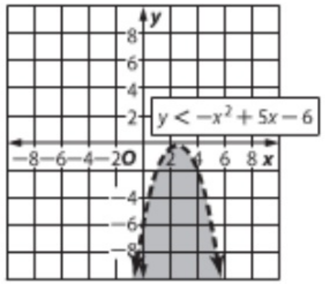



## Study Guide and Review

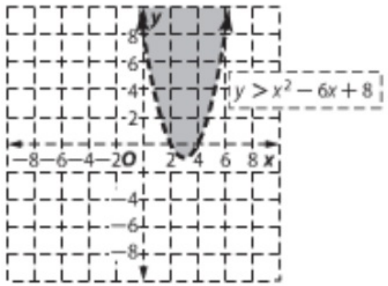
**64.**




**65.**




**66.**




**67.**