What you'll learn about

- Volume As an Integral
- Square Cross Sections
- Circular Cross Sections
- Cylindrical Shells
- Other Cross Sections

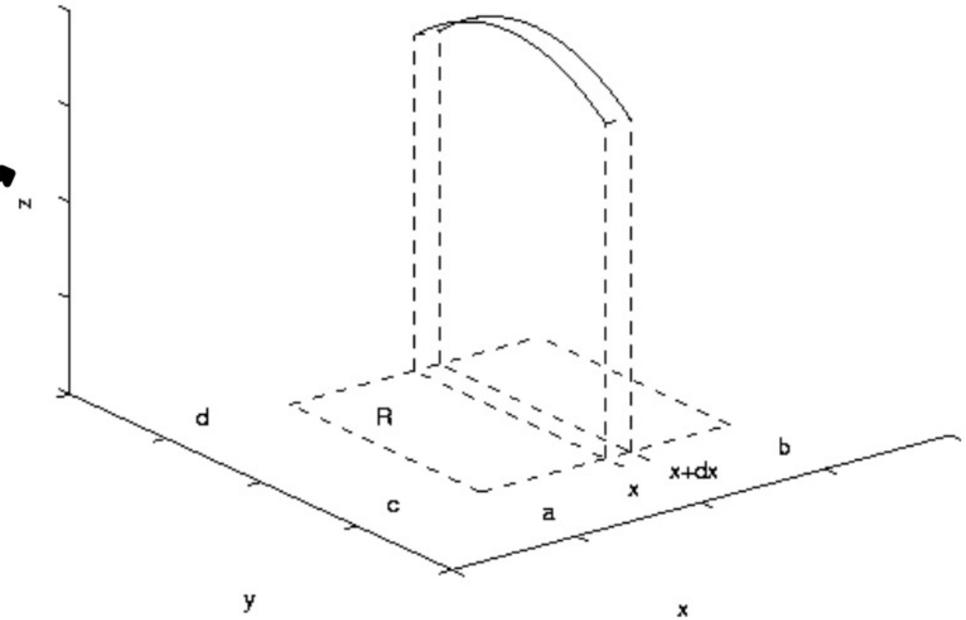
. . . and why

The techniques of this section allow us to compute volumes of certain solids in three dimensions.

7.3 Volumes

In this section, we are going to generate volumes with known cross sections.

First, let's think of it like a loaf of sliced bread. We will find the volume of each "slice," and then add them all up as the thickness approaches 0 (integrate!)



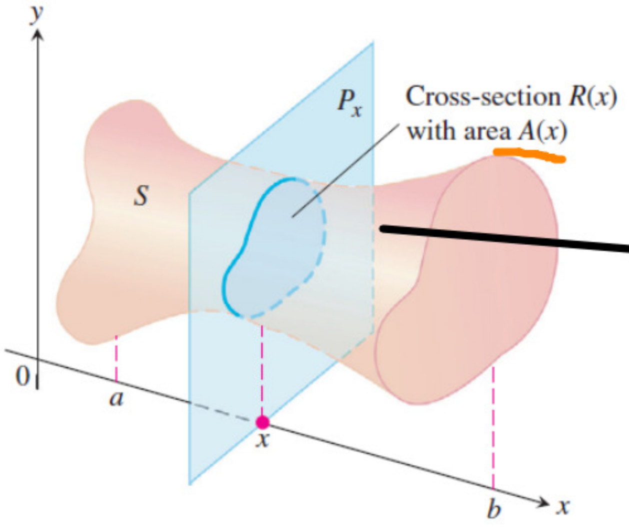


Figure 7.15 The cross section of an arbitrary solid at point x.

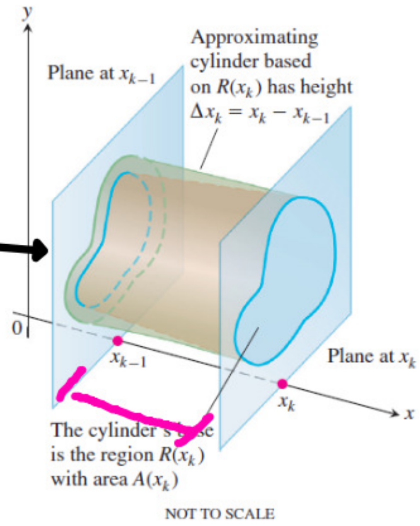


Figure 7.16 Enlarged view of the slice of the solid between the planes at x_{k-1} and x_k.

