

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Evaluate the integral.

$$1) \int x^6(x^7 - 4)^4 dx$$

① $u = x^7 - 4 \quad \frac{1}{7} \int u^4 du \quad 1) \underline{\hspace{2cm}}$

$$du = 7x^6 dx$$

$\frac{1}{7} du = x^6 dx$

$$= \frac{1}{7} \left(\frac{1}{5} u^5 \right) + C$$
$$= \frac{1}{35} (x^7 - 4)^5 + C$$

$$2) \int \frac{dx}{x \ln x^4}$$

$$2) \underline{\hspace{2cm}}$$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Evaluate the integral.

1) $\int x^6(x^7 - 4)^4 dx$

2) $u = \ln x$

1) _____

2) $\int \frac{1}{x \ln x^4} dx$

$$du = \frac{1}{x} dx$$

2) _____

$$\begin{aligned} &= \int \frac{1}{4x \ln x} dx = \frac{1}{4} \int \frac{dx}{x \ln x} = \frac{1}{4} \int \frac{1}{u} du \\ &= \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |\ln x| + C. \end{aligned}$$

$$F(b) - F(a)$$

Evaluate the definite integral.

$$3) \int_0^{\pi/2} x^2 \sin 3x \, dx$$

Give your answer in exact form.

$$\begin{aligned} & -\frac{1}{3}x^2 \cos 3x + \frac{2}{9}x \sin 3x \\ & + \frac{2}{27} \cos 3x \end{aligned}$$

$$\left|_0^{\frac{\pi}{2}} \right. \begin{aligned} & \left(-\frac{1}{3}\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{3\pi}{2}\right) \right. \\ & \left. + \frac{2}{9}\left(\frac{\pi}{2}\right) \sin\left(\frac{3\pi}{2}\right) \right. \\ & \left. + \frac{2}{27} \cos\left(\frac{3\pi}{2}\right) \right) \end{aligned}$$

$$\begin{aligned} u &= x^2 & dv &= \sin 3x \\ du &= 2x \, dx & v &= -\frac{1}{3} \cos 3x \\ & & &+ \frac{1}{9} \sin 3x \\ & & &+ \frac{1}{27} \cos 3x \\ & & & \vdots \\ & & & \left. \left(0 + 0 + \frac{2}{27} \cos 0 \right) \right. \end{aligned}$$

$$3) \quad -\frac{\pi}{9} - \frac{2}{27}$$

$$⑤ \int dy = 22 \left(\frac{1}{x} dx \right)$$

$$y = 22 \ln|x| + C$$

$$21 = 22 \ln(1) + C \quad C = 21$$

$$21 = 22 \ln(\underline{\underline{0}}) + C \quad C = 21$$

$$\begin{aligned} \ln 1 &= x \\ C^x &= 1 \\ x &= 0 \end{aligned}$$

Use separation of variables to solve the initial value problem.

$$5) \frac{dy}{dx} = \frac{22}{x} \text{ and } y = 21 \text{ when } x = 1$$

$$5) \underline{y = 22 \ln x + 21}$$

$$⑥ \quad y' = \frac{dy}{dx} \quad \frac{1}{2} + 1 = \frac{1}{2} + \frac{3}{2}$$

$$\int (y)^{\frac{1}{2}} dx = \int 5x^2 dx \quad \text{so...}$$

$$\frac{2}{3} y^{\frac{3}{2}} = \frac{5}{3} x^3 + C \quad \frac{2}{3} (1)^{\frac{3}{2}} = \frac{5}{3} (0)^3 + C$$

$$6) y' = \frac{5x^2}{\sqrt{y}} \text{ and } y = 1 \text{ when } x = 0$$

$$3 \left(\frac{2}{3} y^{\frac{3}{2}} = \frac{5}{3} x^3 + \frac{2}{3} \right) \quad \frac{2}{3} = C$$

$$\frac{2}{2} y^{\frac{3}{2}} = \frac{5}{2} x^3 + 2 \quad (-)^{\frac{3}{2}} = (-)^{\frac{3}{2}}$$

$$y^{\frac{3}{2}} = \frac{5x^3 + 2}{2}$$