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5-5 Study Guide and Intervention *(continued)*

Solving Polynomial Equations

Solve Polynomial Equations If a polynomial expression can be written in quadratic form, then you can use what you know about solving quadratic equations to solve the related polynomial equation.

Example 1: Solve $x^4 - 40x^2 + 144 = 0$.

$x^4 - 40x^2 + 144 = 0$			Original equation
$(x^2)^2 - 40(x^2) + 144 = 0$			Write the expression on the left in quadratic form.
$(x^2 - 4)(x^2 - 36) = 0$			Factor.
$x^2 - 4 = 0$	or	$x^2 - 36 = 0$	Zero Product Property
$(x - 2)(x + 2) = 0$	or	$(x - 6)(x + 6) = 0$	Factor.
$x - 2 = 0$ or $x + 2 = 0$	or	$x - 6 = 0$ or $x + 6 = 0$	Zero Product Property
$x = 2$ or $x = -2$	or	$x = 6$ or $x = -6$	Simplify.

The solutions are ±2 and ±6.

Example 2: Solve $2x + \sqrt{x} - 15 = 0$.

$2x + \sqrt{x} - 15 = 0$	Original equation
$2(\sqrt{x})^2 + \sqrt{x} - 15 = 0$	Write the expression on the left in quadratic form.
$(2\sqrt{x} - 5)(\sqrt{x} + 3) = 0$	Factor.
$2\sqrt{x} - 5 = 0$ or $\sqrt{x} + 3 = 0$	Zero Product Property
$\sqrt{x} = \frac{5}{2}$ or $\sqrt{x} = -3$	Simplify.

Since the principal square root of a number cannot be negative, $\sqrt{x} = -3$ has no solution. The solution is $\frac{25}{4}$ or $6\frac{1}{4}$.

Exercises

Solve each equation.

1. $x^4 = 49$

$x^4 - 49 = 0$
$(x^2+7)(x^2-7) = 0$
$x^2 = -7$ $\quad x = \pm i\sqrt{7}$
$x^2 = 7$ $\quad x = \pm\sqrt{7}$

2. $x^4 - 6x^2 = -8$

$x^4 - 6x^2 + 8 = 0$
$(x^2-4)(x^2-2) = 0$
$x^2 = 4$ $\quad x^2 = 2$
$x = \pm 2$ $\quad x = \pm\sqrt{2}$

3. $x^4 - 3x^2 = 54$

$x^4 - 3x^2 - 54 = 0$
$(x^2-9)(x^2+6) = 0$
$x^2 = -6$ $\quad x^2 = 9$
$x = \pm i\sqrt{2}$ $\quad x = \pm 3$

4. $3t^6 - 48t^2 = 0$

$3t^2(t^4 - 16) = 0$
$3t^2(t^2-4)(t^2+4) = 0$
$3t^2(t+2)(t-2)(t^2+4) = 0$
$t = \pm 2$ $\quad t^2 = -4$ $\quad t = \pm 2i$

5. $m^6 - 16m^3 + 64 = 0$

$(m^3-8)(m^3+8) = 0$
$(m-2)(m^2-2m+4)($
$(m+2)(m^2+2m+4) = 0$
quadratic formula!
$a = 1$ $\quad b = 2$ $\quad c = 4$
5 (cont.)
$m = \pm 2, \pm 1 \pm i\sqrt{3}$

6. $y^4 - 5y^2 + 4 = 0$

$(y^2-4)(y^2-1) = 0$
$y = \pm 2$ $\quad y = \pm 1$

7. $x^4 - 29x^2 + 100 = 0$

$(x^2-4)(x^2-25) = 0$
$x = \pm 2$ $\quad x = \pm 5$

8. $4x^4 - 73x^2 + 144 = 0$

$(4x^2-9)(x^2-16) = 0$
$4x^2 = 9$ $\quad x^2 = 9/4$ $\quad x^2 = 16$
$x = \pm 3/2, \pm 4$

9. $\frac{1}{x^2} - \frac{7}{x} + 12 = 0$

$(\frac{1}{x} - 3)(\frac{1}{x} - 7) = 0$
$\frac{1}{x} = 3$ $\quad \frac{1}{x} = 7$
$x = 1/3$ $\quad x = 1/7$

10. $x - 5\sqrt{x} + 6 = 0$

$(\sqrt{x} - 2)(\sqrt{x} - 3) = 0$
$\sqrt{x} = 2$ $\quad \sqrt{x} = 3$
$x = 4$ $\quad x = 9$

11. $x - 10\sqrt{x} + 21 = 0$

$(\sqrt{x} - 3)(\sqrt{x} - 7) = 0$
$\sqrt{x} = 3$ $\quad \sqrt{x} = 7$
$x = 9$ $\quad x = 49$

12. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$

$(x^{1/3} - 2)(x^{1/3} - 3) = 0$
$x^{1/3} = 2$ $\quad x^{1/3} = 3$
$x = 8$ $\quad x = 27$

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5-5 Study Guide and Intervention

Solving Polynomial Equations

Factor Polynomials

Techniques for Factoring Polynomials	For any number of terms, check for: greatest common factor
	For two terms, check for: Difference of two squares $a^2 - b^2 = (a + b)(a - b)$ Sum of two cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ Difference of two cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
	For three terms, check for: Perfect square trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$ General trinomials $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
	For four or more terms, check for: Grouping $ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)$

Example: Factor $24x^2 - 42x - 45$.

First factor out the GCF to get $24x^2 - 42x - 45 = 3(8x^2 - 14x - 15)$. To find the coefficients of the x terms, you must find two numbers whose product is $8 \cdot (-15) = -120$ and whose sum is -14. The two coefficients must be -20 and 6. Rewrite the expression using $-20x$ and $6x$ and factor by grouping.

$8x^2 - 14x - 15 = 8x^2 - 20x + 6x - 15$ Group to find a GCF.

$= 4x(2x - 5) + 3(2x - 5)$ Factor the GCF of each binomial.

$= (4x + 3)(2x - 5)$ Distributive Property

Thus, $24x^2 - 42x - 45 = 3(4x + 3)(2x - 5)$.

Exercises

Factor completely. If the polynomial is not factorable, write *prime*. GCF + Factor

1. $14x^2y^2 + 42xy^3$ GCF: $14xy^2$
   $14xy^2(x + 3y)$

2. $6mn + 18m - n - 3$ (Grouping)
   $6m(n+3) - (n+3)$
   $(6m-1)(n+3)$

3. $2x^2 + 18x + 16$
   $2(x^2+9x+8)$
   $2(x+1)(x+8)$

4. $x^4 - 1$ (diff. of squares)
   $(x^2-1)(x^2+1)$ ~~(x+1)(x-1)~~
   $= (x+1)(x-1)(x^2+1)$

5. $35x^3y^4 - 60x^4y$ GCF: $5x^3y$
   $5x^3y(7y^3 - 12x)$

6. $2r^3 + 250$ GCF: 2 + sum of cubes
   $2(r^3+125)$
   $2(r+5)(r^2-5r+25)$

7. $100m^8 - 9$ (diff. of squares)
   $(10m^4-3)(10m^4+3)$

8. $x^2 + x + 1$?
   prime

9. $c^4 + c^3 - c^2 - c$ (Grouping + diff. of squares)
   $c^3(c+1) - c(c+1)$
   $(c^3-c)(c+1) = c(c^2-1)(c+1)$
   $= c(c-1)(c+1)(c+1)$