The volume of the cylinder is

$$V_k = \text{base area} \times \text{height} = A(x_k) \times \Delta x.$$

The sum

$$\sum V_k = \sum A(x_k) \times \Delta x$$

DEFINITION Volume of a Solid

The **volume of a solid** of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$$V = \int_a^b A(x)\, dx.$$

How to Find Volume by the Method of Slicing

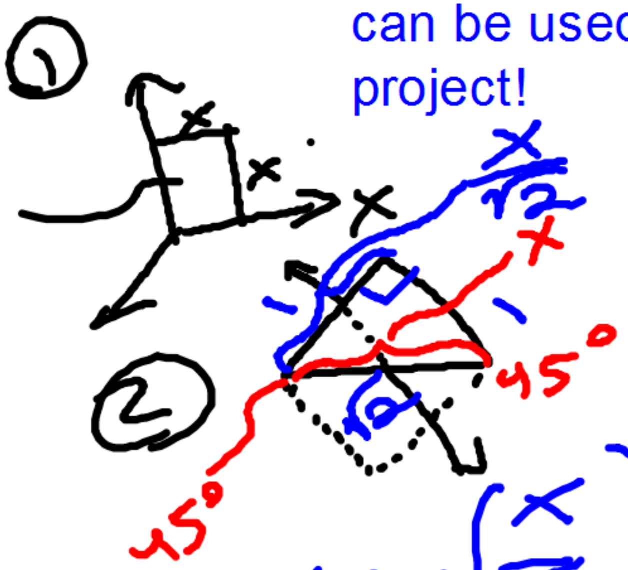
1. Sketch the solid and a typical cross section.
2. Find a formula for $A(x)$.
3. Find the limits of integration.
4. Integrate $A(x)$ to find the volume.

You'll want to write these down- these can be used for your project!

First, let's practice writing formula's for A(x)!

In Exercises 1–10, give a formula for the area of the plane region in terms of the single variable x.

1. a square with sides of length x x^2
2. a square with diagonals of length x $x^2/2$
3. a semicircle of radius x
4. a semicircle of diameter x
5. an equilateral triangle with sides of length x
6. an isosceles right triangle with legs of length x
7. an isosceles right triangle with hypotenuse x
8. an isosceles triangle with two sides of length $2x$ and one side of length x
9. a triangle with sides $3x$, $4x$, and $5x$
10. a regular hexagon with sides of length x



$A(x) = \left(\frac{x}{\sqrt{2}}\right)^2$

In Exercises 1–10, give a formula for the area of the plane region in terms of the single variable x.

1. a square with sides of length x $\quad x^2$

2. a square with diagonals of length x $\quad x^2/2$

3. a semicircle of radius x $\quad \pi x^2/2$

4. a semicircle of diameter x $\quad \pi x^2/8$

5. an equilateral triangle with sides of length x $\quad (\sqrt{3}/4)x^2$

4

3)

$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi x^2$

$r = \frac{1}{2}x$

$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(\frac{1}{2}$

$\frac{1}{8}\pi x^2$

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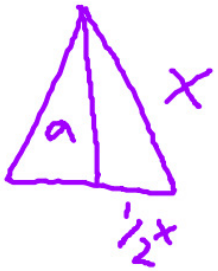


$$x^2 = \left(\tfrac{1}{2}x\right)^2 + a^2$$

$$x^2 = \frac{1}{4}x^2 + a^2$$

$$\tfrac{3}{4}x^2 = a^2$$

$$a = \sqrt{\tfrac{3}{4}x^2}$$

$$a = \frac{\sqrt{3}\,x}{2}.$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(x)\left(\frac{\sqrt{3}\,x}{2}\right)$$

$$= \frac{\sqrt{3}\,x^2}{4}$$

6. an isosceles right triangle with legs of length x $x^2/2$

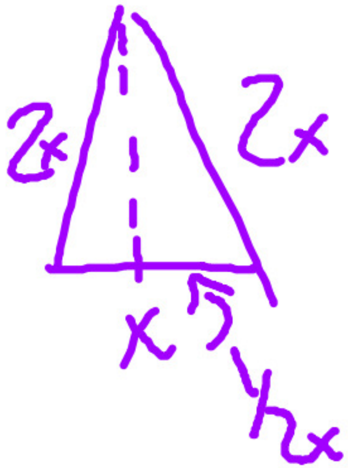
7. an isosceles right triangle with hypotenuse x $x^2/4$

8. an isosceles triangle with two sides of length $2x$ and one side of length x $(\sqrt{15}/4)x^2$

