

Lesson-by-Lesson Review

6-1 Operations on Functions

Find $[f \circ g](x)$ and $[g \circ f](x)$. **10–15. See margin.**

10. $f(x) = 2x + 1$
 $g(x) = 4x - 5$

11. $f(x) = x^2 + 1$
 $g(x) = x - 7$

12. $f(x) = x^2 + 4$
 $g(x) = -2x + 1$

13. $f(x) = 4x$
 $g(x) = 5x - 1$

14. $f(x) = x^3$
 $g(x) = x - 1$

15. $f(x) = x^2 + 2x - 3$
 $g(x) = x + 1$

16. **MEASUREMENT** The formula $f = 3y$ converts yards y to feet f and $f = \frac{n}{12}$ converts inches n to feet f . Write a composition of functions that converts yards to inches.
 $n = 36y$

Example 1

If $f(x) = x^2 + 3$ and $g(x) = 3x - 2$, find $g[f(x)]$ and $f[g(x)]$.

$$\begin{aligned} g[f(x)] &= 3(x^2 + 3) - 2 && \text{Replace } f(x) \text{ with } x^2 + 3. \\ &= 3x^2 + 9 - 2 && \text{Distributive Property} \\ &= 3x^2 + 7 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} f[g(x)] &= (3x - 2)^2 + 3 && \text{Replace } g(x) \text{ with } 3x - 2. \\ &= 9x^2 - 12x + 4 + 3 && \text{Multiply.} \\ &= 9x^2 - 12x + 7 && \text{Simplify.} \end{aligned}$$

Additional Answers

10. $[f \circ g](x) = 8x - 9$
 $[g \circ f](x) = 8x - 1$

11. $[f \circ g](x) = x^2 - 14x + 50$
 $[g \circ f](x) = x^2 - 6$

12. $[f \circ g](x) = 4x^2 - 4x + 5$
 $[g \circ f](x) = -2x^2 - 7$

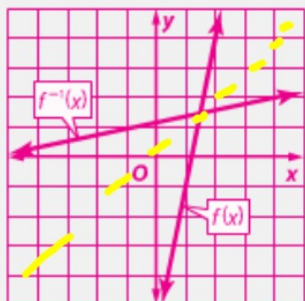
13. $[f \circ g](x) = 20x - 4$
 $[g \circ f](x) = 20x - 1$

14. $[f \circ g](x) = x^3 - 3x^2 + 3x - 1$
 $[g \circ f](x) = x^3 - 1$

15. $[f \circ g](x) = x^2 + 4x$
 $[g \circ f](x) = x^2 + 2x - 2$

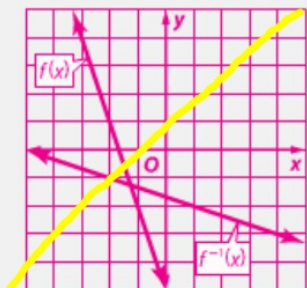
⊙ $(f \circ g)(x) = f(g(x))$
 $= 2(4x - 5) + 1$
 $= 8x - 10 + 1$
 $(g \circ f)(x) = g(f(x))$
 $= 4(2x + 1) - 5$
 $= 8x + 4 - 5$

$$17. f^{-1}(x) = \frac{x+6}{5}$$

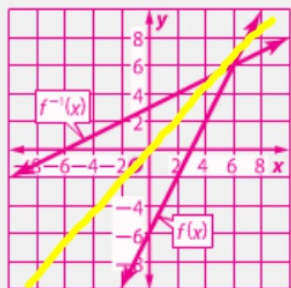


Additional Answers

$$18. f^{-1}(x) = \frac{x+5}{-3}$$



$$19. f^{-1}(x) = 2x - 6$$



6-2 Inverse Functions and Relations

Find the inverse of each function. Then graph the function and its inverse. **17–22. See margin.**

17. $f(x) = 5x - 6$ 18. $f(x) = -3x - 5$
 19. $f(x) = \frac{1}{2}x + 3$ 20. $f(x) = \frac{4x+1}{5}$
 21. $f(x) = x^2$ 22. $f(x) = (2x+1)^2$

