Find the derivative of the given function.

31) $y = \tan^{-1}\sqrt{3x}$

$$\frac{d}{dx}\left(\tan^{-1}u\right) = \frac{1}{1+u^2}u'$$

$u = (3x)^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}(3x)^{-\frac{1}{2}} \cdot 3$

$u' = \frac{1}{2\sqrt{3x}} \cdot 3$

$y' = \frac{1}{1+3x}\left(\frac{3}{2\sqrt{3x}}\right)$

$= \frac{3}{(2\sqrt{3x})(1+3x)}$

ind the extreme values of the function and where they occur.

33) $y = \frac{4x}{x^2 + 1}$

32

$g(x) = -x^2 + 10x - 21$

$g'(x) = -2x + 10 = 0$

$x = 5$

Test $x = 4$ $\quad$ $x = 6$

$-2(4) + 10$ $\quad$ 5 $\quad$ $-2(6) + 10$

$+$ $\quad$ $-$

33) _____

$g(3) = -(3)^2 + 10(3) - 21$

$-9 + 30 - 21 = 0$ min

$g(5) = -(5)^2 + 10(5) - 21$

$-25 + 50 - 21 = 4$ max

$g(7) = -(7)^2 + 10(7) - 21$

$-49 + 70 - 21 = 0$ min

Test $x=-2$, $x=0$, $x=2$

$-$ | $+$ | $-$

$\frac{4-16}{+}$ -1 $\frac{4}{+}$ 1 $\frac{4-16}{+}$ y

min max

min $(-1,-2)$

max $(1,2)$

Find the extreme values of the function and where they occur.

33) $y = \frac{4x}{x^2+1}$ ← $\frac{f}{g}$

33) _____

$f = 4x \quad g = x^2+1$

$f' = 4 \quad g' = 2x$

$$y' = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2} = 0$$

$$y' = \frac{4-4x^2}{(x^2+1)^2}$$

$4x^2+4-8x^2=0$

$4-4x^2=0$

$\frac{4}{4} \quad \frac{4}{4} \quad \frac{0}{4}$

$1-x^2=0$

$(1+x)(1-x)=0$

$x=\pm 1$

$f(-1) = \frac{-4}{1+1} = -2$

$f(1) = \frac{4}{1+1} = 2$

ive an appropriate answer.

34) Find the value or values of c that satisfy $\frac{f(b) - f(a)}{b - a} = f'(c)$ for the function $f(x) = x + \frac{27}{x}$ on the interval [3, 9]

34) _____

$f(3) = 3 + \frac{27}{3}$

$3 + 9 = 12$

$f(9) = 9 + \frac{27}{9} = 12$

$\frac{12-12}{9-3} = 1 - 27x^{-2}$

$0 = 1 - \frac{27}{x^2}$

$f(x) = x + 27x^{-1}$

$f'(x) = 1 - 27x^{-2}$

$\frac{27}{x^2} = 1$

$27 = x^2$

$x = \pm 3\sqrt{3}$

28) $f(x) = 4 \cos 4x$ 28)

is decreasing.

37) $f(x) = x^4 - 2$ 37) _____

37

$f(x) = x^4 - 2$

$f'(x) = 4x^3 = 0$

$x = 0$

Test

$x = -1$ $x = 1$

$-$ 0 $+$

increasing : $(0, \infty)$

decreasing : $(-\infty, 0)$

Find all possible functions with the given derivative.

38) $f'(x) = 4 \cos 4x$ 38) _____

Find the function with the given derivative whose graph passes through the point P.

39) $f'(x) = x^2 + 9$, P(3, 55) 39) _____

$f(x) = \frac{1}{3}x^3 + 9x + 19$

x y

38 $f(x) = \sin(4x) + C$

39 $f(x) = \frac{1}{3}x^3 + 9x + C$

$55 = \frac{1}{3}(27) + 9(3) + C$

$55 = 9 + 27 + C$

$55 = 36 + C$ $C = 19$

Find the points of inflection.

40) $y = \frac{4}{3}x^3 - 12x^2 + 10x + 50$

40) _____

$y' = 4x^2 - 24x + 10$

$y'' = 8x - 24 = 0$

$+24 \quad +24$

$x = 3$

$x = 2 \qquad x = 4$

$- \quad 3 \quad +$ $\qquad y''$

(4) $\frac{4}{3}(27) - 12(9) + 80$

$36 - 108 + 80$

$116 - 108 = 8$

(3, 8)