9. a triangle with sides $3x$, $4x$, and $5x$ $6x^2$

10. a regular hexagon with sides of length x $(3\sqrt{3}/2)x^2$

8



$A = \frac{1}{2}bh$

$A = \frac{1}{2}x\left(\frac{\sqrt{15}\,x}{2}\right)$

$A = \frac{\sqrt{15}}{4}x^2$

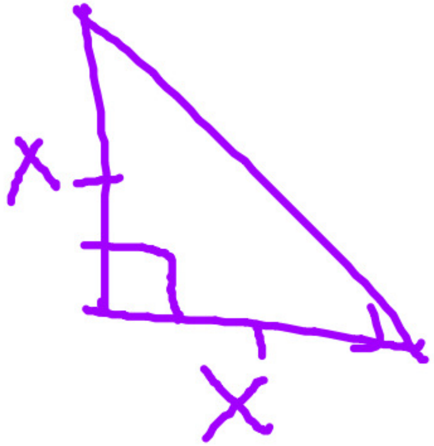
$(2x)^2 = \left(\frac{1}{2}x\right)^2 + c^2$

$4x^2 = \frac{1}{4}x^2 + c^2$

$\frac{15}{4}x^2 = c^2$

$c = \frac{\sqrt{15}\,x}{2}$

6



$A = \frac{1}{2}bh$

$A = \frac{1}{2}(x)(x)$

$A = \frac{1}{2}x^2$

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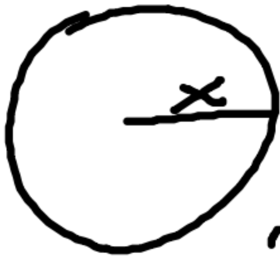
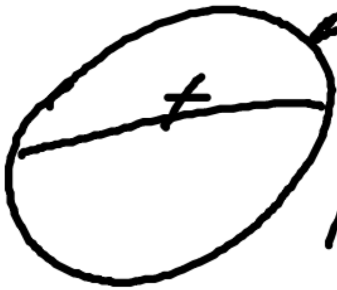
3



$A(x)=\frac{1}{2}\pi x^2$

4



$r=\frac{x}{2}$

$A(x)=\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

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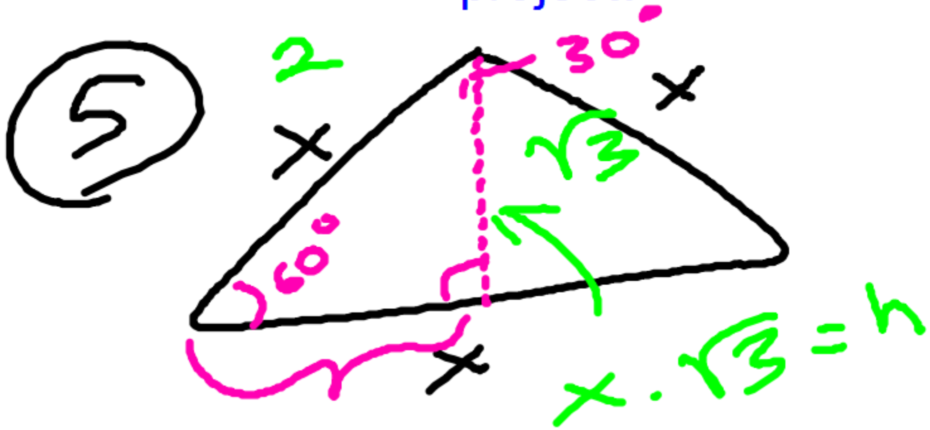

$b = x$

$A = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right)$

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9. a triangle with sides $3x$, $4x$, and $5x$ $\quad 6x^2$

10. a regular hexagon with sides of length x $\quad (3\sqrt{3}/2)x^2$

$P = 6x$

$A = \frac{1}{\cancel{2}}(\cancel{6}^{3}x)\left(\frac{\sqrt{3}}{2}x\right)$

$A = \frac{1}{2}Ph$

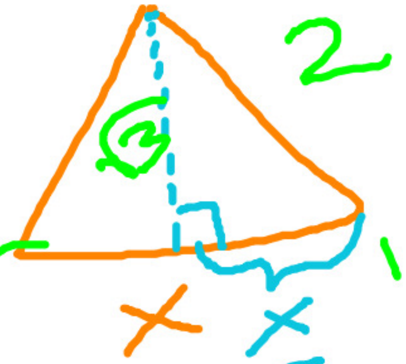
6-sided figures....

$\frac{360}{6} = 60$

$60 + 2x = 180$

$x = 60$

equilateral



$h = \frac{\sqrt{3}}{2}x$