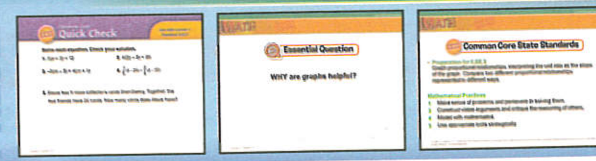
PLAN ONLINE

PD RtI IWB Watch Worksheets Vocab Tutor Tools Check

PRESENT



Lesson 1

Constant Rate of Change

What You'll Learn

Scan the lesson. Write the definitions of linear relationships and constant rate of change. **Sample answers:**

- **Relationships that have straight-line graphs are called linear relationships.**
- **A constant rate of change is when the rate of change between any two points in a relationship is the same or constant.**

Essential Question

WHY are graphs helpful?

Vocabulary

linear relationship
constant rate of change

Common Core State Standards

Content Standards
Preparation for 8.EE.5

Mathematical Practices
1, 3, 4, 5

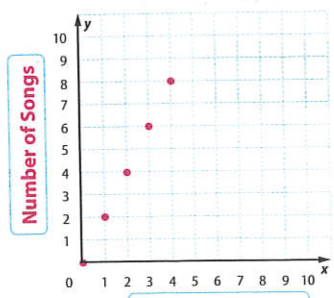
Real-World Link

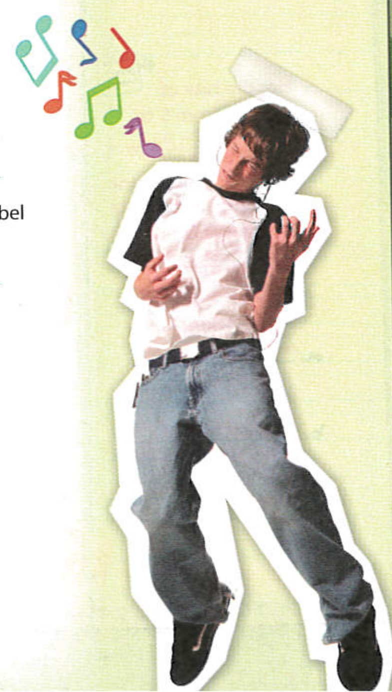
Music Marcus can download two songs from the Internet each minute. This is shown in the table below.

Time (minutes), *x*	0	1	2	3	4
Number of Songs, *y*	0	2	4	6	8

1. Compare the change in the number of songs *y* to the change in time *x*. What is the rate of change?
   Sample answer: The number of songs increases by 2, and the time increases by 1; 2 songs per minute

2. Graph the ordered pairs from the table on the graph shown. Label the axes. Then describe the pattern shown on the graph.
   Sample answer: The points appear to make a line.





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Skills Trace

Focus

Objective Identify proportional and nonproportional linear relationships by finding a constant rate of change.

Coherence

Previous
Students identified proportional and nonproportional relationships in tables and graphs.

Now
Students use tables and graphs to find the rate of change in a linear relationship.

Next
Students will relate constant rate of change to the slope of a line.

Building on the Essential Question

At the end of the lesson, students should be able to answer "How can you use a table to determine if there is a proportional relationship between two quantities?"

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

EL **Think-Pair-Solo** Have students think through their solution to Exercise 1. Then have them discuss their responses with a partner. Have them work alone to complete Exercise 2. Have them trade papers with a partner and each partner checks the other's graph and answer. Then have them discuss and resolve any differences.

Alternate Strategies

AL Have students explain the pattern shown in the table.

BL Have them extend the pattern in the table for 10, 11, 12, 13, 14, and 15 minutes and confirm the rate of change.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

IWB

Example

1. Identify linear relationships.

AL
- *Is the balance increasing as the transactions increase, or decreasing?* decreasing
- *As the number of transactions increases by 3, what happens to the balance?* It decreases by \$30.

OL
- *How do you determine if the relationship between two quantities is linear?* if the rate of change is constant
- *How do you find the constant rate of change, if it exists?* Find the change in the balance per each transaction.

BL
- *How could you find the balance after 5 transactons?* Sample answer: Because the rate of change is −\$10 per transaction, add \$10 to the balance after 6 transactions. After 5 transactions, \$140 + \$10 = \$150
- *Find the balance after 11 transactions.* \$90
- *If you were to graph this relationship, what would the graph look like?* Sample answer: a line sloping down from left to right

Need Another Example?

The amount a babysitter charges is shown. Is the relationship between the number of hours and the amount charged linear? If so, find the constant rate of change. If not, explain your reasoning. yes; The constant rate of change is $\frac{8}{1}$, or \$8 per hour.

