Simplify.

50. $\sqrt[3]{54}$ $3\sqrt[3]{2}$

51. $\sqrt{144a^3b^5}$ $12ab^2\sqrt{ab}$

52. $4\sqrt{6y} \cdot 3\sqrt{7x^2y}$ $12|x|y\sqrt{42}$

53. $6\sqrt{72} + 7\sqrt{98} - \sqrt{50}$ $80\sqrt{2}$

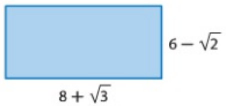
54. $(6\sqrt{5} - 2\sqrt{2})(3\sqrt{5} + 4\sqrt{2})$ $74 + 18\sqrt{10}$

55. $\dfrac{\sqrt{6m^5}}{\sqrt{p^{11}}}$ $\dfrac{m^2\sqrt{6mp}}{p^6}$

56. $\dfrac{3}{5 + \sqrt{2}}$ $\dfrac{15 - 3\sqrt{2}}{23}$

57. $\dfrac{\sqrt{3}}{\sqrt{5} - \sqrt{6}}$ $-\sqrt{15} - 3\sqrt{2}$

58. **GEOMETRY** What are the perimeter and the area of the rectangle?



perimeter $= 28 + 2\sqrt{3} - 2\sqrt{2}$ units;
area $= 48 + 6\sqrt{3} - 8\sqrt{2} - \sqrt{6}$ units^2

Example 7

Simplify $2\sqrt[3]{18a^2b} \cdot 3\sqrt[3]{12ab^5}$.

$$2\sqrt[3]{18a^2b} \cdot 3\sqrt[3]{12ab^5}$$
$$= (2 \cdot 3)\sqrt[3]{18a^2b \cdot 12ab^5} \quad \text{Product Property}$$
$$= 6\sqrt[3]{2^3 3^3 a^3 b^6} \quad \text{Factor.}$$
$$= 6 \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{a^3} \cdot \sqrt[3]{b^6} \quad \text{Product Property}$$
$$= 6 \cdot 2 \cdot 3 \cdot a \cdot b^2 \quad \text{Find cube roots.}$$
$$= 36ab^2 \quad \text{Simplify.}$$

Example 8

Simplify $\sqrt{\dfrac{x^4}{y^5}}$.

$$\sqrt{\frac{x^4}{y^5}} = \frac{\sqrt{x^4}}{\sqrt{y^5}} \quad \text{Quotient Property}$$
$$= \frac{\sqrt{(x^2)^2}}{\sqrt{(y^2)^2} \cdot \sqrt{y}} \quad \text{Factor into squares.}$$
$$= \frac{x^2}{y^2\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} \quad \text{Rationalize the denominator.}$$
$$= \frac{x^2\sqrt{y}}{y^3} \quad \sqrt{y} \cdot \sqrt{y} = y$$

53

$6\sqrt{72}$ $(7\sqrt{98})$ $\sqrt{50}$

$2 \cdot 36$ $\quad 2 \cdot 49$ $\quad 2 \cdot 25$

$6 \cdot 6$ $\quad 7 \cdot 7$ $\quad 5 \cdot 5$

$36\sqrt{2} + 49\sqrt{2} - 5\sqrt{2}$

6-6 Rational Exponents

Simplify each expression.

59. $x^{\frac{1}{2}} \cdot x^{\frac{2}{3}}$ $x^{\frac{7}{6}}$ **60.** $m^{-\frac{3}{4}}$ $\frac{m^{\frac{1}{4}}}{m}$ **61.** $\frac{d^{\frac{1}{6}}}{d^{\frac{3}{4}}}$ $\frac{d^{\frac{5}{12}}}{d}$

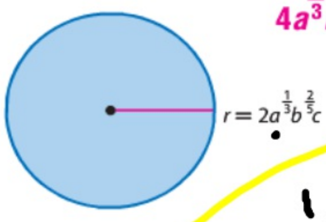
Simplify each expression.

62. $\frac{1}{y^{\frac{1}{4}}}$ $\frac{y^{\frac{3}{4}}}{y}$ **63.** $\sqrt[3]{\sqrt{729}}$ 3 **64.** $\frac{x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}}}{x^{\frac{1}{3}}}$ $x^{\frac{1}{3}} - y^{\frac{2}{3}}$

$\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}}\right)x^{2/3}$

$x^{2/3}$

65. **GEOMETRY** What is the area of the circle? $4a^{\frac{2}{3}}b^{\frac{4}{5}}c^2\pi$ units^2



Example 9

Simplify $a^{\frac{2}{3}} \cdot a^{\frac{1}{5}}$.

$a^{\frac{2}{3}} \cdot a^{\frac{1}{5}} = a^{\frac{2}{3}+\frac{1}{5}}$ Product of Powers

$= a^{\frac{13}{15}}$ Add.