23. **SHOPPING** Samuel bought a computer. The sales tax rate was 6% of the sale price, and he paid \$50 for shipping. Find the sale price if Samuel paid a total of \$1322. **\$1200**

Use the horizontal line test to determine whether the inverse of each function is also a function. **26, 28. no**

24. $f(x) = 3x^2$ **no** 25. $h(x) = x^3 - 3$ **yes**
 26. $g(x) = -3x^4 + 2x - 1$ 27. $g(x) = 4x^3 - 5x$ **no**
 28. $f(x) = -3x^5 + x^2 - 3$ 29. $h(x) = 4x^4 + 7x$ **no**

30. **FINANCIAL LITERACY** During the last month, Jonathan has made two deposits of \$45, made a deposit of double his original balance, and has withdrawn \$35 five times. His balance is now \$189. Write an equation that models this problem. How much money did Jonathan have in his account at the beginning of the month?
 $x + 2(45) + 2x - 5(35) = 189$; **about \$91.33**

Example 2

Find the inverse of $f(x) = -2x + 7$.

Rewrite $f(x)$ as $y = -2x + 7$. Then interchange the variables and solve for y .

$$x = -2y + 7 \quad \text{Interchange the variables.}$$

$$2y = -x + 7 \quad \text{Solve for } y.$$

$$y = \frac{-x+7}{2} \quad \text{Divide each side by 2.}$$

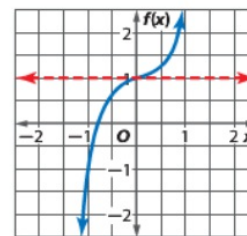
$$f^{-1}(x) = \frac{-x+7}{2} \quad \text{Rewrite using function notation.}$$

Example 3

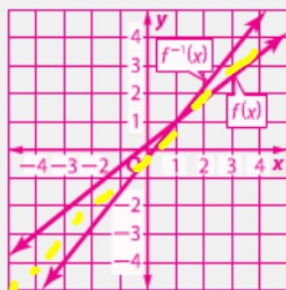
Use the horizontal line test to determine whether the inverse of $f(x) = 2x^3 + 1$ is also a function.

Graph the function.

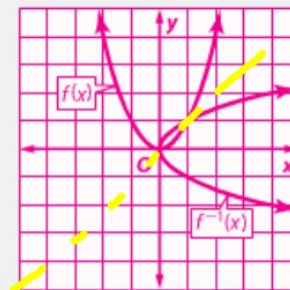
No horizontal line can be drawn so that it passes through more than one point. The inverse of this function is a function.



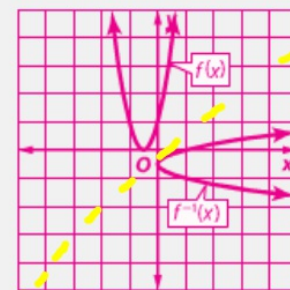
$$20. f^{-1}(x) = \frac{5x-1}{4}$$



$$21. f^{-1}(x) = \pm\sqrt{x}$$



$$22. f^{-1}(x) = \frac{-1 \pm \sqrt{x}}{2}$$



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Rewrite $f(x)$ as $y = -2x + 7$. Then interchange the variables and solve for y .

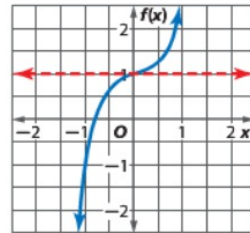
$$\begin{aligned} x &= -2y + 7 && \text{Interchange the variables.} \\ 2y &= -x + 7 && \text{Solve for } y. \\ y &= \frac{-x+7}{2} && \text{Divide each side by 2.} \\ f^{-1}(x) &= \frac{-x+7}{2} && \text{Rewrite using function notation.} \end{aligned}$$

Example 3

Use the horizontal line test to determine whether the inverse of $f(x) = 2x^3 + 1$ is also a function.