Hours	Charge
1	\$10
2	\$18
3	\$26
4	\$34

Work Zone

Show your work.

a. No; the rate of change from 5 to 10 min, $\frac{90-95}{10-5}$ or −1°F per min, is not the same as the rate of change from 10 to 15 min, $\frac{86-90}{15-10}$ or −0.8°F per min, the relationship is not linear

b. Yes; the rate of change is −2.5 min per volunteer.

Linear Relationships

Relationships that have straight-line graphs, like the one on the previous page, are called **linear relationships**. Notice that as the numberxof songs increases by 2, the time in minutes increases by 1.

	+2	+2	+2	+2	
Number of Songs, y	0	2	4	6	8
Time (minutes), x	0	1	2	3	4
	+1	+1	+1	+1	

Rate of Change

$\frac{2}{1}$ = 2 songs per minute

The rate of change between any two points in a linear relationship is the same or *constant*. A linear relationship has a **constant rate of change**.

Example

Tutor

1. The balance in an account after several transactions is shown. Is the relationship between the balance and number of transactions linear? If so, find the constant rate of change. If not, explain your reasoning.

Number of Transactions	Balance (\$)
3	170
6	140
9	110
12	80

+3 +3 +3 −30 −30 −30

As the number of transactions increases by 3, the balance in the account decreases by \$30.

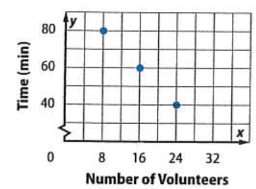
Since the rate of change is constant, this is a linear relationship. The constant rate of change is $\frac{-30}{3}$ or −\$10 per transaction. This means that each transaction involved a \$10 *withdrawal*.

Got It? **Do these problems to find out.**

a.

Cooling Water	
Time (min)	**Temperature (°F)**
5	95
10	90
15	86
20	82

b.



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Proportional Linear Relationships

Key Concept

Words Two quantities a and b have a proportional linear relationship if they have a constant ratio and a constant rate of change.

Symbols $\frac{b}{a}$ is constant and $\frac{\text{change in } b}{\text{change in } a}$ is constant.

To determine if two quantities are proportional, compare the ratio $\frac{b}{a}$ for several pairs of points to determine if there is a constant ratio.

Proportional Relationships
Two quantities are proportional if they have a constant ratio.

Real World

Example

Watch Tutor

2. Use the table to determine if there is a proportional linear relationship between a temperature in degrees Fahrenheit and a temperature in degrees Celsius. Explain your reasoning.

+5 +5 +5 +5

Degrees Celsius	0	5	10	15	20
Degrees Fahrenheit	32	41	50	59	68

+9 +9 +9 +9

Constant Rate of Change

$$\frac{\text{change in °F}}{\text{change in °C}} = \frac{9}{5}$$

Since the rate of change is constant, this is a linear relationship.

To determine if the two scales are proportional, express the relationship between the degrees for several columns as a ratio.

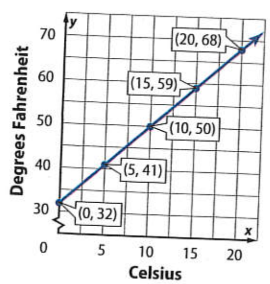
$$\frac{\text{degrees Fahrenheit}}{\text{degrees Celsius}} \longrightarrow \frac{41}{5} = 8.2 \quad \frac{50}{10} = 5 \quad \frac{59}{15} \approx 3.9$$

Since the ratios are not the same, the relationship between degrees Fahrenheit degrees Celsius is *not* proportional.

Check: Graph the points on the coordinate plane. Then connect them with a line.

The points appear to fall in a straight line so the relationship is linear. ✓

The line connecting the points does not pass through the origin so the relationship is not proportional. ✓



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Example

IWB

2. Identify proportional linear relationships.

AL
- *What is a linear relationship?* a relationship that has a constant rate of change
- *What is a proportional linear relationship?* a relationship that has a constant rate of change and the ratios between the two pairs of quantities are the same

OL
- *What are two methods you could use to determine if a proportional linear relationship exists?* Make a table or make a graph.
- *Is there a constant rate of change? Explain.* yes; Sample answer: The degrees Fahrenheit increases by 9 for every 5 degree increase in degrees Celsius. The rate of change is $\frac{9}{5}$.
- *Does this represent a linear relationship?* yes
- *Are the ratios of degrees Fahrenheit to degrees Celsius the same?* no
- *Does this represent a proportional relationship?* no
- *How does the graph illustrate that this relationship is linear, but not proportional?* The graph is a straight line (linear), but it does not pass through the origin (not proportional).

