

Solve the problem analytically.

1) Of all numbers whose sum is 140, find the two that have the maximum product. That is, maximize $Q = xy$, where $x + y = 140$. 1) _____

2) Of all numbers whose difference is 18, find the two that have the minimum product. 2) _____

① $Q = xy$

$Q = (140 - y)y$

$Q = 140y - y^2$

$Q' = 140 - 2y = 0$

$y = 70$

$x + y = 140$

$-y \quad -y$

$x = 140 - y$

$y = 70$

$x - y = 18$

$+y \quad +y$

$x = 18 + y$

$xy = P$


$(18 + y)y = P$

$18y + y^2 = P$

$18 + 2y = 0$

$y = -9$

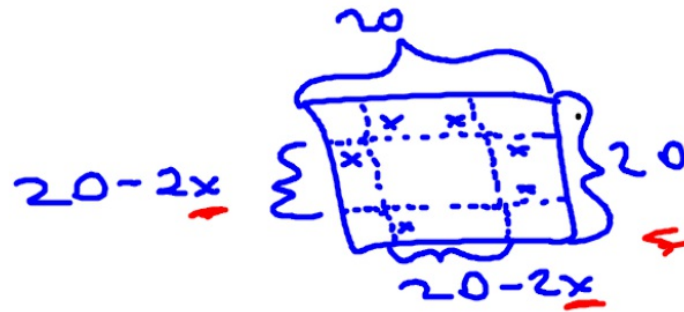
$x = 9$



Solve the problem.

- 3) Find the dimensions that produce the maximum floor area for a one-story house that is rectangular in shape and has a perimeter of 121 ft. Round to the nearest hundredth, if necessary.
- 4) From a thin piece of cardboard 20 in. by 20 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

$$\begin{aligned} \textcircled{3} \quad 2x + 2y &= 121 \rightarrow x = \frac{121 - 2y}{2} \\ xy &= \text{Max} \# \\ \left(\frac{121 - 2y}{2} \right) y &= \# \\ \frac{121}{2} y - y^2 &= \# \end{aligned} \rightarrow \begin{aligned} \frac{121}{2} - 2y &= 0 \\ \frac{121}{4} &= y \quad \text{and } x \end{aligned}$$



4) From a thin piece of cardboard 20 in. by 20 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

4) _____

$V = w \cdot l \cdot h$

$w = 20 - 2x$
 $l = 20 - 2x$
 $h = x$

$V = (20 - 2x)(20 - 2x)x$
 $V = (400 - 80x + 4x^2)x$
 $V = 400x - 80x^2 + 4x^3$
 $V' = 400 - 160x + 12x^2$
 $0 = 12x^2 - 160x + 400$
 $0 = 3x^2 - 40x + 100$

$a = 3$
 $b = -40$
 $c = 100$

$x = \frac{40 \pm \sqrt{1600 - 1200}}{6}$
 $x = \frac{40 \pm \sqrt{400}}{6} = \frac{40 \pm 20}{6}$

$x = \frac{40 + 20}{6} = \frac{60}{6} = 10$
 $x = \frac{40 - 20}{6} = \frac{20}{6} = 3\frac{1}{3}$

Use $x = 3\frac{1}{3}$!



$$\left(\text{price of product} \right) \times \left(\text{\# of items sold} \right) = \text{Revenue}$$

5) If the price charged for a bolt is p cents, then x thousand bolts will be sold in a certain hardware store, where $p = 42 - \frac{x}{18}$. How many bolts must be sold to maximize revenue?

5) _____

$$R = \left(42 - \frac{x}{18} \right) (x)$$
$$R = 42x - \frac{x^2}{18}$$
$$0 = 42 - \frac{x}{9}$$
$$\frac{x}{9} = 42$$
$$x = 378$$

Revenue - cost = profit

$$R(x) - C(x) = P(x)$$

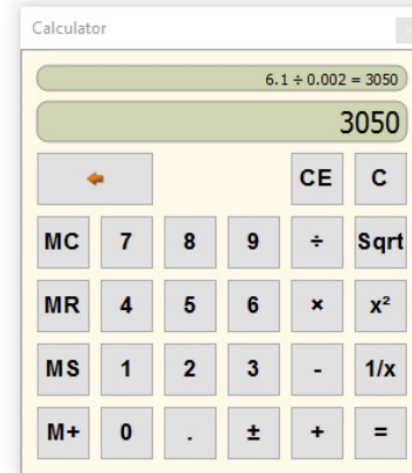
6) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

$$R(x) = 7x$$

$$C(x) = 0.001x^2 + 0.9x + 20.$$

$$\begin{array}{r} 6 \\ 7.50 \\ - 0.9 \\ \hline \end{array}$$

$$7x - (0.001x^2 + 0.9x + 20) = P(x)$$
$$-0.001x^2 + 6.1x - 20 = P(x)$$
$$-0.002x + 6.1 = 0$$
$$-0.002x = -6.1$$
$$\frac{-0.002x}{-0.002} = \frac{-6.1}{-0.002}$$
$$x = 3050$$



6) _____