$\int_a^b f(x) = F(b) - F(a)$

Calculus Practice Final 2017

$\int_1^8 3x^2 + x$ 200 − 4 = 196

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Express the limit as a definite integral.

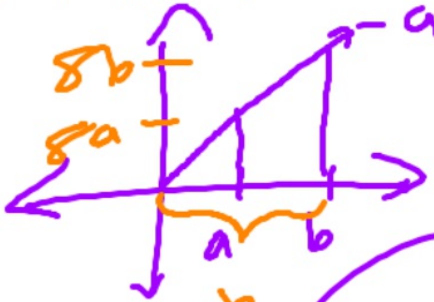
1) $\lim_{n\to\infty} \sum_{k=1}^{n} (3c_k^2 - 6c_k + 16)\,\Delta x_k,\ [-9, 2]$

$\int^{2} (3x^2 - 6x + 16)\,dx \to \frac{196}{7}$

1) ______________

Use areas to evaluate the integral.

2) $\int_a^b 8x\,dx, \quad 0 < a < b$



$\frac{1}{2}(a)(8a)$

$\frac{1}{2}(b)(8b)$

$4b^2 - 4a^2$

2) ______________

Find the average value over the given interval.

3) $y = 6x + 1;\ [1, 8]$

ab

$\frac{1}{8-1}\int_1^8 6x+1$

3) ______________

Find dy/dx.

4) $\int_{\pi/4}^{\cot x} \csc^2 t\,dt$

4) ______________

Calculator

Find dy/dx.

4) $\int_{\pi/4}^{\cot x} \csc^2 t \, dt$ $= \csc^2(\cot x) \cdot -\csc^2(x)$

$\frac{d}{dx} f(g(x))$ chain rule

$= f'(g(x)) \cdot g'(x)$

5) $\int_{0}^{x} \sqrt{4t + 7} \, dt$ $= \sqrt{4x+7}$

$u =$

$u = x^{10} - x^4 + 5x + C$

$2 = 1 - 1 + 5 + C$

$-3 = C$

Solve the initial value problem explicitly.

6) $\frac{du}{dx} = 10x^9 - 4x^3 + 5$ and $u = 2$ when $x = 1$

$u = x^{10} - x^4 + 5x - 3$

Solve the initial value problem using the Fundamental Theorem. Your answer will contain a definite integral.

7) $\frac{dy}{dx} = \cos(x^2)$ and $y = 8$ when $x = 3$ 7) ______________

$y = \int_3^x \cos x^2 + 8$

Solve the initial value problem.

Solve the initial value problem. $\int u\,dv = uv - \int v\,du$

8) $\frac{dy}{dx} = x \sin 3x$ and $y = 6$ when $x = 0$

$u = x \quad dv = \sin 3x$

$du = 1 \quad v = -\frac{1}{3}\cos 3x$

$y = \left(-\frac{1}{3}\cos 3x\right)(x) - \int -\frac{1}{3}\cos 3x\,dx + C$

$y = \frac{-x}{3}\cos 3x + \frac{1}{3}\left(\frac{1}{3}\sin 3x\right) + C$

$y = \frac{-x}{3}\cos 3x + \frac{1}{9}\sin 3x + 6$

$6 = 0 + 0 + C$

$C = 6$

Solve the problem.

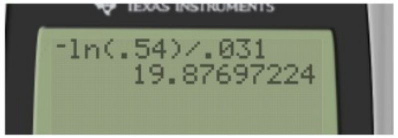
9) The decay equation for a radioactive substance is known to be $y = y_0e^{-0.031t}$, with t in days. About how long will it take for the amount of substance to decay to 54% of its original value?

$$.54\cancel{y_0} = \cancel{y_0}e^{-0.031t}$$

$$.54 = e^{-0.031t}$$

$$\ln .54 = -0.031t$$

$$t = \frac{\ln .54}{-0.031}$$



Evaluate the integral.

10) $\int \frac{2x + 23}{x^2 + 11x + 28}\, dx$

$(x+4)(x+7)$

$\int \frac{2x+23}{(x+4)(x+7)} = \int \frac{A}{x+4} + \frac{B}{x+7}$

$2x + 23 = A(x+7) + B(x+4)$

$x = -7 \ldots$

$9 = 0 + B(-7+4)$

$9 = -3B$ $B = -3$

$x = -4$

$15 = 3A + 0$

$A = 5$

$\frac{u'}{u} = \ln|u|$

$\int \frac{5}{x+4} dx + \int \frac{-3}{x+7} dx$

$5\ln|x+4| - 3\ln|x+7|$

$\ln(x+4)^5 - \ln(x+7)^3$

$= \ln \frac{(x+4)^5}{(x+7)^3} + C$