Graph the function.

No horizontal line can be drawn so that it passes through more than one point. The inverse of this function is a function.



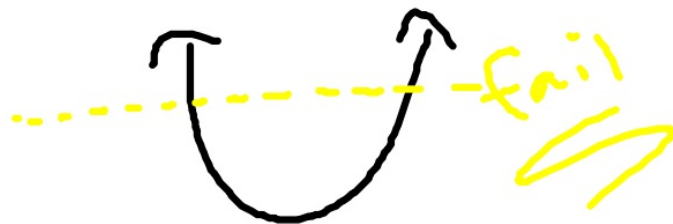
$$\textcircled{1} y = 5x - 6$$

$$\begin{array}{r} x = 5y - 6 \\ + 6 \quad \quad + 6 \\ \hline \end{array}$$

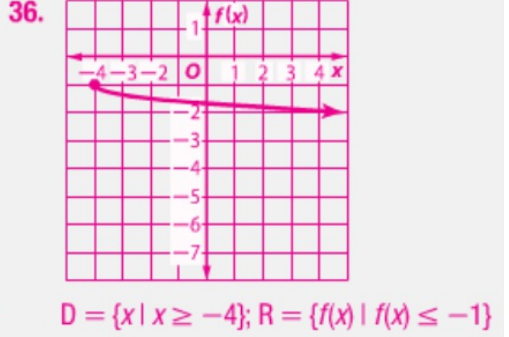
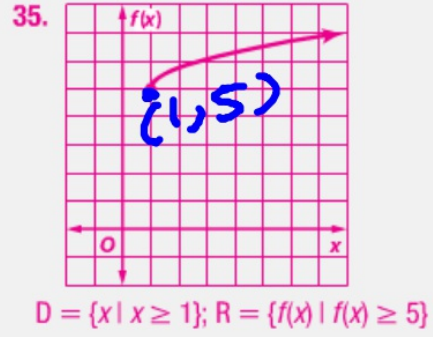
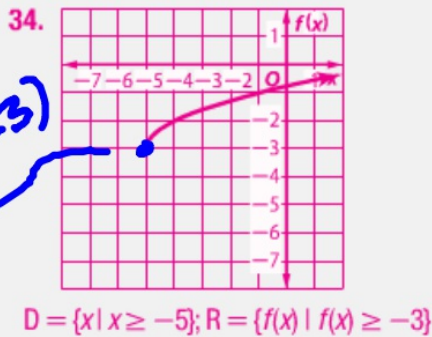
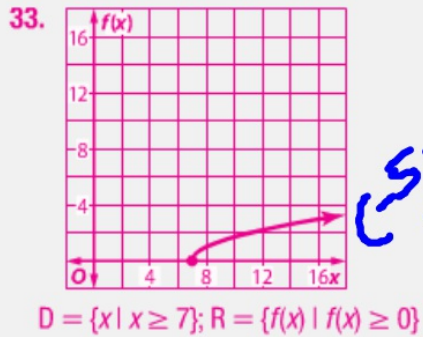
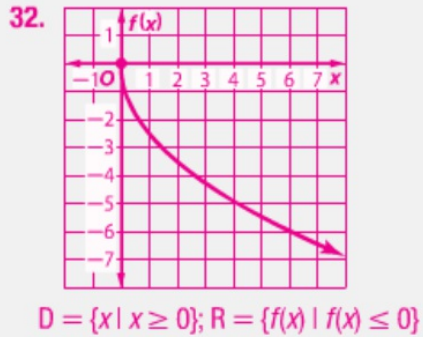
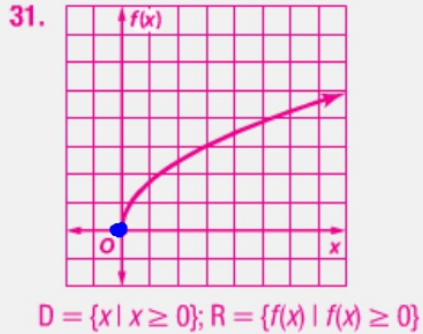
$$\frac{x+6}{5} = \frac{5y}{5}$$

$$f^{-1}(x) = \frac{x+6}{5}$$

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Additional Answers



6-3 Square Root Functions and Inequalities

Graph each function. State the domain and range.