Example 10

Simplify $\frac{2a}{\sqrt[3]{b}}$.

$\frac{2a}{\sqrt[3]{b}} = \frac{2a}{b^{\frac{1}{3}}}$ Rational exponents

$= \frac{2a}{b^{\frac{1}{3}}} \cdot \frac{b^{\frac{2}{3}}}{b^{\frac{2}{3}}}$ Rationalize the denominator.

$= \frac{2ab^{\frac{2}{3}}}{b}$ or $\frac{2a\sqrt[3]{b^2}}{b}$ Rewrite in radical form.

3/4 ? 1

$d \quad d = d$

$x^a \cdot x^b = x^{a+b}$

$? = 1/4$

61 $\frac{d^{1/6} \cdot d^{1/4}}{d^{3/4} \cdot d^{1/4}}$

65 $\pi r^2 = A$

$\pi(2a^{1/3}b^{2/5}c^1)^2$

$4\pi a^{2/3} b^{4/5} c^2$

6-7 Solving Radical Equations and Inequalities

Solve each equation.

66. $\sqrt{x-3}+5=15$ **103**

67. $-\sqrt{x-11}=3-\sqrt{x}$ **$\frac{100}{9}$**

68. $4+\sqrt{3x-1}=8$ **$\frac{17}{3}$**

69. $\sqrt{m+3}=\sqrt{2m+1}$ **2**

70. $\sqrt{2x+3}=3$ **3**

71. $(x+1)^{\frac{1}{4}}=-3$ **no solution**

72. $a^{\frac{1}{3}}-4=0$ **64**

73. $3(3x-1)^{\frac{1}{3}}-6=0$ **3**

74. **PHYSICS** The formula $t=2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where t is the time in seconds for the pendulum to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum

Example 11

Solve $\sqrt{2x+9}-2=5$.

$\sqrt{2x+9}-2=5$	Original equation
$\sqrt{2x+9}=7$	Add 2 to each side.
$(\sqrt{2x+9})^2=7^2$	Square each side.
$2x+9=49$	Evaluate the squares.
$2x=40$	Subtract 9 from each side.
$x=20$	Divide each side by 2.

Example 12

Solve $\sqrt{2x-5}+2>5$.

$(3-\sqrt{x})(3-\sqrt{x})$

67

$(-\sqrt{(x-11)})^2=(3-\sqrt{x})^2$

$x-11=9-6\sqrt{x}+x$

$-x \quad -9 \qquad -9 \qquad -x$

$\frac{-20}{-6}=\frac{-6\sqrt{x}}{-6}$

$(\sqrt{x})^2=\left(\frac{10}{3}\right)^2$

6-7 Solving Radical Equations and Inequalities

Solve each equation.

66. $\sqrt{x-3}+5=15$ **103**
67. $-\sqrt{x-11}=3-\sqrt{x}$ **$\frac{100}{9}$**
68. $4+\sqrt{3x-1}=8$ **$\frac{17}{3}$**
69. $\sqrt{m+3}=\sqrt{2m+1}$ **2**
70. $\sqrt{2x+3}=3$ **3**
71. $(x+1)^{\frac{1}{4}}=-3$ **no solution**
72. $a^{\frac{1}{3}}-4=0$ **64**
73. $3(3x-1)^{\frac{1}{3}}-6=0$ **3**

74. **PHYSICS** The formula $t=2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where t is the time in seconds for the pendulum to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum

Example 11

Solve $\sqrt{2x+9}-2=5$.

$\sqrt{2x+9}-2=5$	Original equation
$\sqrt{2x+9}=7$	Add 2 to each side.
$(\sqrt{2x+9})^2=7^2$	Square each side.
$2x+9=49$	Evaluate the squares.
$2x=40$	Subtract 9 from each side.
$x=20$	Divide each side by 2.

Example 12

Solve $\sqrt{2x-5}+2>5$.

69 $(\sqrt{m+3})^2=(\sqrt{2m+1})^2$

$$m+3=2m+1$$
$$-m-1 \qquad -m-1$$
$$2=m$$

Solve each equation.

66. $\sqrt{x-3}+5=15$ **103**

67. $-\sqrt{x-11}=3-\sqrt{x}$ **$\frac{100}{9}$**

68. $4+\sqrt{3x-1}=8$ **$\frac{17}{3}$**

69. $\sqrt{m+3}=\sqrt{2m+1}$ **2**

70. $\sqrt{2x+3}=3$ **3**

71. $(x+1)^{\frac{1}{4}}=-3$ **no solution**

72. $a^{\frac{1}{3}}-4=0$ **64**

73. $3(3x-1)^{\frac{1}{3}}-6=0$ **3**

74. **PHYSICS** The formula $t=2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where t is the time in seconds for the pendulum to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum

Example 11

Solve $\sqrt{2x+9}-2=5$.