BL
- *Which you rather use a table or a graph to determine proportionality? Explain.* Sample answer: Using a graph is more visual, but I need to use graph paper or a grid. Using a table is not as visual, but I don't need any materials to create a table.

Need Another Example?

Use the table to determine if there is a proportional linear relationship between the speed (meters per second) and the time since a ball has been dropped. Explain your reasoning. yes; The ratio of speed to time is a constant 9.8, so the relationship is proportional.

Speed (m/s)	0	9.8	19.6	29.4	39.2	49.0
Time (s)	0	1	2	3	4	5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

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IWB

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- *Find the balance after 11 transactions.* $90
- *If you were to graph this relationship, what would the graph look like?* Sample answer: a line sloping down from left to right

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Linear Relationships

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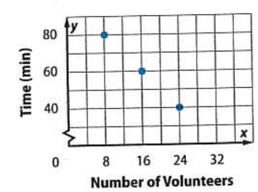

Show your work.

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Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL EL **Teammates Consult** Have students work in small groups to discuss Exercise 1 with Student 1 leading the discussion. When everyone on the team has contributed and a solution is agreed upon, have all students record their answer in their textbooks. Repeat the process for Exercise 2 with Student 2 leading the discussion, and so on.

BL EL **Trade-a-Problem** Have students create their own real-world problem involving a constant rate of change. Have them trade their problems with a partner. Each partner generates a list of ordered pairs that represent the problem and graph them on a coordinate plane. Have them use the graph to verify whether the relationship demonstrates a constant rate of change. Then have them use the graph to determine whether the relationship is proportional. Have them justify their response.

Watch Out!

Common Error Students may have trouble determining if a relationship is proportional. Have students write the ratio in words to compare the quantities. They should make sure they compare the two quantities in the same order and that they do not invert any ratios.

Show your work.

c. It is a proportional linear relationship. The ratio of mass to weight is a constant $\frac{9}{20}$; the rate of change is a constant $\frac{9}{20}$.

Got It? Do this problem to find out.

c. Use the table to determine if there is a proportional linear relationship between mass of an object in kilograms and the weight of the object in pounds. Explain your reasoning.

Weight (lb)	20	40	60	80
Mass (kg)	9	18	27	36

Check

Guided Practice

1. The amount of paint y needed to paint a certain amount of chairs x is shown in the table. Is the relationship between the two quantities linear? If so, find the constant rate of change. If not, explain your reasoning. (Example 1)

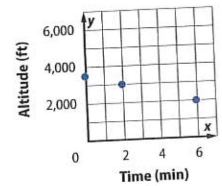
Show your work.

Yes; the rate of change between total cans of paint and number of chairs for each number of chairs is a constant $\frac{6}{5}$ or $1\frac{1}{5}$ cans per chair.

Paint Needed for Chairs	
Chairs, x	Cans of Paint, y
5	6
10	12
15	18

2. The altitude y of a certain airplane after a certain number of minutes x is shown in the graph. Is the relationship linear? If so, find the constant rate of change. If not, explain your reasoning. (Example 1)

yes; −250 ft/min or a decrease of 250 feet each minute



3. Determine whether a proportional relationship exists between the two quantities shown in Exercise 1. Explain your reasoning. (Example 2)

Yes; the ratio of cans of paint to number of chairs is a constant $1\frac{1}{5}$ cans of paint per chair so the relationship is proportional. The rate of change is a constant $1\frac{1}{5}$ cans of paint per chair so the relatonsip is linear.

4. **Building on the Essential Question** How can you use a table to determine if there is a proportional relationship between two quantities? Sample answer: You can write the ratio $\frac{b}{a}$ for each pair of points in the table to determine if a constant ratio exists.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

YES ? NO

For more help, go online to access a Personal Tutor.

Tutor

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Name ______________________ My Homework ______________________

Independent Practice

Go online for Step-by-Step Solutions eHelp

Determine whether the relationship between the two quantities shown in each table or graph is linear. If so, find the constant rate of change. If not, explain your reasoning. (Example 1)

1. Cost of Electricity to Run Personal Computer

Time (h)	Cost (¢)
5	15
8	24
12	36
24	72

Show your work.

Yes; the rate of change between cost and time for each hour is a constant 3¢ per hour.

