3-1 Solving Systems of Equations

Solve each system of equations by graphing.

11. $3x + 4y = 8$ **(0, 2)**
$x - 3y = -6$

12. $x + \frac{8}{3}y = 12$
$\frac{1}{2}x + \frac{4}{3}y = 6$

13. $y - 3x = 13$ **(−3, 4)**
$y = \frac{1}{3}x + 5$

14. $6x - 14y = 5$
$3x - 7y = 5$

15. **LAWN CARE** André and Paul each mow lawns. André charges a $30 service fee and $10 per hour. Paul charges a $10 service fee and $15 per hour. After how many hours will André and Paul charge the same amount? **4 hours**

Solve each system of equations by using either substitution or elimination. **18. (5.25, −1.75)**

16. $x + y = 6$ **(2, 4)**
$3x - 2y = -2$

17. $5x - 2y = 4$ **(−2, −7)**
$-2y + x = 12$

18. $x + y = 3.5$
$x - y = 7$

19. $3y - 5x = 0$ **(3, 5)**
$2y - 4x = -2$

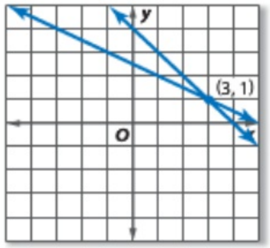
20. **SCHOOL SUPPLIES** At an office supply store, Emilio bought 3 notebooks and 5 pens for $13.75. If a notebook costs $1.25 more than a pen, how much does a notebook cost? How much does a pen cost? **notebook: $2.50; pen: $1.25**

Example 1

Solve the system of equations by graphing.
$x + y = 4$

$x + 2y = 5$

Graph both equations on the coordinate plane.



The solution of the system is (3, 1).

12. infinitely many solutions
14. no solution

Example 2

Solve the system of equations by using either substitution or elimination.
$3x + 2y = 1$

$y = -x + 1$

Substitute $-x + 1$ for y in the first equation. Then solve for y.

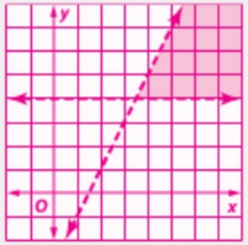
$$\begin{aligned} 3x + 2y &= 1 \\ 3x + 2(-x + 1) &= 1 \\ 3x - 2x + 2 &= 1 \\ x + 2 &= 1 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} y &= -x + 1 \\ &= -(-1) + 1 \\ &= 2 \end{aligned}$$

The solution is (−1, 2).

Additional Answers

21.



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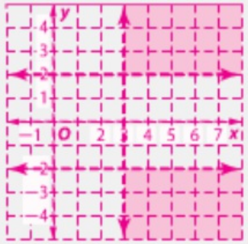

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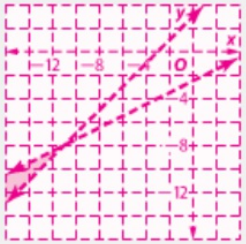


3-2 Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

21. $y < 2x - 3$
$y \geq 4$

22. $|y| > 2$
$x > 3$

23. $y \geq x + 3$
$2y \leq x - 5$

24. $y > x + 1$
$x < -2$

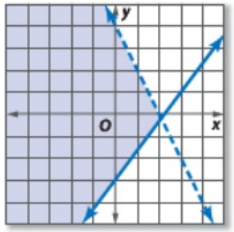
25. **JEWELRY** Payton makes jewelry to sell at her mother's clothing store. She spends no more than 3 hours making jewelry on Saturdays. It takes her 15 minutes to set up her supplies and 25 minutes to make each bracelet. Draw a graph that represents this.
21–25. See margin.

Example 3

Solve the system of inequalities by graphing.

$y \geq \frac{3}{2}x - 3$

$y < 4 - 2x$



The solution of the system is the region that satisfies both inequalities. The solution of this system is the shaded region.