31. $f(x) = \sqrt{3x}$ **31-36. See margin.**

32. $f(x) = -\sqrt{6x}$

33. $f(x) = \sqrt{x-7}$

34. $f(x) = \sqrt{x+5} - 3$

35. $f(x) = \frac{3}{4}\sqrt{x-1} + 5$

36. $f(x) = -\frac{1}{3}\sqrt{x+4} - 1$

37. **GEOMETRY** The area of a circle is given by the formula $A = \pi r^2$. What is the radius of a circle with an area of 300 square inches? **about 9.8 in.**

Graph each inequality. **38-40. See margin.**

38. $y \geq \sqrt{x} + 3$

39. $y < 2\sqrt{x-5}$

40. $y > -\sqrt{x-1} + 2$

Handwritten notes:
 $3x \geq 0 \Rightarrow x \geq 0$
 $x \geq 0$
 $x-7 \geq 0 \Rightarrow x \geq 7$
 $x+5 \geq 0 \Rightarrow x \geq -5$
 $x-1 \geq 0 \Rightarrow x \geq 1$
 $x+4 \geq 0 \Rightarrow x \geq -4$
 $x-5 \geq 0 \Rightarrow x \geq 5$
 $x-1 \geq 0 \Rightarrow x \geq 1$

skip #39!

Example 4

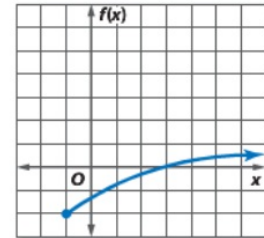
Graph $f(x) = \sqrt{x+1} - 2$. State the domain and range.

Identify the domain.

$x+1 \geq 0$ Write the radicand as greater than or equal to 0.
 $x \geq -1$ Subtract 1 from each side.

Make a table of values for $x \geq -1$ and graph the function.

x	f(x)
-1	-2
0	-1
1	-0.59
2	-0.27
3	0
4	0.24
5	0.45



The domain is $\{x \mid x \geq -1\}$, and the range is $\{f(x) \mid f(x) \geq -2\}$.

6-4 n th Roots

Simplify.

41. $\pm\sqrt{121}$ **± 11**

42. $\sqrt[3]{-125}$ **-5**

43. $\sqrt{(-6)^2}$ **6**

44. $\sqrt{-(x+3)^4}$ **$i(x+3)^2$**

45. $\sqrt[6]{(x^2+2)^{18}}$ **$(x^2+2)^3$**

46. $\sqrt[3]{27(x+3)^3}$ **$3(x+3)$**

47. $\sqrt[4]{a^8b^{12}}$ **$a^2|b^3|$**

48. $\sqrt[5]{243x^{10}y^{25}}$ **$3x^2y^5$**

49. **PHYSICS** The velocity v of an object can be defined as $v = \sqrt{\frac{2K}{m}}$, where m is the mass of an object and K is the kinetic energy in joules. Find the velocity in meters per second of an object with a mass of 17 grams and a kinetic energy of 850 joules. **10 m/s**

Example 5

Simplify $\sqrt{64x^6}$.

$$\begin{aligned}\sqrt{64x^6} &= \sqrt{(8x^3)^2} & 64x^6 &= (8x^3)^2 \\ &= 8|x^3| & \text{Simplify.}\end{aligned}$$

Use absolute value symbols because x could be negative.

Example 6

Simplify $\sqrt[6]{4096x^{12}y^{24}}$.

$$\begin{aligned}\sqrt[6]{4096x^{12}y^{24}} &= \sqrt[6]{(4x^2y^4)^6} & 4096x^{12}y^{24} &= (4x^2y^4)^6 \\ &= 4x^2y^4 & \text{Simplify.}\end{aligned}$$