2. Distance Traveled by Falling Object

Time (s)	1	2	3	4
Distance (m)	4.9	19.6	44.1	78.4

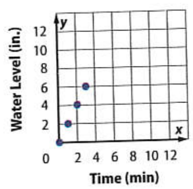
No; the rate of change from 1 to 2 seconds, 14.7 m/s, is not the same as the rate of change from 2 to 3 seconds, 24.5 m/s, so the rate of change is not constant.

3. Italian Dressing Recipe

Oil (c)	2	4	6	8
Vinegar (c)	$\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	3

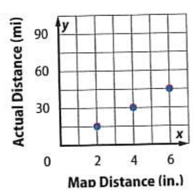
Yes; the rate of change between vinegar and oil for each cup of oil is a constant $\frac{3}{8}$ cup vinegar per cup of oil.

4.



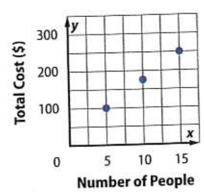
Yes; the rate of change between water level and time for each minute is a constant 2 in./min.

5.



Yes; the rate of change between the actual distance and the map distance for each inch on the map is a constant 7.5 mi/in.

6.



Yes; the rate of change between the total cost and the number of people is a constant $15/person.

Determine whether a proportional relationship exists between the two quantities shown in the following Exercises. Explain your reasoning. (Example 2)

7. Exercise 1

Yes; the ratio of the cost to time is a constant 3¢ per hour, so the relationship is proportional.

8. Exercise 3

Yes; the ratio of vinegar to oil is a constant $\frac{3}{8}$ cup vinegar per cup of oil, so the relationship is proportional.

9. Exercise 5

Yes; the ratio of actual distance to map distance is a constant $\frac{15}{2}$ miles per inch, so the relationship is proportional.

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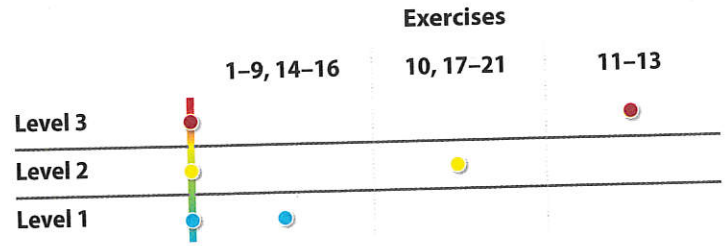
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 12, 13, 20, 21
OL	On Level	1–9 odd, 10, 12, 13, 20, 21
BL	Beyond Level	10–13, 20, 21

CCSS MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
2 Reason abstractly and quantitatively.	17, 18, 19
3 Construct viable arguments and critique the reasoning of others.	13
4 Model with mathematics.	12
5 Use appropriate tools strategically.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students give a real-world example of a relationship that has a constant rate of change. Sample answer: the distance traveled at a constant speed

10. CCSS **Use Math Tools** Match each table with its rate of change.

2.4 ft/min

10 ft/min

−0.8 ft/min

0.25 ft/min

Time (minutes)	20	30	40
Altitude (feet)	170	162	154

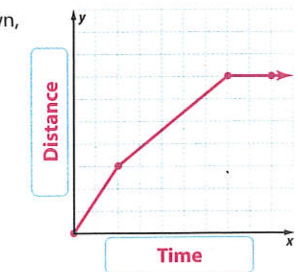
Time (minutes)	1	2	3
Distance (feet)	20	30	40

Time (minutes)	4	6	8
Height (feet)	1	1.5	2

Time (minutes)	5	10	15
Depth (feet)	12	24	36

H.O.T. Problems Higher Order Thinking

11. CCSS **Persevere with Problems** A dog starts walking, slows down, and then sits down to rest. Sketch a graph of the situation to represent the different rates of change. Label the *x*-axis "Time" and the *y*-axis "Distance". Sample answer:



12. CCSS **Model with Mathematics** Describe a situation with two quantities that have a proportional linear relationship.

Sample answer: Josiah can read at a constant rate of 1.5 pages per minute. The number of pages read and the amount of time in minutes is proportional.

13. CCSS **Justify Conclusions** Each table shows a relationship with a constant rate of change. Is each relationship proportional? Justify your reasoning.

a.

Cost of Play Tickets ($)				
t	1	2	3	4
c	3.50	4.00	4.50	5.00

no; Sample answer: $\frac{3.50}{1} \neq \frac{4.00}{2}$

b.

Cost of Play Tickets ($)				
t	1	2	3	4
c	2.50	5.00	7.50	10.00

yes; Sample answer: $\frac{2.50}{1} = \frac{5.00}{2} = \frac{7.50}{3} = \frac{10.00}{4}$

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