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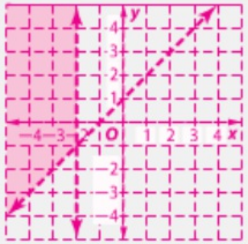

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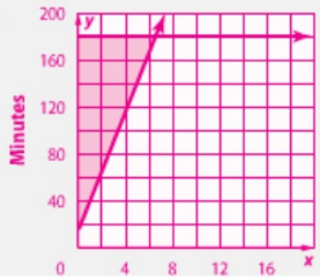


3-3 Optimization with Linear Programming

26. **FLOWERS** A florist can make a grand arrangement in 18 minutes or a simple arrangement in 10 minutes. The florist makes at least twice as many of the simple arrangements as the grand arrangements. The florist can work only 40 hours per week. The profit on the simple arrangements is $10 and the profit on the grand arrangements is $25. Find the number and type of arrangements that the florist should produce to maximize profit. **126 simple and 63 grand**

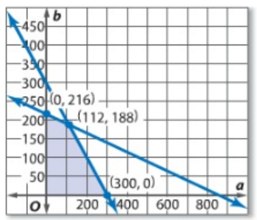
27. **MANUFACTURING** A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one and 1 hour in step two, and produces a profit of $20. Each pair of indoor shoes requires 1 hour in step one and 3 hours in step two, and produces a profit of $15. The company has 40 hours of labor available per day for step one and 60 hours available for step two. What is the manufacturer's maximum profit? What is the combination of shoes for this profit?
$480; 12 outdoor, 16 indoor

Example 4

A gardener is planting two types of herbs in a 5184-square-inch garden. Herb A requires 6 square inches of space, and herb B requires 24 square inches of space. The gardener will plant no more than 300 plants. If herb A can be sold for $8 and herb B can be sold for $12, how many of each herb should be sold to maximize income?

Let a = the number of herb A and b = the number of herb B.

$a \geq 0$, $b \geq 0$,
$6a + 24b \leq 5184$,
and $a + b \leq 300$



Graph the inequalities. The vertices of the feasible region are (0, 0), (300, 0), (0, 216), and (112, 188).

The profit function is $f(a, b) = 8a + 12b$.

The maximum value of $3152 occurs at (112, 188). So the gardener should plant 112 of herb A and 188 of herb B.

3-4 Systems of Equations in Three Variables

Solve each system of equations.

28. $a - 4b + c = 3$
$b - 3c = 10$
$3b - 8c = 24$
(−23, −8, −6)

29. $2x - z = 14$
$3x - y + 5z = 0$
$4x + 2y + 3z = -2$
(5, −5, −4)

30. **AMUSEMENT PARKS** Dustin, Luis, and Marci went to an amusement park. They purchased snacks from the same vendor. Their snacks and how much they paid are listed in the table. How much did each snack cost?

Name	Hot Dogs	Popcorn	Soda	Price
Dustin	1	2	3	$15.25
Luis	2	0	3	$14.00
Marci	1	2	1	$10.25

hot dog: $3.25; popcorn: $2.25; soda: $2.50

Example 5

Solve the system of equations.

$x + y + 2z = 6$
$2x + 5z = 12$
$x + 2y + 3z = 9$

$$\begin{array}{rll} & 2x + 2y + 4z = 12 & \text{Equation 1} \times 2 \\ (-) & x + 2y + 3z = 9 & \text{Equation 3} \\ \hline & x + z = 3 & \text{Subtract.} \end{array}$$

Solve the system of two equations.

$$\begin{array}{rll} & 2x + 5z = 12 & \text{Equation 2} \\ (-) & 2x + 2z = 6 & 2 \times (x + z = 3) \\ \hline & 3z = 6 & \text{Subtract.} \\ & z = 2 & \text{Divide each side by 3.} \end{array}$$

Substitute 2 for z in one of the equations with two variables, and solve for y. Then, substitute 2 for z and the value you got for y into an equation from the original system to solve for x.

The solution is (1, 1, 2).

Additional Answers

33a. buying price: $\begin{bmatrix} 15 \\ 25 \\ 30 \end{bmatrix}$;

33b. selling price: $\begin{bmatrix} 35 \\ 55 \\ 85 \end{bmatrix}$;

33c. $\begin{bmatrix} 35 \\ 55 \\ 85 \end{bmatrix} - \begin{bmatrix} 15 \\ 25 \\ 30 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 55 \end{bmatrix}$

3-5 Operations with Matrices

Perform the indicated operations. If the matrix does not exist, write *impossible*.

31. $3\left(\begin{bmatrix} -2 & 0 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -4 \end{bmatrix}\right)$ $\begin{bmatrix} -3 & 27 \\ 9 & 12 \end{bmatrix}$

32. $\begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 11 \\ -8 \end{bmatrix}$

33. **RETAIL** Current Fashions buys shirts, jeans, and shoes from a manufacturer, marks them up, and then sells them. The table shows the purchase price and the selling price.