$\sqrt{2x+9}-2=5$	Original equation
$\sqrt{2x+9}=7$	Add 2 to each side.
$(\sqrt{2x+9})^2=7^2$	Square each side.
$2x+9=49$	Evaluate the squares.
$2x=40$	Subtract 9 from each side.
$x=20$	Divide each side by 2.

Example 12

Solve $\sqrt{2x-5}+2>5$.

73

$$3(3x-1)^{1/3}-6=0$$
$$+6 \quad +6$$
$$3(3x-1)^{1/3}=6$$
$$\frac{3(3x-1)^{1/3}}{3}=\frac{6}{3}$$
$$\left((3x-1)^{1/3}\right)^3=(2)^3$$
$$3x-1=8$$
$$+1 \quad +1$$
$$3x=9$$
$$\frac{3x}{3}=\frac{9}{3}$$
$$x=3$$

swing of a pendulum, where T is the time in seconds for the pendulum to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 2.75 seconds. **about 6.13 ft**

Solve each inequality.

75. $2 + \sqrt{3x - 1} < 5$

76. $\sqrt{3x + 13} - 5 \geq 5$ **$x \geq 29$**

77. $6 - \sqrt{3x + 5} \leq 3$

78. $\sqrt{-3x + 4} - 5 \geq 3$ **$x \leq -20$**

79. $5 + \sqrt{2y - 7} < 5$

80. $3 + \sqrt{2x - 3} \geq 3$ **$x \geq \frac{3}{2}$**

81. $\sqrt{3x + 1} - \sqrt{6 + x} > 0$ **$x > \frac{5}{2}$**

75. $\frac{1}{3} \leq x < \frac{10}{3}$
77. $x \geq \frac{4}{3}$
79. no solution

Example 12

Solve $\sqrt{2x - 5} + 2 > 5$.

$\sqrt{2x - 5} \geq 0$	Radicand must be ≥ 0.
$2x - 5 \geq 0$	Square each side.
$2x \geq 5$	Add 5 to each side.
$x \geq 2.5$	Divide each side by 2.

The solution must be greater than or equal to 2.5 to satisfy the domain restriction.

$\sqrt{2x - 5} + 2 > 5$	Original inequality
$\sqrt{2x - 5} > 3$	Subtract 2 from each side.
$(\sqrt{2x - 5})^2 > 3^2$	Square each side.
$2x - 5 > 9$	Evaluate the squares.
$2x > 14$	Add 5 to each side.
$x > 7$	Divide each side by 2.

Since $x \geq 2.5$ contains $x > 7$, the solution of the inequality is $x > 7$.

75

$$2 + \sqrt{3x-1} < 5$$
$$-2 \qquad -2$$
$$\sqrt{3x-1} < 3$$
$$3x - 1 < 9$$
$$+1 \qquad +1$$
$$3x < 10/3$$
$$x < 10/3$$

$$3x - 1 \geq 0$$
$$+1 \qquad +1$$
$$3x \geq 1$$
$$/3 \qquad /3$$
$$x \geq 1/3$$

swing of a pendulum, where T is the time in seconds for the pendulum to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 2.75 seconds. **about 6.13 ft**

Solve each inequality.

75. $2 + \sqrt{3x - 1} < 5$

76. $\sqrt{3x + 13} - 5 \geq 5$ $\quad x \geq 29$

77. $6 - \sqrt{3x + 5} \leq 3$

78. $\sqrt{-3x + 4} - 5 \geq 3$ $\quad x \leq -20$

79. $5 + \sqrt{2y - 7} < 5$

80. $3 + \sqrt{2x - 3} \geq 3$ $\quad x \geq \frac{3}{2}$

81. $\sqrt{3x + 1} - \sqrt{6 + x} > 0$ $\quad x > \frac{5}{2}$

75. $\frac{1}{3} \leq x < \frac{10}{3}$

77. $x \geq \frac{4}{3}$

79. no solution

Example 12

Solve $\sqrt{2x - 5} + 2 > 5$.

$\sqrt{2x - 5} \geq 0$	Radicand must be ≥ 0.
$2x - 5 \geq 0$	Square each side.
$2x \geq 5$	Add 5 to each side.
$x \geq 2.5$	Divide each side by 2.

The solution must be greater than or equal to 2.5 to satisfy the domain restriction.

$\sqrt{2x - 5} + 2 > 5$	Original inequality
$\sqrt{2x - 5} > 3$	Subtract 2 from each side.
$(\sqrt{2x - 5})^2 > 3^2$	Square each side.
$2x - 5 > 9$	Evaluate the squares.
$2x > 14$	Add 5 to each side.
$x > 7$	Divide each side by 2.

Since $x \geq 2.5$ contains $x > 7$, the solution of the inequality is $x > 7$.

79 $\quad 5 + \sqrt{2y-7} < 5$

$\quad -5 \qquad\qquad -5$

$\sqrt{2y-7} < 0$

can't happen!