Item	Purchase Price	Selling Price
shirts	$15	$35
jeans	$25	$55
shoes	$30	$85

a. Write a matrix for the purchase price. **a–c. See margin.**

b. Write a matrix for the selling price.

c. Use matrix operations to find the profit on 1 shirt, 1 pair of jeans, and 1 pair of shoes.

Example 6

Find $2A + 3B$ if $A = \begin{bmatrix} 9 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix}$.

$2B = 2\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix}$ or $\begin{bmatrix} 2 & 8 \\ 6 & 14 \end{bmatrix}$

$3A = 3\begin{bmatrix} 9 & 1 \\ 1 & 2 \end{bmatrix}$ or $\begin{bmatrix} 27 & 3 \\ 3 & 6 \end{bmatrix}$

$2B + 3A = \begin{bmatrix} 2 & 8 \\ 6 & 14 \end{bmatrix} + \begin{bmatrix} 27 & 3 \\ 3 & 6 \end{bmatrix}$ or $\begin{bmatrix} 29 & 11 \\ 9 & 20 \end{bmatrix}$

Example 7

Find $3C - 5D$ if $C = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$ and $D = [9 \quad 8]$.

$3C - 5D = 3\begin{bmatrix} 3 \\ -7 \end{bmatrix} - 5\,[9 \quad 8]$.

Because the dimensions are different, you cannot subtract the matrices.

3-7 Solving Systems of Equations Using Cramer's Rule

Evaluate each determinant.

38. $\begin{vmatrix} 2 & 4 \\ 7 & -3 \end{vmatrix}$ **−34**

39. $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$ **−44**

Use Cramer's Rule to solve each system of equations.

40. $3x - y = 0$ **(2, 6)**
$5x + 2y = 22$

41. $5x + 2y = 4$
$3x + 4y + 2z = 6$
$7x + 3y + 4z = 29$ **(2, −3, 6)**

42. **JEWELRY** Alana paid $98.25 for 3 necklaces and 2 pairs of earrings. Petra paid $133.50 for 2 necklaces and 4 pairs of earrings. Use Cramer's Rule to find out how much 1 necklace costs and how much 1 pair of earrings costs. **necklace: $15.75; pair of earrings: $25.50**

Example 9

Evaluate $\begin{vmatrix} 4 & -6 \\ 2 & 5 \end{vmatrix}$.

$\begin{vmatrix} 4 & -6 \\ 2 & 5 \end{vmatrix} = 4(5) - (-6)(2)$ — Definition of determinant

$= 20 + 12$ or 32 — Simplify.

Example 10

Use Cramer's Rule to solve $2a + 6b = -1$ and $a + 8b = 2$.

$a = \dfrac{\begin{vmatrix} -1 & 6 \\ 2 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 6 \\ 1 & 8 \end{vmatrix}}$ Cramer's Rule $b = \dfrac{\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 6 \\ 1 & 8 \end{vmatrix}}$

$= \dfrac{-8 - 12}{16 - 6}$ Evaluate each determinant. $= \dfrac{4 + 1}{16 - 6}$

$= \dfrac{-20}{10}$ or -2 Simplify. $= \dfrac{5}{10}$ or $\dfrac{1}{2}$

The solution is $\left(-2, \frac{1}{2}\right)$.

3-8 Solving Systems of Equations Using Inverse Matrices

Find the inverse of each matrix, if it exists.

43. $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ $\frac{1}{2}\begin{bmatrix} 2 & -4 \\ -3 & 7 \end{bmatrix}$

44. $\begin{bmatrix} 2 & 5 \\ -5 & -13 \end{bmatrix}$ $\begin{bmatrix} 13 & 5 \\ -5 & -2 \end{bmatrix}$

45. $\begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix}$ **No inverse exists.**

Use a matrix equation to solve each system of equations.

46. $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **(8, −12)**

47. $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ **(2, 1)**

48. **HEALTH FOOD** Heath sells nuts and raisins by the pound. Sonia bought 2 pounds of nuts and 2 pounds of raisins for $23.50. Drew bought 3 pounds of nuts and 1 pound of raisins for $22.25. What is the cost of 1 pound of nuts and 1 pound of raisins? **nuts: $5.25 per pound; raisins: $6.50 per pound**

Example 11

Solve $\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 36 \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$A^{-1} = \dfrac{1}{-12 - (-15)}\begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix}$ or $\dfrac{1}{3}\begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix}$

Step 2 Multiply each side by the inverse matrix.

$$\frac{1}{3}\begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 36 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 90 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$